Canadian
Mathematics
Competition
An activity of the Centre for Education
in Mathematics and Computing,
University of Waterloo, Waterloo, Ontario

Grade 8 solutions follow the Grade 7 solutions

2010 Gauss Contests

(Grades 7 and 8)

Wednesday, May 12, 2010

Solutions

Centre for Education in Mathematics and Computing Faculty and Staff

Ed Anderson
Lloyd Auckland
Terry Bae
Janet Baker
Steve Brown
Ersal Cahit
Karen Cole
Jennifer Couture
Frank DeMaio
Fiona Dunbar
Jeff Dunnett
Mike Eden
Barry Ferguson
Judy Fox
Steve Furino
Sandy Graham
Angie Hildebrand
Judith Koeller
Joanne Kursikowski
Angie Murphy
Dean Murray
Jen Nissen
J.P. Pretti
Linda Schmidt
Kim Schnarr
Jim Schurter
Carolyn Sedore
Ian VanderBurgh
Troy Vasiga

Gauss Contest Committee

Mark Bredin (Chair), St. John's Ravenscourt School, Winnipeg, MB
Kevin Grady (Assoc. Chair), Cobden District P.S., Cobden, ON
John Grant McLoughlin, University of New Brunswick, Fredericton, NB
JoAnne Halpern, Thornhill, ON
David Matthews, University of Waterloo, Waterloo, ON
Allison McGee, All Saints C.H.S., Kanata, ON
Kim Stenhouse, William G. Davis P.S., Cambridge, ON
David Switzer, Sixteenth Ave. P.S., Richmond Hill, ON
Tanya Thompson, Nottawa, ON
Chris Wu, Amesbury M.S., Toronto, ON

Grade 7

1. Reading the number on the vertical axis corresponding to the pet *fish*, we find that 40 students chose fish as their favourite pet.

   Answer: (D)

2. By dividing, we find the fraction $\frac{20}{25}$ is equivalent to the decimal 0.80.
   We convert this to a percent by multiplying by 100%.
   Thus, Tanya scored $0.80 \times 100\% = 80\%$ on her math quiz.

   Answer: (C)

3. Using the correct order of operations, $4 \times 5 + 5 \times 4 = 20 + 20 = 40$.

   Answer: (E)

4. To find the location of the point $(-2, -3)$, we begin at the origin, $(0, 0)$, and move left 2 units and down 3 units.
   The point $(-2, -3)$ is located at D.

   Answer: (D)

5. Going down 2 floors from the 11th floor brings Chaz to the 9th floor.
   Going down 4 floors from the 9th floor brings Chaz to the 5th floor.
   Thus, Chaz gets off the elevator on the 5th floor.

   Answer: (D)

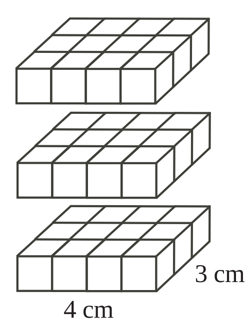
6. The answer, 10000.3, is 1000 times bigger than 10.0003. This can be determined either by dividing 10000.3 by 10.0003 or by recognizing that the decimal point in 10.0003 is moved three places to the right to obtain 10000.3. Thus, the number that should replace the □ is 1000.

   Answer: (B)

7. The four angles shown, 150°, 90°, x°, and 90°, form a complete rotation, a 360° angle.
   Thus, $150^\circ + 90^\circ + x^\circ + 90^\circ = 360^\circ$, or $x^\circ = 360^\circ - 150^\circ - 90^\circ - 90^\circ = 30^\circ$.

   Answer: (D)

8. *Solution 1*
   To build the solid rectangular prism, we could first construct the 4 cm by 3 cm base using $4 \times 3 = 12$ of the 1 cm × 1 cm × 1 cm blocks.
   Two more layers identical to the first layer, placed on top of the first layer, would give the prism its required 3 cm height.
   This would require 12 more 1 cm × 1 cm × 1 cm blocks in layer two and 12 more in layer three, or $12 \times 3 = 36$ blocks in total.



   Solution 2
   Equivalently, this question is asking for the volume of the rectangular prism.
   The volume of a prism is the area of the base times the height,
   or $V = 4 \text{ cm} \times 3 \text{ cm} \times 3 \text{ cm} = 36 \text{ cm}^3$.
   Since the volume of each of the 1 cm × 1 cm × 1 cm blocks is 1 cm^3,
   then 36 blocks are needed to build the solid rectangular prism.
   (The prism can actually be built with 36 blocks as seen in Solution 1.)

   Answer: (E)

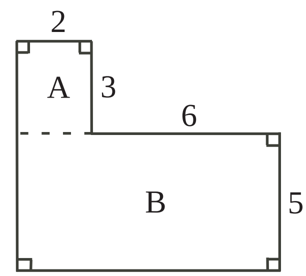
9. If the time reads 3:33, the next time that all of the digits on the clock are equal to one another is 4:44. Since the amount of time between 3:33 and 4:44 is 1 hour and 11 minutes, the shortest length of time in minutes is $60 + 11 = 71$.

