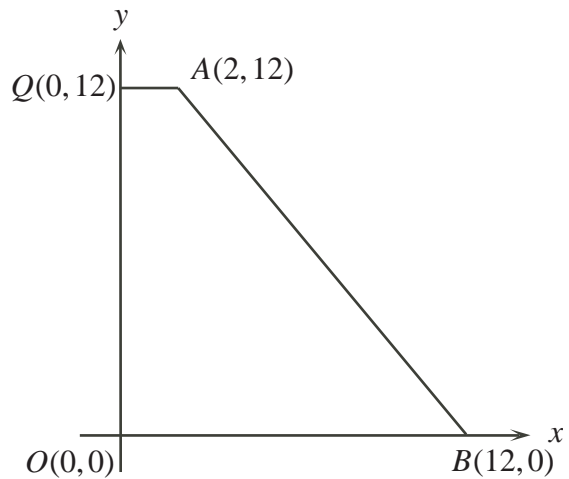


**2010 Galois Contest (Grade 10)**  
**Friday, April 9, 2010**

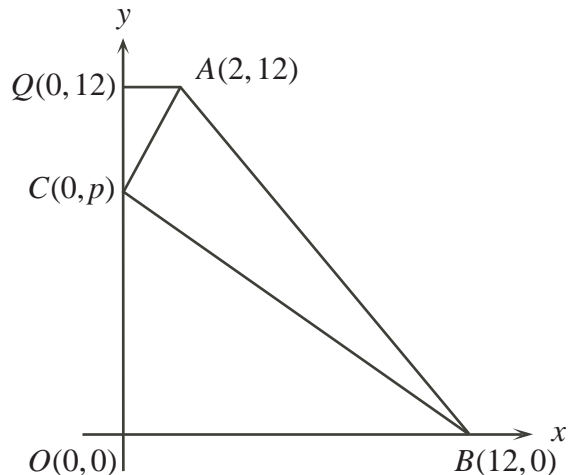
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1. Emily's old showerhead used 18 L of water per minute. She installs a new showerhead that uses 13 L per minute.
  - (a) When Emily takes a bath, she uses 260 L of water. Using the new showerhead, what length of shower, in minutes, uses 260 L of water?
  - (b) How much *less* water is used for a 10 minute shower with the new showerhead than with the old showerhead?
  - (c) Emily is charged 8 cents per 100 L of water that she uses. Using the new showerhead instead of the old showerhead saves water and so saves Emily money. How much money does Emily *save* in water costs for a 15 minute shower?
  - (d) How many minutes of showering, using the new showerhead, will it take for Emily to have saved \$30 in water costs?

2. (a) Quadrilateral  $QABO$  is constructed as shown. Determine the area of  $QABO$ .



- (b) Point  $C(0,p)$  lies on the  $y$ -axis between  $Q(0,12)$  and  $O(0,0)$  as shown. Determine an expression for the area of  $\triangle COB$  in terms of  $p$ .
- (c) Determine an expression for the area of  $\triangle QCA$  in terms of  $p$ .
- (d) If the area of  $\triangle ABC$  is 27, determine the value of  $p$ .



3. (a) Solve the system of equations algebraically for  $(x, y)$ :

$$x + y = 42$$

$$x - y = 10$$

- (b) Suppose that  $p$  is an even integer and that  $q$  is an odd integer. Explain why the system of equations

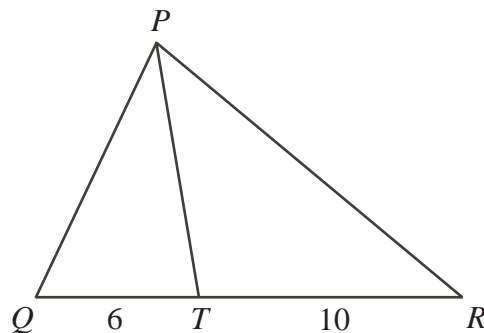
$$x + y = p$$

$$x - y = q$$

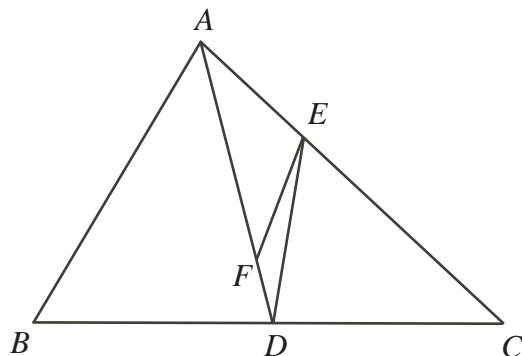
has no positive integer solutions  $(x, y)$ .

- (c) Determine all pairs of positive integers  $(x, y)$  that satisfy the equation  $x^2 - y^2 = 420$ .

4. (a) In  $\triangle PQR$ , point  $T$  is on side  $QR$  such that  $QT = 6$  and  $TR = 10$ . Explain why the ratio of the area of  $\triangle PQT$  to the area of  $\triangle PTR$  is  $3 : 5$ .



- (b) In  $\triangle ABC$ , point  $D$  is the midpoint of side  $BC$ . Point  $E$  is on  $AC$  such that  $AE : EC = 1 : 2$ . Point  $F$  is on  $AD$  such that  $AF : FD = 3 : 1$ . If the area of  $\triangle DEF$  is 17, determine the area of  $\triangle ABC$ .



- (c) In the diagram, points  $X, Y$  and  $Z$  are on the sides of  $\triangle UVW$ , as shown. Line segments  $UY, VZ$  and  $WX$  intersect at  $P$ . Point  $Y$  is on  $VW$  such that  $VY : YW = 4 : 3$ . If  $\triangle PYW$  has an area of 30 and  $\triangle PZW$  has an area of 35, determine the area of  $\triangle UXP$ .

