The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

2011 Cayley Contest

(Grade 10)

Thursday, February 24, 2011

Solutions

1. We regroup $(5+2)+(8+6)+(4+7)+(3+2)$ as $5+(2+8)+(6+4)+(7+3)+2$, which equals $5+10+10+10+2$ or 37.
   We could add the numbers directly instead.

   Answer: (B)

2. Since $(-1)(2)(x)(4) = 24$, then $-8x = 24$ or $x = \dfrac{24}{-8} = -3$.

   Answer: (B)

3. *Solution 1*
   Since $\angle PRS$ is an exterior angle to $\triangle PQR$, then $\angle PQR + \angle QPR = \angle PRS$ or $x^\circ + 75^\circ = 125^\circ$.
   Thus, $x + 75 = 125$ or $x = 50$.

   Solution 2
   Since QRS is a straight line, then $\angle PRQ = 180^\circ - \angle PRS = 180^\circ - 125^\circ = 55^\circ$.
   Since the sum of the angles in a triangle is 180°, then $x^\circ + 75^\circ + 55^\circ = 180^\circ$ or $x + 130 = 180$ or $x = 50$.

   Answer: (A)

4. *Solution 1*
   To get back to the original number, we undo the given operations.
   We add 5 to 16 to obtain 21 and then divide by 3 to obtain 7.
   These are the "inverse" operations of decreasing by 5 and multiplying by 3.

   Solution 2
   Suppose that the original number is x.
   After tripling and decreasing by 5, the result is $3x - 5$.
   Therefore, $3x - 5 = 16$ or $3x = 21$ or $x = 7$.

   Answer: (C)

5. We evaluate from the inside towards the outside:

$$\sqrt{13+\sqrt{7+\sqrt{4}}} = \sqrt{13+\sqrt{7+2}} = \sqrt{13+\sqrt{9}} = \sqrt{13+3} = \sqrt{16} = 4$$

   Answer: (D)

6. A graph that is linear with a slope of 0 is a horizontal straight line. This is Graph Q.

   Answer: (B)

7. With a fair die that has faces numbered from 1 to 6, the probability of rolling each of 1 to 6 is $\frac{1}{6}$.
   We calculate the probability for each of the five choices.
   There are 4 values of x that satisfy $x > 2$, so the probability is $\frac{4}{6} = \frac{2}{3}$.
   There are 2 values of x that satisfy $x = 4$ or $x = 5$, so the probability is $\frac{2}{6} = \frac{1}{3}$.
   There are 3 values of x that are even, so the probability is $\frac{3}{6} = \frac{1}{2}$.
   There are 2 values of x that satisfy $x < 3$, so the probability is $\frac{2}{6} = \frac{1}{3}$.
   There is 1 value of x that satisfies $x = 3$, so the probability is $\frac{1}{6}$.
   Therefore, the most likely of the five choices is that x is greater than 2.

   Answer: (A)

8. When 2.4×10^8 is doubled, the result is $2 \times 2.4 \times 10^8 = 4.8 \times 10^8$.

ANSWER: (C)

9. Since the face of a foonie has area 5 cm^2 and its thickness is 0.5 cm, then the volume of one foonie is $5 \times 0.5 = 2.5 \text{ cm}^3$.
If a stack of foonies has a volume of 50 cm^3 and each foonie has a volume of 2.5 cm^3, then there are $50 \div 2.5 = 20$ foonies in the stack.

ANSWER: (D)

10. In order to make the playoffs, the Athenas must win at least 60% of their 44 games. That is, they must win at least $0.6 \times 44 = 26.4$ games.
Since they must win an integer number of games, then the smallest number of games that they can win to make the playoffs is the smallest integer larger than 26.4, or 27.
Since they have won 20 games so far, then they must win $27 - 20 = 7$ of their remaining games to make the playoffs.

ANSWER: (E)

11. From the definition, $(3,1)\nabla(4,2) = (3)(4) + (1)(2) = 12 + 2 = 14$.

ANSWER: (D)

12. Since the angle in the sector representing cookies is 90°, then this sector represents $\frac{1}{4}$ of the total circle.
Therefore, 25% of the students chose cookies as their favourite food.
Thus, the percentage of students who chose sandwiches was $100\% - 30\% - 25\% - 35\% = 10\%$.
Since there are 200 students in total, then $200 \times \frac{10}{100} = 20$ students said that their favourite food was sandwiches.

