Canadian Mathematics Competition

An activity of the Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Fermat Contest *(Grade 11)*

Thursday, February 25, 2010

Time: 60 minutes

Calculators are permitted

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. **Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.**
6. This is a multiple-choice test. Each question is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are *not* drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have *sixty* minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.

Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{1}{2} + \frac{1}{2} + \frac{1}{3} + \frac{1}{3} + \frac{1}{3}$ is

   (A) 2 **(B)** $\frac{5}{13}$ **(C)** $\frac{5}{6}$ **(D)** 1 **(E)** $\frac{13}{6}$

2. The quantity "2% of 1" is equal to

   (A) $\frac{2}{100}$ **(B)** $\frac{2}{10}$ **(C)** 2 **(D)** 20 **(E)** 200

3. In the diagram, points P, Q, R, and S are arranged in order on a line segment. If $PQ = 1$, $QR = 2PQ$ and $RS = 3QR$, then the length of PS is

   (A) 7 **(B)** 6 **(C)** 9
   (D) 8 **(E)** 10

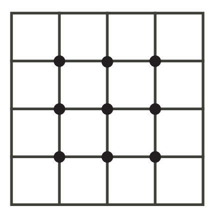
4. If $u = -6$ and $x = \frac{1}{3}(3 - 4u)$, then x equals

   (A) -23 **(B)** -7 **(C)** 9 **(D)** 2 **(E)** 25

5. If $2^x = 16$, then 2^{x+3} equals

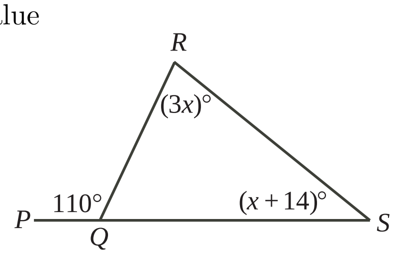
   (A) 19 **(B)** 48 **(C)** 22 **(D)** 128 **(E)** 2048

6. The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?



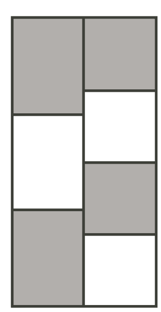
   (A) 100 **(B)** 121 **(C)** 132
   (D) 144 **(E)** 169

7. In the diagram, PQS is a straight line. What is the value of x?



   (A) 19 **(B)** 62 **(C)** 21.5
   (D) 24 **(E)** 32

8. A rectangle is divided into two vertical strips of equal width. The strip on the left is divided into three equal parts and the strip on the right is divided into four equal parts. Parts of the rectangle are then shaded as shown. What fraction of the original rectangle is shaded?



   (A) $\frac{3}{5}$ **(B)** $\frac{2}{7}$ **(C)** $\frac{4}{7}$
   (D) $\frac{7}{6}$ **(E)** $\frac{7}{12}$

9. The value of $k \nabla m$ is defined to be $k(k-m)$. For example, $7 \nabla 2 = 7(7-2) = 35$. What is the value of $(5 \nabla 1) + (4 \nabla 1)$?

(A) 9 **(B)** 84 **(C)** 20 **(D)** 32 **(E)** 72

10. If $2x^2 = 9x - 4$ and $x \neq 4$, then the value of $2x$ is

(A) 4 **(B)** 1 **(C)** -1 **(D)** 0 **(E)** 2

Part B: Each correct answer is worth 6.

11. A loonie is a \$1 coin and a dime is a \$0.10 coin. One loonie has the same mass as 4 dimes. A bag of dimes has the same mass as a bag of loonies. The coins in the bag of loonies are worth \$400 in total. How much are the coins in the bag of dimes worth?

(A) \$40 **(B)** \$100 **(C)** \$160 **(D)** \$1000 **(E)** \$1600

12. When k candies were distributed among seven people so that each person received the same number of candies and each person received as many candies as possible, there were 3 candies left over. If instead, $3k$ candies were distributed among seven people in this way, then the number of candies left over would have been

(A) 1 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 9

13. Fifty numbers have an average of 76. Forty of these numbers have an average of 80. The average of the other ten numbers is

(A) 60 **(B)** 4 **(C)** 72 **(D)** 40 **(E)** 78

14. Four friends went fishing one day and caught a total of 11 fish.
Each person caught at least one fish.
All of the following statements *could* be true.
Which one of the statements *must* be true?

(A) At least one person caught exactly one fish.
(B) At least one person caught exactly three fish.
(C) At least one person caught more than three fish.
(D) At least one person caught fewer than three fish.
(E) At least two people each caught more than one fish.

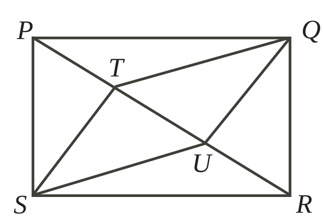
15. The number of positive integers p for which $-1 < \sqrt{p} - \sqrt{100} < 1$ is

(A) 19 **(B)** 21 **(C)** 38 **(D)** 39 **(E)** 41

16. Positive integers a and b satisfy $ab = 2010$. If $a > b$, the smallest possible value of $a - b$ is

(A) 37 **(B)** 119 **(C)** 191 **(D)** 1 **(E)** 397

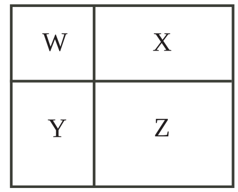
17. In the diagram, $PQRS$ is a rectangle with $PQ = 5$ and $QR = 3$. PR is divided into three segments of equal length by points T and U. The area of quadrilateral $STQU$ is



(A) $\frac{17}{3}$ **(B)** 5 **(C)** $\frac{5}{2}$

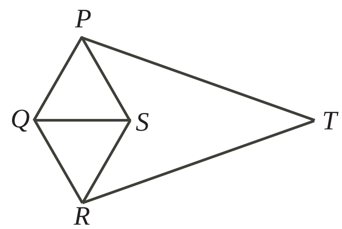
(D) $\frac{\sqrt{34}}{3}$ **(E)** $\sqrt{34}$

18. A rectangle is divided into four smaller rectangles, labelled W, X, Y, and Z, as shown. The perimeters of rectangles W, X and Y are 2, 3 and 5, respectively. What is the perimeter of rectangle Z?