Answer: (A)

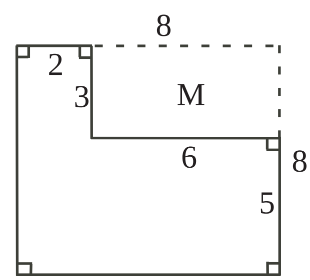
10. Since 700 is the product of 35 and y, then $35 \times y = 700$ or $y = 700 \div 35 = 20$.
Since 20 is the product of 5 and x, then $5 \times x = 20$ or $x = 20 \div 5 = 4$.

Answer: (B)

11. *Solution 1*
We divide the shape into two rectangles, A and B, by constructing the dotted line segment of length 2 units shown.
The area of rectangle A is $2 \times 3 = 6$ square units.
The length of rectangle B is 6 units plus the length of the dotted line segment, or $6 + 2 = 8$.
Thus, the area of rectangle B is $8 \times 5 = 40$ square units.
The area of the entire figure is the sum of the areas of rectangles A and B, or $6 + 40 = 46$ square units.



Solution 2
By constructing the dotted lines shown, we form a rectangle with length $2 + 6 = 8$ units and width $5 + 3 = 8$ units (in fact, this large rectangle is a square).
We find the required area by subtracting the area of rectangle M from the area of the 8 by 8 square.
Thus, the area is $(8 \times 8) - (6 \times 3) = 64 - 18 = 46$ square units.



Answer: (C)

12. If 4 schools each recycle $\frac{3}{4}$ of a tonne of paper, then combined, they recycle $4 \times \frac{3}{4} = \frac{12}{4} = 3$ tonnes of paper.
Since recycling 1 tonne of paper will save 24 trees, recycling 3 tonnes of paper will save $3 \times 24 = 72$ trees.

Answer: (B)

13. *Solution 1*
The mean of 5 consecutive integers is equal to the number in the middle.
Since the numbers have a mean of 21, if we were to distribute the quantities equally, we would have 21, 21, 21, 21, and 21.
Since the numbers are consecutive, the second number is 1 less than the 21 in the middle, while the fourth number is 1 more than the 21 in the middle.
Similarly, the first number is 2 less than the 21 in the middle, while the fifth number is 2 more than the 21 in the middle.
Thus, the numbers are $21 - 2$, $21 - 1$, 21, $21 + 1$, $21 + 2$.
The smallest of 5 consecutive integers having a mean of 21, is 19.

Solution 2
Since 21 is the mean of five consecutive integers, the smallest of these five integers must be less than 21.
Suppose that the smallest number is 20.

The mean of $20, 21, 22, 23$, and 24 is $\dfrac{20+21+22+23+24}{5} = 22$.
This mean of 22 is greater than the required mean of 21; thus, the smallest of the 5 consecutive integers must be less than 20.
Suppose that the smallest number is 19.
The mean of $19, 20, 21, 22$, and 23, is $\dfrac{19+20+21+22+23}{5} = 21$, as required.
Thus, the smallest of the 5 consecutive integers is 19.

Answer: (E)

14. *Solution 1*
Since the bag contains green mints and red mints only, the remaining $100\% - 75\% = 25\%$ of the mints must be red.
Thus, the ratio of the number of green mints to the number of red mints is $75 : 25 = 3 : 1$.

Solution 2
Since 75% of the mints are green, then $\frac{3}{4}$ of the mints are green.
Since the bag contains only green mints and red mints, then $1 - \frac{3}{4} = \frac{1}{4}$ of the mints in the bag are red.
Thus, there are 3 times as many green mints as red mints.
The ratio of the number of green mints to the number of red mints is $3 : 1$.

Answer: (B)

15. The area of square N is four times the area of square M or $4 \times 100\text{ cm}^2 = 400\text{ cm}^2$.
Thus, each side of square N has length $\sqrt{400} = 20$ cm.
The perimeter of square N is $4 \times 20\text{ cm} = 80$ cm.

Answer: (C)

16. First we must find the *magic constant*, that is, the sum of each row, column and diagonal.
From column one, we find that the magic constant is $(+1) + (-4) + (-3) = -6$.
In the diagonal extending from the top left corner to the bottom right corner, the two existing numbers $+1$ and -5 have a sum of -4.
Thus, to obtain the magic constant of -6 in this diagonal, -2 must occupy the centre square.
In the diagonal extending from the bottom left corner to the top right corner, the two numbers -3 and -2, have a sum of -5.
Thus, to obtain the magic constant of -6 in this diagonal, Y must equal -1.
The completed magic square is shown below.

