



Canadian Mathematics Competition

An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Cayley Contest (Grade 10)

Thursday, February 25, 2010



STRONGER COMMUNITIES TOGETHER™



Time: 60 minutes

©2009 Centre for Education in Mathematics and Computing

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A, B, C, D, and E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, <http://www.cemc.uwaterloo.ca>.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

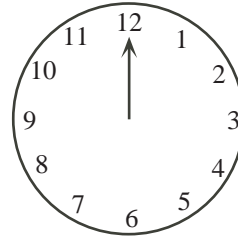
Part A: Each correct answer is worth 5.

1. The value of $6 + 4 \div 2$ is

(A) 5 (B) 6 (C) 7 (D) 8 (E) 9

2. The minute hand on a clock points at the 12. The minute hand then rotates 120° clockwise. Which number will it be pointing at?

(A) 6 (B) 2 (C) 4
(D) 3 (E) 5



3. If $x + \sqrt{25} = \sqrt{36}$, then x equals

(A) 1 (B) 2 (C) 3 (D) 4 (E) 11

4. When simplified, $\frac{1}{2 + \frac{2}{3}}$ is equal to

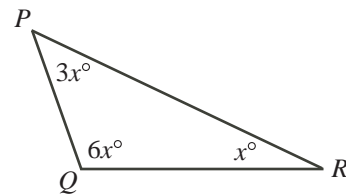
(A) $\frac{1}{8}$ (B) $\frac{5}{2}$ (C) $\frac{5}{8}$ (D) $\frac{1}{2}$ (E) $\frac{3}{8}$

5. A rectangle has a length of $\frac{3}{5}$ and an area of $\frac{1}{3}$. What is the width of the rectangle?

(A) $\frac{1}{5}$ (B) $\frac{5}{9}$ (C) $\frac{14}{15}$ (D) $\frac{15}{14}$ (E) $\frac{9}{5}$

6. What is the measure of the largest angle in $\triangle PQR$?

(A) 144° (B) 96° (C) 120°
(D) 60° (E) 108°

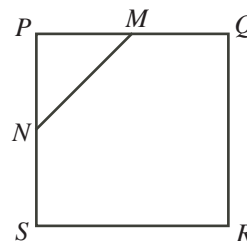


7. The mean (average) of 5 consecutive integers is 9. What is the smallest of these 5 integers?

(A) 4 (B) 5 (C) 6 (D) 7 (E) 8

8. Square $PQRS$ has an area of 900. M is the midpoint of PQ and N is the midpoint of PS . What is the area of triangle PMN ?

(A) 100 (B) 112.5 (C) 150
(D) 225 (E) 180

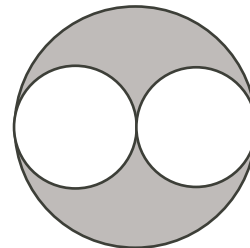


9. Which of the following lines, when drawn together with the x -axis and the y -axis, encloses an isosceles triangle?
- (A) $y = 4x + 4$ (B) $y = \frac{1}{2}x + 4$ (C) $y = -x + 4$
 (D) $y = 2x + 4$ (E) $y = -3x + 4$
10. There are 400 students at Pascal H.S., where the ratio of boys to girls is 3 : 2. There are 600 students at Fermat C.I., where the ratio of boys to girls is 2 : 3. When considering all the students from both schools, what is the ratio of boys to girls?
- (A) 2 : 3 (B) 12 : 13 (C) 1 : 1 (D) 6 : 5 (E) 3 : 2

Part B: Each correct answer is worth 6.

11. If x and y are positive integers with $x + y = 31$, then the largest possible value of xy is
- (A) 240 (B) 238 (C) 255 (D) 248 (E) 242
12. The price of each item at the Gauss Gadget Store has been reduced by 20% from its original price. An MP3 player has a sale price of \$112. What would the same MP3 player sell for if it was on sale for 30% off of its original price?
- (A) \$78.40 (B) \$100.80 (C) \$89.60 (D) \$168.00 (E) \$98.00

13. In the diagram, the smaller circles touch the larger circle and touch each other at the centre of the larger circle. The radius of the larger circle is 6. What is the area of the shaded region?



- (A) 27π (B) 6π (C) 9π
 (D) 18π (E) 36π
14. How many ordered pairs (a, b) of positive integers satisfy $a^2 + b^2 = 50$?
- (A) 0 (B) 1 (C) 3 (D) 5 (E) 7
15. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?
- (A) \$40 (B) \$100 (C) \$160 (D) \$1000 (E) \$1600

16. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of x ?

