



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING

www.cemc.uwaterloo.ca

Canadian Senior Mathematics Contest

Sample Contest

Time: 2 hours

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Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.

There are two parts to this paper.

PART A

1. This part consists of 6 questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of 3 questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, it will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert the information sheet inside the answer booklet.

The names of some top-scoring students will be published on the CEMC website, <http://www.cemc.uwaterloo.ca>.

Canadian Senior Mathematics Contest

- NOTE:
1. Please read the instructions on the front cover of this booklet.
 2. Write solutions in the answer booklet provided.
 3. It is expected that all calculations and answers will be expressed as exact numbers such as 4π , $2 + \sqrt{7}$, etc., rather than as $12.566\dots$ or $4.646\dots$
 4. **Calculators are permitted**, provided they are non-programmable and without graphic displays.
 5. Diagrams are not drawn to scale. They are intended as aids only.

Note: The problems for this sample contest have been taken from past CEMC contests to demonstrate the level of difficulty that the 2011 contest will have.

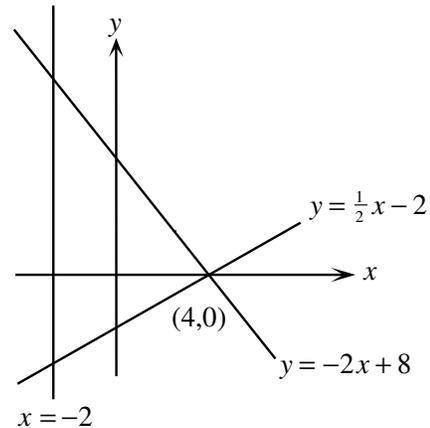
PART A

1. In the sequence $32, 8, _, _, x$, each term after the second is the average of the two terms immediately before it. Determine the value of x .

{2005 Pascal #10}

2. The lines $y = -2x + 8$ and $y = \frac{1}{2}x - 2$ meet at $(4, 0)$, as shown. Determine the area of the triangle formed by these two lines and the line $x = -2$.

{2005 Cayley #12}



3. A certain four-digit number has four distinct digits. If the product of these four digits is 810, then determine the sum of the four digits.

{2003 Fermat #16}

4. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes N marbles all at once. If he is to be sure that, no matter which choice of N marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of N ?

{2006 Fermat #20}

5. Solve the system of equations:

$$\log_{10}(x^3) + \log_{10}(y^2) = 11$$

$$\log_{10}(x^2) - \log_{10}(y^3) = 3$$

{2003 Euclid #6(b)}

6. Three thin metal rods of lengths 9, 12 and 15 are welded together to form a right-angled triangle, which is held in a horizontal position. A solid sphere of radius 5 rests in the triangle so that it is tangent to each of the three sides. Assuming that the thickness of the rods can be neglected, how high above the plane of the triangle is the top of the sphere?

{2002 Euclid #9(b)}

PART B

1. For numbers a and b , the notation $a\Delta b$ means $2a + b^2 + ab$.
For example, $1\Delta 2 = 2(1) + 2^2 + 1(2) = 8$.

- (a) Determine the value of $3\Delta 2$.
- (b) If $x\Delta(-1) = 8$, determine the value of x .
- (c) If $4\Delta y = 20$, determine the two possible values of y .
- (d) If $(w - 2)\Delta w = 14$, determine all possible values of w .

{2008 Hypatia #1}

2. (a) Two fair dice, each having six faces numbered 1 to 6, are thrown. What is the probability that the product of the two numbers on the top faces is divisible by 5?

{2004 Euclid #4(a)}

- (b) Six tickets numbered 1 through 6 are placed in a box. Two tickets are randomly selected and removed together. What is the probability that the smaller of the two numbers on the tickets selected is less than or equal to 4?

{2005 Euclid #4(a)}

- (c) Oi-Lam tosses three fair coins and removes all of the coins that come up heads. George then tosses the coins that remain, if any. Determine the probability that George tosses exactly one head.

{2010 Euclid #8(a)}

3. A *Skolem sequence* of order n is a sequence $(s_1, s_2, \dots, s_{2n})$ of $2n$ integers satisfying the conditions:

- i) for every k in $\{1, 2, 3, \dots, n\}$, there exist exactly two elements s_i and s_j with $s_i = s_j = k$, and
- ii) if $s_i = s_j = k$ with $i < j$, then $j - i = k$.

For example, $(4, 2, 3, 2, 4, 3, 1, 1)$ is a Skolem sequence of order 4.

- (a) List all Skolem sequences of order 4.
- (b) Determine, with justification, all Skolem sequences of order 9 which satisfy all of the following three conditions:
 - I) $s_3 = 1$,
 - II) $s_{18} = 8$, and
 - III) between any two equal even integers, there is exactly one odd integer.
- (c) Prove that there is no Skolem sequence of order n , if n is of the form $4k + 2$ or $4k + 3$, where k is a non-negative integer.

{2004 Euclid #10}

Sample
Canadian
Senior
Mathematics
Challenge
(English)