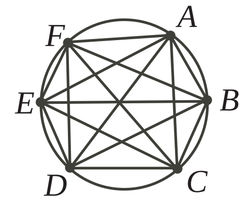
+1	−6	−1
−4	−2	0
−3	+2	−5

Answer: (A)

17. The smallest possible three-digit integer that is 17 more than a two-digit integer is 100 (100 is 17 more than 83 and 100 is in fact the smallest possible three-digit integer).
Notice that 101 is 17 more than 84, 102 is 17 more than 85, and so on. This continues until we reach 117 which is 17 more than 100, but 100 is not a two-digit integer. Thus, 116 is the largest possible three-digit integer that is 17 more than a two-digit integer (116 is 17 more than 99).
Therefore, all of the integers from 100 to 116 inclusive, or 17 three-digit integers, are exactly 17 more than a two-digit integer.

Answer: (A)

18. *Solution 1*



We label the 6 points A through F as shown and proceed to connect the points in all possible ways.
From point A, 5 line segments are drawn, 1 to each of the other points, B through F.
From point B, 4 new line segments are drawn, 1 to each of the points C through F, since the segment AB has already been drawn.
This continues, with 3 line segments drawn from point C, 2 from point D, 1 from point E, and 0 from point F since it will have already been joined to each of the other points.
In total, there are $5 + 4 + 3 + 2 + 1 = 15$ line segments.

Solution 2
Label the 6 points A through F as shown above.
From each of the 6 points, 5 line segments can be drawn leaving the point, 1 to each of the other 5 points.
Thus, the total number of line segments leaving the 6 points is $6 \times 5 = 30$.
However, this counts each of the line segments twice, since each segment will be counted as leaving both of its ends.
For example, the segment leaving point A and ending at point D is also counted as a segment leaving point D and ending at point A.
Thus, the actual number of line segments is $30 \div 2 = 15$.

ANSWER: (D)

19. The value of any positive fraction is increased by increasing the numerator and/or decreasing the denominator.
Thus, to obtain the largest possible sum, we choose 6 and 7 as the numerators, and 3 and 4 as the denominators.
We then calculate:

$$\frac{7}{3} + \frac{6}{4} = \frac{28}{12} + \frac{18}{12} = \frac{46}{12} = \frac{23}{6}$$

We recognize that $\frac{23}{6}$ is the largest of the 5 possible answers, and thus is the correct response.
(This means that we do not need to try $\frac{7}{4} + \frac{6}{3}$.)

ANSWER: (E)

20. *Solution 1*
To determine who *cannot* be sitting in the middle seat, we may eliminate the 4 people who *can* be sitting in the middle seat.
First, assume that Sally and Mike, who must be beside one another, are in seats 1 and 2, or in seats 2 and 1.
Since Andy and Jen are not beside each other, either Andy is in seat 3 (the middle seat) and Jen is in seat 5, or vice versa.
Thus, Andy and Jen can each be sitting in the middle seat and are eliminated as possible choices.
Next, assume that Sally and Mike are in seats 2 and 3, or in seats 3 and 2.
That is, either Sally is in the middle (seat 3), or Mike is.
In either case, seats 1, 4 and 5 are empty, allowing either Andy or Jen to choose seat 1 and hence, they are not next to one another.
This demonstrates that Sally and Mike can each be sitting in the middle seat.

Having eliminated Andy, Jen, Sally, and Mike, it must be Tom who cannot be sitting in the middle seat.

Solution 2
Assume that Tom is sitting in the middle (seat 3).
Since Sally and Mike are seated beside each other, they are either sitting in seats 4 and 5 or seats 1 and 2.
In either case, seats 1 and 2 remain empty or seats 4 and 5 remain empty.
However, Andy and Jen cannot sit beside each other.
Therefore, this arrangement is not possible.
Thus, Tom cannot be sitting in the middle seat.
Since the question implies that there is a unique answer, then Tom is the answer.

ANSWER: (E)

21. Traveling at a constant speed of 15 km/h, in 3 hours the bicycle will travel $15 \times 3 = 45$ km.
At the start, the bicycle was 195 km ahead of the bus.
Therefore, in order to catch up to the bicycle, the bus must travel 195 km plus the additional 45 km that the bicycle travels, or $195 + 45 = 240$ km.
To do this in 3 hours, the bus must travel at an average speed of $240 \div 3 = 80$ km/h.

ANSWER: (B)

22. When tossing a single coin, there are two possible outcomes, a head (H) or a tail (T).
When tossing 2 coins, there are $2 \times 2 = 4$ possible outcomes.
These are HH, HT, TH, and TT.
When tossing 3 coins, there are $2 \times 2 \times 2 = 8$ possible outcomes.
These are HHH, HHT, HTH, HTT, THH, THT, TTH, and TTT.
Of these 8 possible outcomes, there are 2 winning outcomes, HHH and TTT.
Thus, the probability of winning the *Coin Game* is $\frac{2}{8} = \frac{1}{4}$.