ANSWER: (B)

13. *Solution 1*
We work from right to left as we would if doing this calculation by hand.
In the units column, we have $L - 1$ giving 1. Thus, $L = 2$. (There is no borrowing required.)
In the tens column, we have $3 - N$ giving 5.
Since 5 is larger than 3, we must borrow from the hundreds column. Thus, $13 - N$ gives 5, which means $N = 8$. This gives

$$\begin{array}{ccccc} & & {\scriptstyle K-1} & {\scriptstyle 13} & \\ & 5 & \cancel{K} & \cancel{3} & 2 \\ - & M & 4 & 8 & 1 \\ \hline & 4 & 4 & 5 & 1 \end{array}$$

In the hundreds column, we have $(K - 1) - 4$ giving 4, which means $K = 9$. This gives

$$\begin{array}{ccccc} & 5 & 9 & 3 & 2 \\ - & M & 4 & 8 & 1 \\ \hline & 4 & 4 & 5 & 1 \end{array}$$

In the thousands column, we have 5 (with nothing borrowed) minus M giving 4.
Thus, $5 - M = 4$ or $M = 1$.
This gives $5932 - 1481 = 4451$, which is correct.
Finally, $K + L + M + N = 9 + 2 + 1 + 8 = 20$.

Solution 2

Since $5K3L - M4N1 = 4451$, then

$$\begin{array}{cccc} & M & 4 & N & 1 \\ + & 4 & 4 & 5 & 1 \\ \hline & 5 & K & 3 & L \end{array}$$

We start from the units column and work towards the left.

Considering the units column, the sum $1 + 1$ has a units digit of L. Thus, $L = 2$. (There is no carry to the tens column.)

Considering the tens column, the sum $N + 5$ has a units digit of 3. Thus, $N = 8$. (There is a carry of 1 to the hundreds column.) This gives

$$\begin{array}{ccccc} & & \scriptstyle 1 & & \\ & M & 4 & 8 & 1 \\ + & 4 & 4 & 5 & 1 \\ \hline & 5 & K & 3 & 2 \end{array}$$

Considering the hundreds column, the sum $4 + 4$ plus the carry of 1 from the tens column has a units digit of K. Therefore, $K = 4 + 4 + 1 = 9$. There is no carry from the hundreds column to the thousands column.

Considering the thousands column, the sum $M + 4$ equals 5. Therefore, $M = 1$. This gives

$$\begin{array}{ccccc} & 1 & 4 & 8 & 1 \\ + & 4 & 4 & 5 & 1 \\ \hline & 5 & 9 & 3 & 2 \end{array}$$

which is equivalent to $5932 - 4451 = 1481$, which is correct.

Finally, $K + L + M + N = 9 + 2 + 1 + 8 = 20$.

Answer: (A)

14. The difference between $\frac{1}{6}$ and $\frac{1}{12}$ is $\frac{1}{6} - \frac{1}{12} = \frac{2}{12} - \frac{1}{12} = \frac{1}{12}$, so $LP = \frac{1}{12}$.

Since LP is divided into three equal parts, then this distance is divided into three equal parts, each equal to $\frac{1}{12} \div 3 = \frac{1}{12} \times \frac{1}{3} = \frac{1}{36}$.

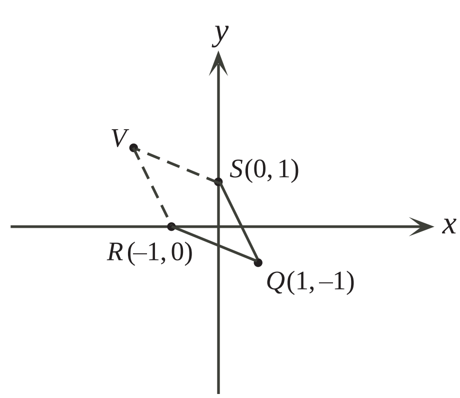
Therefore, M is located $\frac{1}{36}$ to the right of L.

