The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
www.cemc.uwaterloo.ca

Canadian Senior Mathematics Contest

Tuesday, November 22, 2011
(in North America and South America)

Wednesday, November 23, 2011
(outside of North America and South America)

UNIVERSITY OF WATERLOO

WATERLOO MATHEMATICS

Time: 2 hours

Calculators are permitted, provided they are non-programmable and without graphic displays.

Do not open this booklet until instructed to do so.
There are two parts to this paper.

PART A

1. This part consists of six questions, each worth 5 marks.
2. **Enter the answer in the appropriate box in the answer booklet.**
   For these questions, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded **only if relevant work** is shown in the space provided in the answer booklet.

PART B

1. This part consists of three questions, each worth 10 marks.
2. **Finished solutions must be written in the appropriate location in the answer booklet.** Rough work should be done separately. If you require extra pages for your finished solutions, they will be supplied by your supervising teacher. Insert these pages into your answer booklet. Be sure to write your name, school name and question number on any inserted pages.
3. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

NOTES:

At the completion of the contest, insert your student information form inside your answer booklet.

The names of some top-scoring students will be published on the CEMC website, http://www.cemc.uwaterloo.ca.

Canadian Senior Mathematics Contest

NOTE:	1.	Please read the instructions on the front cover of this booklet.
	2.	Write solutions in the answer booklet provided.
	3.	All calculations and answers should be expressed as exact numbers such as 4π, $2+\sqrt{7}$, etc., rather than as $12.566\ldots$ or $4.646\ldots$.
	4.	**Calculators are permitted**, provided they are non-programmable and without graphic displays.
	5.	Diagrams are not drawn to scale. They are intended as aids only.

PART A

For each question in Part A, full marks will be given for a correct answer which is placed in the box. Part marks will be awarded only if relevant work is shown in the space provided in the answer booklet.

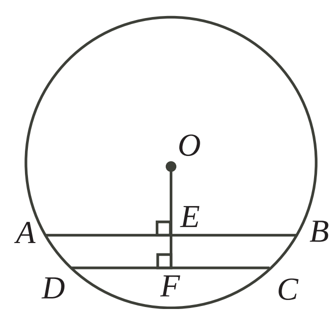
1. Determine the value of $2^4\left(1+\dfrac{1}{2}+\dfrac{1}{2^2}+\dfrac{1}{2^3}+\dfrac{1}{2^4}\right)$.

2. Four years ago, Daryl was three times as old as Joe was.
   In five years, Daryl will be twice as old as Joe will be.
   How old is Daryl now?

3. A die is a cube with its faces numbered 1 through 6. One red die and one blue die are rolled. The sum of the numbers on the top two faces is determined. What is the probability that this sum is a perfect square?

4. Determine the number of positive divisors of 18 800 that are divisible by 235.

5. In the diagram, the circle has centre O. OF is perpendicular to DC at F and is perpendicular to AB at E. If $AB=8$, $DC=6$ and $EF=1$, determine the radius of the circle.



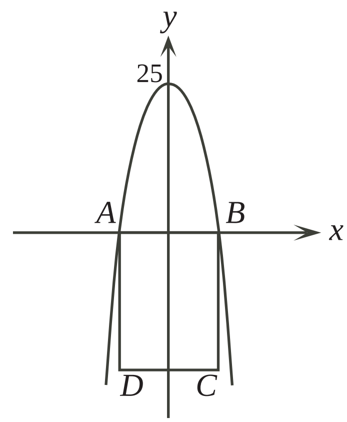
6. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. Given the magic square shown with $a,b,c,x,y,z>0$, determine the product xyz in terms of a, b and c.

$\log a$	$\log b$	$\log x$
p	$\log y$	$\log c$
$\log z$	q	r

PART B

For each question in Part B, your solution must be well organized and contain words of explanation or justification when appropriate. Marks are awarded for completeness, clarity, and style of presentation. A correct solution, poorly presented, will not earn full marks.

1. The parabola with equation $y = 25 - x^2$ intersects the x-axis at points A and B, as shown.



   (a) Determine the length of AB.

   (b) Rectangle $ABCD$ is formed as shown with C and D below the x-axis and $BD = 26$. Determine the length of BC.

   (c) If CD is extended in both directions, it meets the parabola at points E and F. Determine the length of EF.

2. (a) First, determine two positive integers x and y with $\dfrac{2x + 11y}{3x + 4y} = 1$.

   Now, let u and v be two positive rational numbers with $u < v$.
   If we write u and v as fractions $u = \dfrac{a}{b}$ and $v = \dfrac{c}{d}$, not necessarily in lowest terms and with a, b, c, d positive integers, then the fraction $\dfrac{a+c}{b+d}$ is called a *mediant* of u and v. Since u and v can be written in many different forms, there are many different mediants of u and v.
   In (a), you showed that 1 is a mediant of $\dfrac{2}{3}$ and $\dfrac{11}{4}$.
   Also, 2 is a mediant of $\dfrac{2}{3}$ and $\dfrac{11}{4}$ because $\dfrac{2}{3} = \dfrac{6}{9}$ and $\dfrac{11}{4} = \dfrac{44}{16}$ and $\dfrac{6+44}{9+16} = 2$.

   (b) Prove that the average of u and v, namely $\frac{1}{2}(u + v)$, is a mediant of u and v.
   (c) Prove that every mediant, m, of u and v satisfies $u < m < v$.

3. Suppose that $n \geq 3$. A sequence $a_1, a_2, a_3, \ldots, a_n$ of n integers, the first m of which are equal to -1 and the remaining $p = n - m$ of which are equal to 1, is called an *MP sequence.*

   (a) The sequence $-1, -1, 1, 1, 1$ is the MP sequence a_1, a_2, a_3, a_4, a_5 with $m = 2$ and $p = 3$. Consider all of the possible products $a_i a_j a_k$ (with $i < j < k$) that can be calculated using the terms from this sequence. Determine how many of these products are equal to 1.
   (b) Consider all of the products $a_i a_j a_k$ (with $i < j < k$) that can be calculated using the terms from an MP sequence $a_1, a_2, a_3, \ldots, a_n$. Determine the number of pairs (m, p) with $1 \leq m \leq p \leq 1000$ and $m + p \geq 3$ for which exactly half of these products are equal to 1.

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Contest
(English)