ANSWER: (B)

23. Since $\text{M} \times \text{O} \times \text{M} = 49$, either M = 7 and O = 1 or M = 1 and O = 49.
However, since the value of the word TOTE is 18, O cannot have a value of 49 because 18 is not divisible by 49.
Thus, M = 7 and O = 1.
Since $\text{T} \times \text{O} \times \text{T} \times \text{E} = 18$ and O = 1, we have $\text{T} \times \text{T} \times \text{E} = 18$.
Therefore, either T = 3 and E = 2 or T = 1 and E = 18.
However, O = 1, and since every letter has a different value, T cannot be equal to 1.
Thus, T = 3 and E = 2.
The value of the word TEAM is 168, so $\text{T} \times \text{E} \times \text{A} \times \text{M} = 168$, or $3 \times 2 \times \text{A} \times 7 = 168$.
Thus, $42 \times \text{A} = 168$ or $\text{A} = 168 \div 42 = 4$.
The value of the word HOME is 70, so $\text{H} \times \text{O} \times \text{M} \times \text{E} = 70$, or $\text{H} \times 1 \times 7 \times 2 = 70$.
Thus, $14 \times \text{H} = 70$ or $\text{H} = 70 \div 14 = 5$.
Finally, the value of the word MATH is $\text{M} \times \text{A} \times \text{T} \times \text{H} = 7 \times 4 \times 3 \times 5 = 420$.

ANSWER: (C)

24. The sum of two even numbers is even. The sum of two odd numbers is even.
The sum of an odd number and an even number is odd.
Thus, for the sum $m+n$ to be even, both m and n must be even, or they must both be odd.
If $m=2$, then n must be even and greater than 2.
Thus, if $m=2$ then n can be $4,6,8,10,12,14,16,18$, or 20.
This gives 9 different pairs (m,n) when $m=2$.
If $m=4$, then n must be even and greater than 4.
Thus, if $m=4$ then n can be $6,8,10,12,14,16,18$, or 20.
This gives 8 different pairs (m,n) when $m=4$.
Continuing in this manner, each time we increase m by 2, the number of choices for n, and thus for (m,n), decreases by 1.
This continues until $m=18$, at which point there is only one choice for n, namely $n=20$.
Therefore, the total number of different pairs (m,n) where both m and n are even is,
$9+8+7+6+5+4+3+2+1=45$.
Similarly, if $m=1$, then n must be odd and greater than 1.
Thus, if $m=1$, then n can be $3,5,7,9,11,13,15,17$, or 19.
This gives 9 different pairs (m,n) when $m=1$.
If $m=3$, then n must be odd and greater than 3.
Thus, if $m=3$ then n can be $5,7,9,11,13,15,17$, or 19.
This gives 8 different pairs (m,n) when $m=3$.
Continuing in this manner, each time we increase m by 2, the number of choices for n, and thus for (m,n), decreases by 1.
This continues until $m=17$, at which point there is only one choice for n, namely $n=19$.
Therefore, the total number of different pairs (m,n) where both m and n are odd is,
$9+8+7+6+5+4+3+2+1=45$.
Thus, the total number of different pairs (m,n) using numbers from the list $\{1,2,3,\ldots,20\}$ such that $m<n$ and $m+n$ is even is $45+45=90$.

Answer: (B)

25. Together, Hose A and Hose B fill the pool in 6 hours.
Thus, it must take Hose A more than 6 hours to fill the pool when used by itself.
Therefore, $a \geq 7$, since a is a positive integer.
Similarly, it must take Hose B more than 6 hours to fill the pool when used by itself.
Therefore, $b \geq 7$, since b is a positive integer.
When used by itself, the fraction of the pool that Hose A fills in 6 hours is $\frac{6}{a}$.
When used by itself, the fraction of the pool that Hose B fills in 6 hours is $\frac{6}{b}$.
When used together, Hose A and Hose B fill the pool *once* in 6 hours. Thus, $\frac{6}{a}+\frac{6}{b}=1$.
Since $a \geq 7$, $b \geq 7$, and both a and b are integers, then we can find values for a and b that satisfy the equation $\frac{6}{a}+\frac{6}{b}=1$ by using systematic trial and error.
For example, let $a=7$. Then $\frac{6}{7}+\frac{6}{b}=1$, or $\frac{6}{b}=1-\frac{6}{7}$, or $\frac{6}{b}=\frac{1}{7}$.
Since $\frac{6}{42}=\frac{1}{7}$, then $b=42$ and $a=7$ is one possible solution to the equation $\frac{6}{a}+\frac{6}{b}=1$.
Compare this to what happens when we let $a=11$.
We have $\frac{6}{11}+\frac{6}{b}=1$, or $\frac{6}{b}=1-\frac{6}{11}$, or $\frac{6}{b}=\frac{5}{11}$, or $5b=66$.
Since there is no integer value for b that makes $5b$ equivalent to 66, then $a=11$ does not give a possible solution.
The possible solutions found by systematic trial and error are shown below.