Thus, the value at M is $\frac{1}{12} + \frac{1}{36} = \frac{3}{36} + \frac{1}{36} = \frac{4}{36} = \frac{1}{9}$.

Answer: (C)

15. We plot the first three vertices on a graph.

We see that one possible location for the fourth vertex, V, is in the second quadrant:



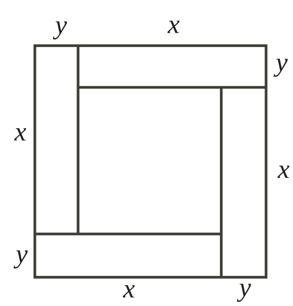
If $VSQR$ is a parallelogram, then SV is parallel and equal to QR.
To get from Q to R, we go left 2 units and up 1 unit.
Therefore, to get from S to V, we also go left 2 units and up 1 unit.
Since the coordinates of S are $(0, 1)$, then the coordinates of V are $(0 - 2, 1 + 1) = (-2, 2)$.
This is choice (A).
There are two other possible locations for the fourth vertex, which we can find in a similar way.
These are $U(0, -2)$ and $W(2, 0)$.
Using these points, we can see that $SQUR$ and $SWQR$ are parallelograms. But $(0, -2)$ and $(2, 0)$ are not among the possible answers.
Therefore, of the given choices, the only one that completes a parallelogram is $(-2, 2)$.

ANSWER: (A)

16. It is possible that after buying 7 gumballs, Wally has received 2 red, 2 blue, 1 white, and 2 green gumballs.
This is the largest number of each colour that he could receive without having three gumballs of any one colour.
If Wally buys another gumball, he will receive a blue or a green or a red gumball.
In each of these cases, he will have at least 3 gumballs of one colour.
In summary, if Wally buys 7 gumballs, he is not guaranteed to have 3 of any one colour; if Wally buys 8 gumballs, he is guaranteed to have 3 of at least one colour.
Therefore, the least number that he must buy to guarantee receiving 3 of the same colour is 8.

ANSWER: (E)

17. Suppose that each of the smaller rectangles has a longer side of length x cm and a shorter side of length y cm.



Since the perimeter of each of the rectangles is 40 cm, then $2x + 2y = 40$ or $x + y = 20$.
But the side length of the large square is $x + y$ cm.
Therefore, the area of the large square is $(x + y)^2 = 20^2 = 400$ cm^2.

ANSWER: (C)

18. *Solution 1*
When a number n is divided by x, the remainder is the difference between n and the largest multiple of x less than n. When 100 is divided by a positive integer x, the remainder is 10.
This means that $100 - 10 = 90$ is exactly divisible by x. It also means that x is larger than 10, otherwise the remainder would be smaller than 10.
Since 90 is exactly divisible by x, then $11 \times 90 = 990$ is also exactly divisible by x.
Since $x > 10$, then the next multiple of x is $990 + x$, which is larger than 1000.
Thus, 990 is the largest multiple of x less than 1000, and so the remainder when 1000 is divided by x is $1000 - 990 = 10$.

Solution 2

When 100 is divided by a positive integer x, the remainder is 10. The remainder is the difference between 100 and the largest multiple of x less than 100.

Therefore, the largest multiple of x less than 100 is $100 - 10 = 90$.

It also means that x is larger than 10, otherwise the remainder would be smaller than 10.

We can choose $x = 15$, since $6 \times 15 = 90$ and 15 is larger than 10.

What is the remainder when 1000 is divided by 15? Using a calculator, $1000 \div 15 = 66.666\ldots$ and $66 \times 15 = 990$.

Thus, the difference between 1000 and the largest multiple of 15 less than 1000 (that is, 990) equals 10 and so the remainder is equal to 10.

Answer: (A)

19. Let $\angle XYW = \theta$.
Since $\triangle XYW$ is isosceles with $WX = WY$, then $\angle YXW = \angle XYW = \theta$.
Since the sum of the angles in $\triangle XYW$ is 180°, then $\angle XWY = 180^\circ - 2\theta$.
Since $\angle XWY + \angle ZWY = 180^\circ$, then $\angle ZWY = 180^\circ - (180^\circ - 2\theta) = 2\theta$.
Since $\triangle YWZ$ is isosceles with $YW = YZ$, then $\angle YZW = \angle ZWY = 2\theta$.
Since $\triangle XYZ$ is isosceles, with $XY = XZ$, then $\angle XYZ = \angle XZY = 2\theta$.
Since the sum of the angles in $\triangle XYZ$ is 180°, then $\angle XYZ + \angle XZY + \angle YXZ = 180^\circ$ or $2\theta + 2\theta + \theta = 180^\circ$, or $5\theta = 180^\circ$, or $\theta = 36^\circ$.