	5	
9		17
x		

- (A) 7 (B) 11 (C) 13
 (D) 15 (E) 19

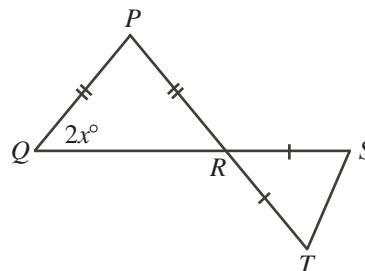
17. In the diagram, the number line is marked at consecutive integers, but the numbers themselves are not shown. The four larger dots represent two numbers that are multiples of 3 and two numbers that are multiples of 5. Which point represents a number which is a multiple of 15?



- (A) A (B) B (C) C (D) D (E) E

18. In the diagram, R is the point of intersection of PT and QS , $PQ = PR$, and $RS = RT$. If $\angle PQR = 2x^\circ$, then the measure of $\angle RST$, in degrees, is

- (A) $45 - x$ (B) $90 + \frac{1}{2}x$ (C) $90 - \frac{1}{2}x$
 (D) $45 + 2x$ (E) $90 - x$



19. How many 3-digit positive integers have exactly one even digit?

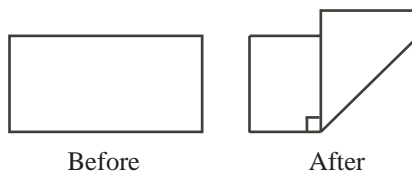
- (A) 350 (B) 450 (C) 375 (D) 75 (E) 125

20. What is the largest positive integer n that satisfies $n^{200} < 3^{500}$?

- (A) 13 (B) 14 (C) 15 (D) 16 (E) 17

Part C: Each correct answer is worth 8.

21. A rectangular piece of paper measures $17\frac{1}{2}$ cm by 8 cm. It is folded so that a right angle is formed between the two segments of the original bottom edge, as shown. What is the area of the new figure?



- (A) 104 cm^2 (B) 81 cm^2 (C) 72 cm^2 (D) 168 cm^2 (E) 64 cm^2

22. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

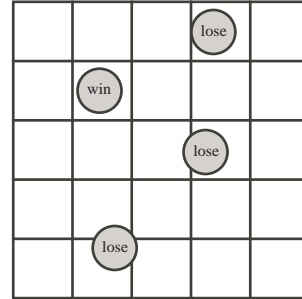
- (A) 2155 (B) 2153 (C) 2151 (D) 2149 (E) 2147

23. Connie has a number of gold bars, all of different weights. She gives the 24 lightest bars, which weigh 45% of the total weight, to Brennan. She gives the 13 heaviest bars, which weigh 26% of the total weight, to Maya. She gives the rest of the bars to Blair. How many bars did Blair receive?

(A) 14 (B) 15 (C) 16 (D) 17 (E) 18

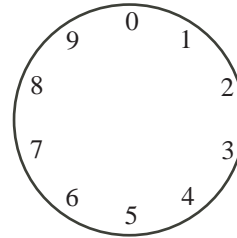
24. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

(A) $\frac{25}{441}$ (B) $\frac{1}{25}$ (C) $\frac{1}{49}$
 (D) $\frac{5}{147}$ (E) $\frac{4\pi}{25}$



25. Steve places a counter at 0 on the diagram. On his first move, he moves the counter 1^1 step clockwise to 1. On his second move, he moves 2^2 steps clockwise to 5. On his third move, he moves 3^3 steps clockwise to 2. He continues in this manner, moving n^n steps clockwise on his n th move. At which position will the counter be after 1234 moves?

(A) 1 (B) 3 (C) 5
 (D) 7 (E) 9





The CENTRE for EDUCATION in MATHEMATICS and COMPUTING



For students...

Thank you for writing the 2010 Cayley Contest!
In 2009, more than 84000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Check out the CEMC's group on Facebook, called "Who is The Mathiest?".

Encourage your teacher to register you for the Galois Contest which will be written on April 9, 2010.

Visit our website

www.cemc.uwaterloo.ca

to find

- More information about the Galois Contest
- Free copies of past contests
- Workshops to help you prepare for future contests
- Information about our publications for mathematics enrichment and contest preparation

For teachers...

Visit our website

www.cemc.uwaterloo.ca

to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 9, 2010
- Learn about workshops and resources we offer for teachers
- Find your school results