a	7	8	9	10	12	15	18	24	42
b	42	24	18	15	12	10	9	8	7

Any value for a larger than 42 requires b to be smaller than 7, but we know that $b \geq 7$.
Thus, there are only 9 different possible values for a.
Note:
We can reduce the time it takes to complete this trial and error above by recognizing that in the equation $\frac{6}{a} + \frac{6}{b} = 1$, the a and b are interchangeable.
That is, interchanging a and b in the equation, gives $\frac{6}{b} + \frac{6}{a} = 1$, which is the same equation.
For example, this tells us that since $a = 7$, $b = 42$ satisfies the equation, then $a = 42$, $b = 7$ satisfies the equation as well.
Moreover, if the pair (a, b) satisfies the equation, then (b, a) satisfies the equation, and if (a, b) does not satisfy the equation, then (b, a) does not satisfy the equation.
This interchangeability of a and b is seen in the symmetry of the list of possible solutions above.
Recognizing that this symmetry must exist allows us to quickly determine the 4 remaining solutions that follow after $a = 12$, $b = 12$.

ANSWER: (C)

Grade 8

1. Using the correct order of operations, $2 + 3 \times 4 + 10 = 2 + 12 + 10 = 24$.

   Answer: (A)

2. The athlete who won the race is the one who had the shortest running time.
   Thus, athlete C won the race.

   Answer: (C)

3. Substituting $x = 2$ and $y = 1$ into the expression $2x - 3y$, we have $2 \times 2 - 3 \times 1$.
   Using the correct order of operations, $2 \times 2 - 3 \times 1 = 4 - 3 = 1$.

   Answer: (B)

4. *Solution 1*
   Evaluating the left side of the equation, we get $44 \times 25 = 1100$.
   Thus, the number that should replace the $\square$ is $1100 \div 100 = 11$.

   Solution 2
   The left side of the equation, 44×25, can be rewritten as $11 \times 4 \times 25$.
   Since $11 \times 4 \times 25 = 11 \times 100$, then we can write $11 \times 100 = \square \times 100$, so the number that should replace the $\square$ is 11.

   Answer: (A)

5. The area of the rectangle, 12, is found by multiplying its length by its width.
   Since the length and width must be whole numbers, the only possible dimensions are:
   12 by 1, 6 by 2, and 4 by 3.
   (We recognize that 1 by 12, 2 by 6, and 3 by 4 are also possibilities, but the perimeter of the rectangle is unchanged by reversing the dimensions.)
   The perimeter of a rectangle is found by doubling the sum of the length and width.
   The results are shown in the table below.

Length	Width	Perimeter
12	1	26
6	2	16
4	3	14

   Therefore, the smallest possible perimeter of a rectangle with the given conditions is 14.

   Answer: (D)

6. We first recognize that $\frac{1}{4}$ is a common fraction in each of the five sums, and so the relative size of the sums depends only on the other fractions.
   Since $\frac{1}{3}$ is the largest of the fractions $\frac{1}{5}, \frac{1}{6}, \frac{1}{3}, \frac{1}{8}, \frac{1}{7}$, we conclude that $\frac{1}{4} + \frac{1}{3}$ is the largest sum.

   Answer: (C)

7. Since 1 gram is the approximate weight of 15 seeds, 300 grams is the approximate weight of $300 \times 15 = 4500$ seeds.
   Therefore, there are approximately 4500 seeds in the container.

   Answer: (B)

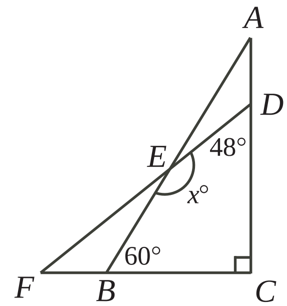
8. The first time after 10:25 at which all of the digits on the clock will be equal to one another is 11:11.
   Thus, the shortest length of time required is the elapsed time between 10:25 and 11:11,
   or 46 minutes (because the time from 10:25 to 11:00 is 35 minutes and the time from 11:00 to 11:11 is 11 minutes).

   Answer: (D)

9. Chris was given $\frac{1}{3}$ of 84 cookies, or $\frac{1}{3} \times 84 = \frac{84}{3} = 28$ cookies.
He ate $\frac{3}{4}$ of the 28 cookies he was given, or $\frac{3}{4} \times 28 = \frac{84}{4} = 21$ cookies.

Answer: (E)

10. *Solution 1*
In $\triangle ABC$ shown below, $\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 60^\circ - 90^\circ = 30^\circ$.
Since $\angle ADC$ is a straight angle, $\angle ADE = 180^\circ - \angle CDE = 180^\circ - 48^\circ = 132^\circ$.
In $\triangle AED$, $\angle AED = 180^\circ - \angle ADE - \angle EAD = 180^\circ - 132^\circ - 30^\circ = 18^\circ$.
Since $\angle AEB$ is a straight angle, $\angle DEB = 180^\circ - \angle AED = 180^\circ - 18^\circ = 162^\circ$.
Thus, the value of x is 162.