(A) 6 **(B)** 7 **(C)** 4 **(D)** 8 **(E)** 7.5

19. In the diagram, $PQ = QR = RS = SP = SQ = 6$ and $PT = RT = 14$. The length of ST is



(A) $4\sqrt{10} - 3$ **(B)** 11 **(C)** $7\sqrt{3} - 3$ **(D)** 10 **(E)** $\sqrt{232 - 84\sqrt{3}}$

20. A square has side length 5. In how many different locations can point X be placed so that the distances from X to the four sides of the square are 1, 2, 3, and 4?

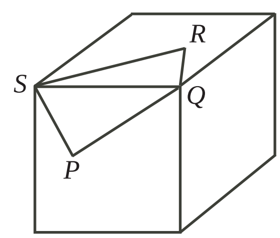
(A) 0 **(B)** 12 **(C)** 4 **(D)** 8 **(E)** 16

Part C: Each correct answer is worth 8.

21. If $\dfrac{x-y}{z-y} = -10$, then the value of $\dfrac{x-z}{y-z}$ is

(A) 11 **(B)** -10 **(C)** 9 **(D)** -9 **(E)** 10

22. A rectangular piece of paper, $PQRS$, has $PQ = 20$ and $QR = 15$. The piece of paper is glued flat on the surface of a large cube so that Q and S are at vertices of the cube. (Note that $\triangle QPS$ and $\triangle QRS$ lie flat on the front and top faces of the cube, respectively.) The shortest distance from P to R, as measured through the cube, is closest to

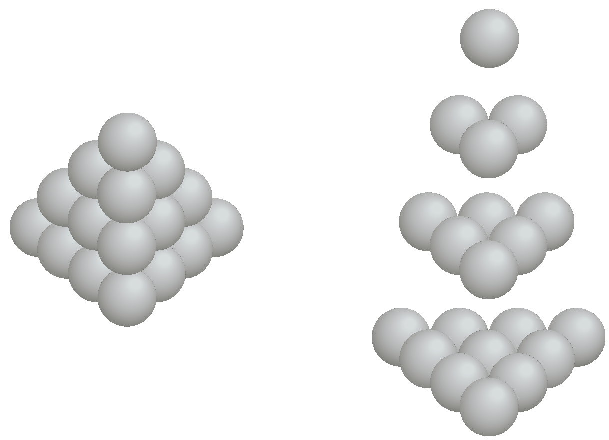


(A) 17.0 **(B)** 25.0 **(C)** 31.0 **(D)** 17.7 **(E)** 18.4

23. Let t_n equal the integer closest to $\sqrt{n}$.
For example, $t_1 = t_2 = 1$ since $\sqrt{1} = 1$ and $\sqrt{2} \approx 1.41$ and $t_3 = 2$ since $\sqrt{3} \approx 1.73$.
The sum $\dfrac{1}{t_1} + \dfrac{1}{t_2} + \dfrac{1}{t_3} + \dfrac{1}{t_4} + \cdots + \dfrac{1}{t_{2008}} + \dfrac{1}{t_{2009}} + \dfrac{1}{t_{2010}}$ equals

(A) $88\frac{1}{6}$ **(B)** $88\frac{1}{2}$ **(C)** $88\frac{2}{3}$ **(D)** $88\frac{1}{3}$ **(E)** 90

24. Spheres can be stacked to form a tetrahedron by using triangular layers of spheres. Each sphere touches the three spheres below it. The diagrams show a tetrahedron with four layers and the layers of such a tetrahedron. An *internal sphere* in the tetrahedron is a sphere that touches exactly three spheres in the layer above. For example, there is one internal sphere in the fourth layer, but no internal spheres in the first three layers.



A tetrahedron of spheres is formed with thirteen layers and each sphere has a number written on it. The top sphere has a 1 written on it and each of the other spheres has written on it the number equal to the sum of the numbers on the spheres in the layer above with which it is in contact. For the whole thirteen layer tetrahedron, the sum of the numbers on all of the internal spheres is

(A) 772 588 **(B)** 772 566 **(C)** 772 156 **(D)** 772 538 **(E)** 772 626

25. Alex chose positive integers a, b, c, d, e, f and completely multiplied out the polynomial product

$$(1-x)^a(1+x)^b(1-x+x^2)^c(1+x^2)^d(1+x+x^2)^e(1+x+x^2+x^3+x^4)^f$$

After she simplified her result, she discarded any term involving x to any power larger than 6 and was astonished to see that what was left was $1-2x$. If $a > d+e+f$ and $b > c+d$ and $e > c$, what value of a did she choose?

(A) 17 **(B)** 19 **(C)** 20 **(D)** 21 **(E)** 23

The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

For students...

Thank you for writing the 2010 Fermat Contest!
In 2009, more than 84 000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Check out the CEMC's group on Facebook, called "Who is The Mathiest?".

Encourage your teacher to register you for the Hypatia Contest which will be written on April 9, 2010.
Visit our website

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to find

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