Answer: (D)

20. For $n^3 + 5n^2$ to be the square of an integer, $\sqrt{n^3 + 5n^2}$ must be an integer.
We know that $\sqrt{n^3 + 5n^2} = \sqrt{n^2(n+5)} = \sqrt{n^2}\sqrt{n+5} = n\sqrt{n+5}$.
For $n\sqrt{n+5}$ to be an integer, $\sqrt{n+5}$ must be an integer. In other words, $n+5$ must be a perfect square.
Since n is between 1 and 100, then $n+5$ is between 6 and 105.
The perfect squares in this range are $3^2 = 9$, $4^2 = 16$, $\ldots$, $10^2 = 100$.
Thus, there are 8 perfect squares in this range.
Therefore, there are 8 values of n for which $\sqrt{n+5}$ is an integer, and thus for which $n^3 + 5n^2$ is the square of an integer.

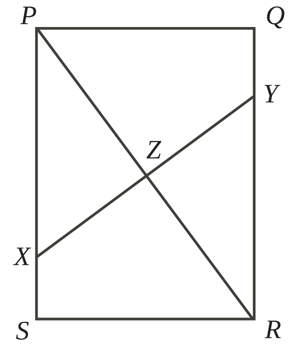
Answer: (B)

21. *Solution 1*
If we multiply the second and third equations together, we obtain $x(y+1)y(y+1) = \frac{7}{9} \cdot \frac{5}{18}$ or $xy(x+1)(x+1) = \frac{35}{162}$.
From the first equation, $xy = \frac{1}{9}$.
Therefore, $\frac{1}{9}(x+1)(y+1) = \frac{35}{162}$ or $(x+1)(y+1) = 9(\frac{35}{162}) = \frac{35}{18}$.

Solution 2
If we expand the left side of the second equation, we obtain $xy + x = \frac{7}{9}$.
Since $xy = \frac{1}{9}$ (from the first equation), then $x = \frac{7}{9} - xy = \frac{7}{9} - \frac{1}{9} = \frac{2}{3}$.
If we expand the left side of the third equation, we obtain $xy + y = \frac{5}{18}$.
Since $xy = \frac{1}{9}$ (from the first equation), then $y = \frac{5}{18} - xy = \frac{5}{18} - \frac{1}{9} = \frac{3}{18} = \frac{1}{6}$.
Therefore, $(x+1)(y+1) = (\frac{2}{3} + 1)(\frac{1}{6} + 1) = \frac{5}{3} \cdot \frac{7}{6} = \frac{35}{18}$.

Answer: (E)

22. Suppose that the crease intersects PS at X, QR at Y, and the line PR at Z. We want to determine the length of XY.



Since P folds on top of R, then line segment PZ folds on top of line segment RZ, since after the fold Z corresponds with itself and P corresponds with R. This means that $PZ = RZ$ and PR must be perpendicular to XY at point Z.
Since $PS = RQ$ and $SR = QP$, then right-angled triangles $\triangle PSR$ and $\triangle RQP$ are congruent (side-angle-side).
Therefore, $\angle XPZ = \angle YRZ$.
Since $PZ = RZ$, then right-angled triangles $\triangle PZX$ and $\triangle RZY$ are congruent too (angle-side-angle).
Thus, $XZ = ZY$ and so $XY = 2XZ$.
Since $\triangle PSR$ is right-angled at S, then by the Pythagorean Theorem,

$$PR = \sqrt{PS^2 + SR^2} = \sqrt{8^2 + 6^2} = \sqrt{100} = 10$$

since $PR > 0$.
Since $PZ = RZ$, then $PZ = \frac{1}{2}PR = 5$.
Now $\triangle PZX$ is similar to $\triangle PSR$ (common angle at P and right angle), so $\dfrac{XZ}{PZ} = \dfrac{RS}{PS}$ or $XZ = \frac{5\cdot 6}{8} = \frac{30}{8} = \frac{15}{4}$.
Therefore, $XY = 2XZ = \frac{15}{2}$, so the length of the fold is $\frac{15}{2}$ or 7.5.