Solution 2
The sum of the interior angles of a quadrilateral is 360°.
In quadrilateral $BCDE$ shown above,

$$\angle DEB = 360^\circ - \angle EDC - \angle DCB - \angle CBE = 360^\circ - 48^\circ - 90^\circ - 60^\circ = 162^\circ.$$

Thus, the value of x is 162.

Answer: (E)

11. *Solution 1*
The mean of 5 consecutive integers is equal to the number in the middle.
Since the numbers have a mean of 21, if we were to distribute the quantities equally, we would have 21, 21, 21, 21, and 21.
Since the numbers are consecutive, the second number is 1 less than the 21 in the middle, while the fourth number is 1 more than the 21 in the middle.
Similarly, the first number is 2 less than the 21 in the middle, while the fifth number is 2 more than the 21 in the middle.
Thus, the numbers are $21 - 2$, $21 - 1$, 21, $21 + 1$, $21 + 2$.
The smallest of 5 consecutive integers having a mean of 21, is 19.

Solution 2
Since 21 is the mean of five consecutive integers, the smallest of these five integers must be less than 21.
Suppose the smallest integer is 20.
The mean of $20, 21, 22, 23$, and 24 is $\dfrac{20 + 21 + 22 + 23 + 24}{5} = 22$.
This mean of 22 is greater than the required mean of 21; thus, the smallest of the 5 consecutive integers must be less than 20.
Suppose the smallest integer is 19.
The mean of $19, 20, 21, 22$, and 23 is $\dfrac{19 + 20 + 21 + 22 + 23}{5} = 21$, as required.
Thus, the smallest of 5 consecutive integers having a mean of 21, is 19.

Answer: (E)

12. For every 3 white balls in the jar, there are 2 red balls in the jar.
Since there are 9 white balls in the jar, which is 3 groups of 3 white balls, there must be 3 groups of 2 red balls in the jar.
Thus, there are $3 \times 2 = 6$ red balls in the jar.

Answer: (D)

13. Evaluating, $\left(\frac{11}{12}\right)^2 = \left(\frac{11}{12}\right) \times \left(\frac{11}{12}\right) = \frac{11\times11}{12\times12} = \frac{121}{144}$.
Since $\frac{121}{144} > \frac{72}{144} = \frac{1}{2}$ and $\frac{121}{144} < \frac{144}{144} = 1$, the value of $\left(\frac{11}{12}\right)^2$ is between $\frac{1}{2}$ and 1.

Answer: (B)

14. During the 5 games, Gina faced $10 + 13 + 7 + 11 + 24 = 65$ shots in total.
During the 5 games, Gina saved $7 + 9 + 6 + 9 + 21 = 52$ of the 65 shots.
Thus, the percentage of total shots saved is $\frac{52}{65} \times 100\% = 0.80 \times 100\% = 80\%$.

Answer: (C)

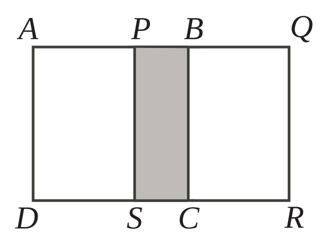
15. To find the smallest possible sum, we first choose the tens digit of each number to be as small as possible.
Therefore, we choose 5 and 6 as the two tens digits.
Next, we choose the units digits to be as small as possible.
Since 7 and 8 are each less than 9, we choose 7 and 8 as the two units digits.
Using 5 and 6 as the tens digits, 7 and 8 as the units digits, we evaluate the only two possibilities.

$$\begin{array}{rr} & 57 \\ + & 68 \\ \hline & 125 \end{array} \qquad \begin{array}{rr} & 58 \\ + & 67 \\ \hline & 125 \end{array}$$

(Can you see why these two sums should be equal?)
The smallest possible sum is 125.

Answer: (B)

16. *Solution 1*
Since $AQ = 20$ and $AB = 12$, then $BQ = AQ - AB = 20 - 12 = 8$.
Thus, $PB = PQ - BQ = 12 - 8 = 4$.
Since $PS = 12$, the area of rectangle $PBCS$ is $12 \times 4 = 48$.

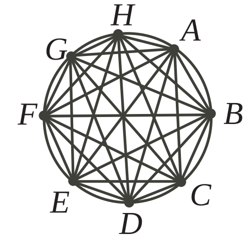


Solution 2
The sum of the areas of squares $ABCD$ and $PQRS$ is $2 \times (12 \times 12) = 2 \times 144 = 288$.
The area of rectangle $AQRD$ is $12 \times 20 = 240$.
The sum of the areas of $ABCD$ and $PQRS$ is equal to the sum of the areas of $APSD$, $PBCS$, $PBCS$, and $BQRC$.
The area of rectangle $AQRD$ is equal to the sum of the areas of $APSD$, $PBCS$, and $BQRC$.
Therefore, the sum of the areas of $ABCD$ and $PQRS$, minus the area of $AQRD$, is the area of $PBCS$.
Thus, the shaded rectangle $PBCS$ has area $288 - 240 = 48$.

Answer: (C)

17. *Solution 1*

Label the 8 points A through H as shown and proceed to connect the points in all possible ways.



From point A, 7 line segments are drawn, 1 to each of the other points, B through H.
From point B, 6 new line segments are drawn, 1 to each of the points C through H, since the segment AB has already been drawn.
This continues, with 5 line segments drawn from point C, 4 from D, 3 from E, 2 from F, 1 from G and 0 from point H since it will already be joined to each of the other points.
In total, there are $7 + 6 + 5 + 4 + 3 + 2 + 1 = 28$ line segments.

Solution 2

Label the 8 points A through H as shown above and proceed to connect the points in all possible ways.
From each of the 8 points, 7 line segments can be drawn leaving the point, 1 to each of the other 7 points.
Thus, the total number of line segments leaving the 8 points is $8 \times 7 = 56$.
However, this counts each of the line segments twice, since each segment will be counted as leaving both of its ends.
For example, the segment leaving point A and ending at point D is also counted as a segment leaving point D and ending at point A.
Thus, the actual number of line segments is half of 56 or $56 \div 2 = 28$.

Answer: (E)

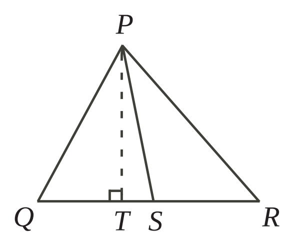
18. Traveling at a constant speed of 15 km/h, in 3 hours the bicycle will travel $15 \times 3 = 45$ km.
At the start, the bicycle was 195 km ahead of the bus.
Therefore, in order to catch up to the bicycle, the bus must travel 195 km plus the additional 45 km that the bicycle travels, or $195 + 45 = 240$ km.
To do this in 3 hours, the bus must travel at an average speed of $240 \div 3 = 80$ km/h.

Answer: (B)

19. Figure 1 is formed with 1 square.
Figure 2 is formed with $4 + 1$ squares.
Figure 3 is formed with $4 + 4 + 1 = 2 \times 4 + 1$ squares.
Figure 4 is formed with $4 + 4 + 4 + 1 = 3 \times 4 + 1$ squares.
Figure 5 is formed with $4 + 4 + 4 + 4 + 1 = 4 \times 4 + 1$ squares.
Thus, the number of groups of 4 squares needed to help form the Figure is increasing by 1.
Also, in each case the number of groups of 4 squares needed is one less than the Figure number.
For example, Figure 6 will be formed with 5 groups of 4 squares plus 1 additional square.
In general, we can say that Figure N will be formed with $N - 1$ groups of 4 squares, plus 1 additional square.
Thus, Figure 2010 will be formed with $2009 \times 4 + 1 = 8036 + 1 = 8037$ squares.

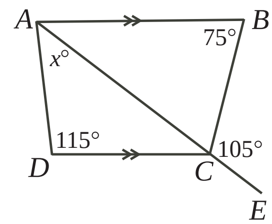
Answer: (A)

20. Position point T on QR such that PT is perpendicular to QR.
Line segment PT is the height of $\triangle PQS$ with base QS, and is also the height of $\triangle PRS$ with base SR.
Since $\triangle PQS$ and $\triangle PRS$ have equal heights and equal areas, their bases must be equal. Thus, $QS = SR$.



Answer: (D)

21. Since $\angle ACE$ is a straight angle, $\angle ACB = 180^\circ - 105^\circ = 75^\circ$.
In $\triangle ABC$, $\angle BAC = 180^\circ - \angle ABC - \angle ACB = 180^\circ - 75^\circ - 75^\circ = 30^\circ$.
Since AB is parallel to DC, $\angle ACD = \angle BAC = 30^\circ$ (alternate angles).
In $\triangle ADC$, $\angle DAC = 180^\circ - \angle ADC - \angle ACD = 180^\circ - 115^\circ - 30^\circ = 35^\circ$.
Thus, the value of x is 35.



ANSWER: (A)

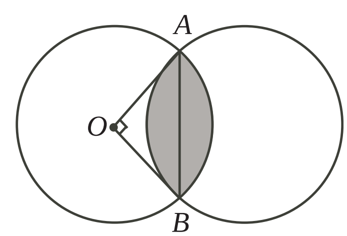
22. We first recognize that in the products, $r \times s$, $u \times r$ and $t \times r$, r is the only variable that occurs in all three.
Thus, to make $r \times s + u \times r + t \times r$ as large as possible, we choose $r = 5$, the largest value possible.
Since each of s, u and t is multiplied by r once only, and the three products are then added, it does not matter which of s, u or t we let equal 2, 3 or 4, as the result will be the same.
Therefore, let $s = 2$, $u = 3$ and $t = 4$.
Thus, the largest possible value of $r \times s + u \times r + t \times r$ is $5 \times 2 + 3 \times 5 + 4 \times 5 = 10 + 15 + 20 = 45$.