Answer: (C)

23. First, we calculate the number of pairs that can be formed from the integers from 1 to n.
One way to form a pair is to choose one number to be the first item of the pair (n choices) and then a different number to be the second item of the pair ($n - 1$ choices).
There are $n(n - 1)$ ways to choose these two items in this way.
But this counts each pair twice; for example, we could choose 1 then 3 and we could also choose 3 then 1.
So we have double-counted the pairs, meaning that there are $\frac{1}{2}n(n-1)$ pairs that can be formed.

Next, we examine the number of rows in the table.
Since each row has three entries, then each row includes three pairs (first and second numbers, first and third numbers, second and third numbers).
Suppose that the completed table has r rows.
Then the total number of pairs in the table is $3r$.
Since each pair of the numbers from 1 to n appears exactly once in the table and the total number of pairs from these numbers is $\frac{1}{2}n(n - 1)$, then $3r = \frac{1}{2}n(n - 1)$, which tells us that $\frac{1}{2}n(n - 1)$ must be divisible by 3, since $3r$ is divisible by 3.
We make a table listing the possible values of n and the corresponding values of $\frac{1}{2}n(n - 1)$:

n	3	4	5	6	7	8	9	10	11	12
$\frac{1}{2}n(n-1)$	3	6	10	15	21	28	36	45	55	66

Since $\frac{1}{2}n(n-1)$ must be divisible by 3, then the possible values of n are 3, 4, 6, 7, 9, 10, and 12.
Next, consider a fixed number m from the list 1 to n.
In each row that m appears, it will belong to 2 pairs (one with each of the other two numbers in its row).
If the number m appears in s rows, then it will belong to $2s$ pairs.
Therefore, each number m must belong to an even number of pairs.
But each number m from the list of integers from 1 to n must appear in $n-1$ pairs (one with each other number in the list), so $n-1$ must be even, and so n is odd.
Therefore, the possible values of n are 3, 7, 9.
Finally, we must verify that we can create a Fano table for each of these values of n. We are given the Fano table for $n=7$.
Since the total number of pairs when $n=3$ is 3 and when $n=9$ is 36, then a Fano table for $n=3$ will have $3 \div 3 = 1$ row and a Fano table for $n=9$ will have $36 \div 3 = 12$ rows.
For $n=3$ and $n=9$, possible tables are shown below:

1	2	3

1	2	3
1	4	5
1	6	7
1	8	9
2	4	7
2	5	8
2	6	9
3	4	9
3	5	6
3	7	8
4	6	8
5	7	9

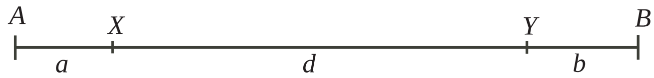
In total, there are 3 values of n in this range for which a Fano table can be created.

Answer: (B)

24. First, we note that the three people are interchangeable in this problem, so it does not matter who rides and who walks at any given moment. We abbreviate the three people as D, M and P.
We call their starting point A and their ending point B.
Here is a strategy where all three people are moving at all times and all three arrive at B at the same time:

D and M get on the motorcycle while P walks.
D and M ride the motorcycle to a point Y before B.
D drops off M and rides back while P and M walk toward B.
D meets P at point X.
D picks up P and they drive back to B meeting M at B.
Point Y is chosen so that D, M and P arrive at B at the same time.

Suppose that the distance from A to X is a km, from X to Y is d km, and the distance from Y to B is b km.



In the time that it takes P to walk from A to X at 6 km/h, D rides from A to Y and back to X at 90 km/h.
The distance from A to X is a km.
The distance from A to Y and back to X is $a + d + d = a + 2d$ km.
Since the time taken by P and by D is equal, then $\frac{a}{6} = \frac{a + 2d}{90}$ or $15a = a + 2d$ or $7a = d$.

In the time that it takes M to walk from Y to B at 6 km/h, D rides from Y to X and back to B at 90 km/h.
The distance from Y to B is b km, and the distance from Y to X and back to B is $d+d+b = b+2d$ km.
Since the time taken by M and by D is equal, then $\frac{b}{6} = \frac{b + 2d}{90}$ or $15b = b + 2d$ or $7b = d$.

Therefore, $d = 7a = 7b$, and so we can write $d = 7a$ and $b = a$.
Thus, the total distance from A to B is $a + d + b = a + 7a + a = 9a$ km.
However, we know that this total distance is 135 km, so $9a = 135$ or $a = 15$.
Finally, D rides from A to Y to X to B, a total distance of $(a + 7a) + 7a + (7a + a) = 23a$ km.
Since $a = 15$ km and D rides at 90 km/h, then the total time taken for this strategy is $\frac{23\times 15}{90} = \frac{23}{6} \approx 3.83$ h.
Since we have a strategy that takes 3.83 h, then the smallest possible time is no more than 3.83 h. Can you explain why this is actually the smallest possible time?

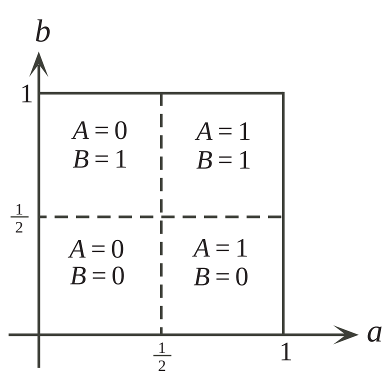
If we didn't think of this strategy, another strategy that we might try would be:

D and M get on the motorcycle while P walks.
D and M ride the motorcycle to B.
D drops off M at B and rides back to meet P, who is still walking.
D picks up P and they drive back to B. (M rests at B.)

This strategy actually takes 4.125 h, which is longer than the strategy shown above, since M is actually sitting still for some of the time.

Answer: (A)

25. By definition, if $0 \leq a < \frac{1}{2}$, then $A = 0$ and if $\frac{1}{2} \leq a \leq 1$, then $A = 1$.
Similarly, if $0 \leq b < \frac{1}{2}$, then $B = 0$ and if $\frac{1}{2} \leq b \leq 1$, then $B = 1$.
We keep track of our information on a set of axes, labelled a and b.



The area of the region of possible pairs (a, b) is 1, since the region is a square with side length 1. Next, we determine the sets of points where $C = 2A + 2B$ and calculate the combined area of these regions.
We consider the four sub-regions case by case. In each case, we will encounter lines of the form $a + b = Z$ for some number Z. We can rewrite these equations as $b = -a + Z$ which shows that this equation is the equation of the line with slope -1 and b-intercept Z. Since the slope is -1, the a-intercept is also Z.

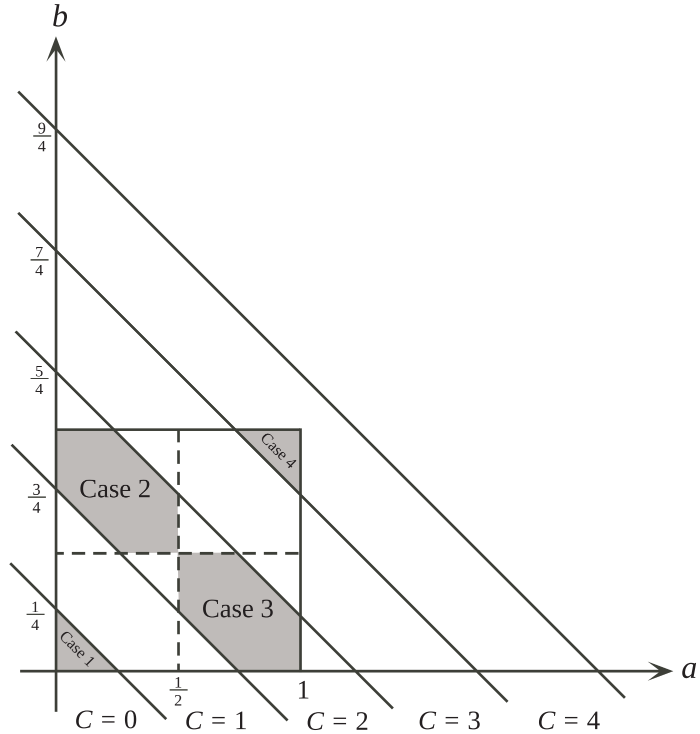
<u>Case 1: $A = 0$ and $B = 0$</u>
For C to equal $2A + 2B$, we need $C = 0$.
Since C is obtained by rounding c, then we need $0 \leq c < \frac{1}{2}$.
Since $c = 2a + 2b$ by definition, this is true when $0 \leq 2a + 2b < \frac{1}{2}$ or $0 \leq a + b < \frac{1}{4}$.
This is the set of points in this subregion above the line $a + b = 0$ and below the line $a + b = \frac{1}{4}$.