ANSWER: (B)

23. Since Kevin needs 12 hours to shovel all of his snow, he shovels $\frac{1}{12}$ of his snow every hour.
Since Dave needs 8 hours to shovel all of Kevin's snow, he shovels $\frac{1}{8}$ of Kevin's snow every hour.
Similarly, John shovels $\frac{1}{6}$ of Kevin's snow every hour, and Allison shovels $\frac{1}{4}$ of Kevin's snow every hour.
Together, Kevin, Dave, John, and Allison can shovel $\frac{1}{12} + \frac{1}{8} + \frac{1}{6} + \frac{1}{4} = \frac{2}{24} + \frac{3}{24} + \frac{4}{24} + \frac{6}{24} = \frac{15}{24}$ of Kevin's snow every hour.
Therefore, together they can shovel $\frac{15}{24} \div 60 = \frac{15}{24} \times \frac{1}{60} = \frac{15}{1440} = \frac{1}{96}$ of Kevin's snow every minute.
Thus, by shoveling $\frac{1}{96}$ of Kevin's snow per minute, together they will shovel all of Kevin's snow in 96 minutes.

ANSWER: (D)

24. Label points A and B, the points of intersection of the two circles, and point O, the centre of the left circle.
Construct line segment AB, which by symmetry divides the shaded area in half.
Construct radii OA and OB with $OA = OB = 10$ cm.
Since each circle contains 25% or $\frac{1}{4}$ of the other circle's circumference, $\angle AOB = \frac{1}{4} \times 360^\circ = 90^\circ$.
Thus, the area of sector AOB is $\frac{1}{4}$ of the area of the entire circle, or $\frac{1}{4}\pi r^2 = \frac{1}{4}\pi 10^2 = 25\pi$ cm^2.
The area of $\triangle AOB$ is $\dfrac{OA \times OB}{2} = \dfrac{10 \times 10}{2} = 50$ cm^2.
The area remaining after $\triangle AOB$ is subtracted from sector AOB is equal to half of the shaded area. Thus, the shaded area is $2 \times (25\pi - 50) \approx 2 \times (28.5398) = 57.0796$ cm^2.
The area of the shaded region is closest to 57.08 cm^2.



ANSWER: (A)

25. We are given that the first two terms of a 10 term sequence are 1 and x.
Since each term after the second is the sum of the previous two terms, then the third term is $1 + x$.
Since the fourth term is the sum of the second and third terms, then the fourth term is $x + (1 + x) = 1 + 2x$.
Continuing in this manner, we construct the 10 term sequence:

$$1, x, 1 + x, 1 + 2x, 2 + 3x, 3 + 5x, 5 + 8x, 8 + 13x, 13 + 21x, 21 + 34x.$$

Each of the second through tenth terms is dependent on the value of x, and thus, any one of these terms could potentially equal 463.
For the second term to equal 463, we need $x = 463$, which is possible since the only requirement is that x is a positive integer.
Thus, if $x = 463$ then 463 appears as the second term in the sequence.
For the third term to equal 463, we need $1 + x = 463$, or $x = 462$.
Thus, if $x = 462$ then 463 appears as the third term in the sequence.
For the fourth term to equal 463, we need $1 + 2x = 463$, or $2x = 462$ or $x = 231$.
Thus, if $x = 231$ then 463 appears as the fourth term in the sequence.
For the fifth term to equal 463, we need $2 + 3x = 463$, or $3x = 461$ or $x = \frac{461}{3}$.
However, $\frac{461}{3}$ is not an integer, and thus, 463 cannot appear as the fifth term in the sequence.
We continue in this manner and summarize all the results in the table below.

Term	Expression	Equation	Value of x	Is x an integer?
2nd	x	$x = 463$	$x = 463$	Yes
3rd	$1 + x$	$1 + x = 463$	$x = 462$	Yes
4th	$1 + 2x$	$1 + 2x = 463$	$x = 231$	Yes
5th	$2 + 3x$	$2 + 3x = 463$	$x = \frac{461}{3}$	No
6th	$3 + 5x$	$3 + 5x = 463$	$x = 92$	Yes
7th	$5 + 8x$	$5 + 8x = 463$	$x = \frac{458}{8}$	No
8th	$8 + 13x$	$8 + 13x = 463$	$x = 35$	Yes
9th	$13 + 21x$	$13 + 21x = 463$	$x = \frac{450}{21}$	No
10th	$21 + 34x$	$21 + 34x = 463$	$x = 13$	Yes

Therefore, the sum of all possible integer values of x for which 463 appears in the sequence is $463 + 462 + 231 + 92 + 35 + 13 = 1296$.

ANSWER: (B)