<u>Case 2: $A = 0$ and $B = 1$</u>
For C to equal $2A + 2B$, we need $C = 2$.
Since C is obtained by rounding c, then we need $\frac{3}{2} \leq c < \frac{5}{2}$.
Since $c = 2a + 2b$ by definition, this is true when $\frac{3}{2} \leq 2a + 2b < \frac{5}{2}$ or $\frac{3}{4} \leq a + b < \frac{5}{4}$.
This is the set of points in this subregion above the line $a + b = \frac{3}{4}$ and below the line $a + b = \frac{5}{4}$.

<u>Case 3: $A = 1$ and $B = 0$</u>
For C to equal $2A + 2B$, we need $C = 2$.
Since C is obtained by rounding c, then we need $\frac{3}{2} \leq c < \frac{5}{2}$.
Since $c = 2a + 2b$ by definition, this is true when $\frac{3}{2} \leq 2a + 2b < \frac{5}{2}$ or $\frac{3}{4} \leq a + b < \frac{5}{4}$.
This is the set of points in this subregion above the line $a + b = \frac{3}{4}$ and below the line $a + b = \frac{5}{4}$.

<u>Case 4: $A = 1$ and $B = 1$</u>
For C to equal $2A + 2B$, we need $C = 4$.
Since C is obtained by rounding c, then we need $\frac{7}{2} \leq c < \frac{9}{2}$.
Since $c = 2a + 2b$ by definition, this is true when $\frac{7}{2} \leq 2a + 2b < \frac{9}{2}$ or $\frac{7}{4} \leq a + b < \frac{9}{4}$.
This is the set of points in this subregion above the line $a + b = \frac{7}{4}$ and below the line $a + b = \frac{9}{4}$.

We shade the appropriate set of points in each of the subregions:



The shaded regions are the regions of points (a, b) where $2A + 2B = C$. To determine the required probability, we calculate the combined area of these regions and divide by the total area of the set of all possible points (a, b). This total area is 1, so the probability will actually be equal to the combined area of the shaded regions.

The region from Case 1 is a triangle with height $\frac{1}{4}$ and base $\frac{1}{4}$, so has area $\frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{32}$.
The region from Case 4 is also a triangle with height $\frac{1}{4}$ and base $\frac{1}{4}$. This is because the line $a + b = \frac{7}{4}$ intersects the top side of the square (the line $b = 1$) when $a = \frac{3}{4}$ and the right side of the square (the line $a = 1$) when $b = \frac{3}{4}$.
The regions from Case 2 and Case 3 have identical shapes and so have the same area. We calculate the area of the region from Case 2 by subtracting the unshaded area from the area of the entire subregion (which is $\frac{1}{4}$).
Each unshaded portion of this subregion is a triangle with height $\frac{1}{4}$ and base $\frac{1}{4}$. We can confirm this by calculating points of intersection as in Case 4.
Therefore, the area of the shaded region in Case 2 is $\frac{1}{4} - 2 \times \frac{1}{2} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$.

Therefore, the combined area of the shaded regions is $\frac{1}{32} + \frac{1}{32} + \frac{3}{16} + \frac{3}{16} = \frac{14}{32} = \frac{7}{16}$.
Thus, the required probability is $\frac{7}{16}$.

(Note that because calculating the probability is equivalent to calculating areas in this problem, we do not have to pay special attention as to whether points on the boundaries of the various regions are included or excluded.)

ANSWER: (D)