The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
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2014 Canadian Intermediate Mathematics Contest

Thursday, November 20, 2014
(in North America and South America)

Friday, November 21, 2014
(outside of North America and South America)

Solutions

Part A

1. Since there are 200 people at the beach and 65% of these people are children, then there are $\frac{65}{100} \times 200 = 65 \times 2 = 130$ children at the beach.
   Since there are 130 children at the beach and 40% of them are swimming, then there are $\frac{40}{100} \times 130 = 0.4 \times 130 = 52$ children swimming.

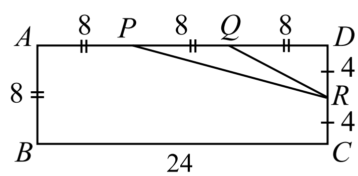
   Answer: 52

2. *Solution 1*
   Since $x + 2y = 14$ and $y = 3$, then $2x + 3y = (2x + 4y) - y = 2(x + 2y) - y = 2(14) - 3 = 25$.

   Solution 2
   Since $x+2y = 14$ and $y = 3$, then $x = 14-2y = 14-2(3) = 8$, and so $2x+3y = 2(8)+3(3) = 25$.

   Answer: 25

3. *Solution 1*
   Since $ABCD$ is a rectangle and $BC = 24$, then $AD = 24$.
   Since AD is divided into three equal parts $AP = PQ = QD$, then each part has length $24 \div 3 = 8$.
   Thus, $AB = AP = PQ = QD = 8$.
   Since $ABCD$ is a rectangle and $AB = 8$, then $DC = 8$.
   Since R is the midpoint of DC, then $DR = \frac{1}{2}(8) = 4$.



   Now $\triangle PQR$ can be viewed as having base PQ. In this case, DR is the height of $\triangle PQR$, since it is the perpendicular distance from R to the line through P and Q.
   Therefore, the area of $\triangle PQR$ is $\frac{1}{2}(PQ)(DR) = \frac{1}{2}(8)(4) = 16$.

   Solution 2
   We begin as in Solution 1 and determine that $AB = AP = PQ = QD = 8$ and $DR = 4$.
   Now, we note that the area of $\triangle PQR$ equals the area of $\triangle PDR$ minus the area of $\triangle QDR$.
   Each of these triangles is right-angled at D.
   $\triangle PDR$ has $PD = 8 + 8 = 16$ and $DR = 4$, and so has area $\frac{1}{2}(16)(4) = 32$.
   $\triangle QDR$ has $QD = 8$ and $DR = 4$, and so has area $\frac{1}{2}(8)(4) = 16$.
   Thus, the area of $\triangle PQR$ is $32 - 16 = 16$.

   Answer: 16

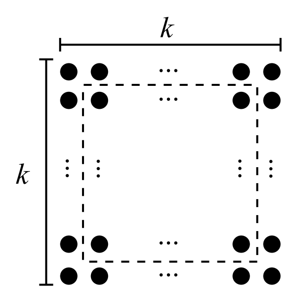
4. The initial depth of snow in Kingston is 12.1 cm and it snows at a rate of 2.6 cm per hour, so after 13 hours, the depth of snow in Kingston is $(12.1 + 2.6(13))$ cm.
The initial depth of snow in Hamilton is 18.6 cm and it snows at a rate of x cm per hour, so after 13 hours, the depth of snow in Hamilton is $(18.6 + 13x)$ cm.
Since the final depths are equal, then

$$\begin{aligned} 12.1 + 2.6(13) &= 18.6 + 13x \\ 45.9 &= 18.6 + 13x \\ 13x &= 27.3 \\ x &= 2.1 \end{aligned}$$

Therefore, $x = 2.1$.

ANSWER: 2.1

5. *Solution 1*
Suppose that the pyramid has n layers.
At each layer, the golfballs that form part of the triangular faces of the pyramid are the golfballs that are part of the perimeter of the layer.
At each layer, the golfballs that are not part of the perimeter are not part of the triangular faces.
The top layer is 1×1 and this 1 golfball is a part of the triangular faces.
The next layer is 2×2 and all 4 of these golfballs form part of the triangular faces.
The next layer is 3×3. There is 1 golfball in the middle of this layer that is not part of the triangular faces.
The next layer is 4×4. There is a 2×2 square of golfballs in the middle of this layer, each of which is not part of the triangular faces.
For the layer which is $k \times k$ (with $k > 2$), there is a $(k - 2) \times (k - 2)$ square of golfballs in the middle of this layer, each of which is not part of the triangular faces.



If the entire pyramid has n layers, then the total number of golfballs in the pyramid is

$$1^2 + 2^2 + 3^2 + 4^2 \cdots + (n-2)^2 + (n-1)^2 + n^2$$

From above, the total number of golfballs that do not form a part of the triangular faces is

$$0 + 0 + 1^2 + 2^2 + \cdots + (n-4)^2 + (n-3)^2 + (n-2)^2$$

The total number of golfballs that do form a part of the triangular faces is the difference between these two totals, which equals

$$(1^2+2^2+3^2+4^2\cdots+(n-2)^2+(n-1)^2+n^2)-(0+0+1^2+2^2+\cdots+(n-4)^2+(n-3)^2+(n-2)^2)$$

which equals $(n-1)^2+n^2$ since all other terms are added and then subtracted.
We want to determine the value of n for which the pyramid with n layers has 145 golfballs on its triangular faces.
In other words, we want to determine the value of n for which $(n-1)^2+n^2=145$.
When $n=9$, we obtain $(n-1)^2+n^2=8^2+9^2=64+81=145$.
Therefore, the pyramid has 9 layers.
(Note that when $n>2$ and as n increases, each of $(n-1)^2$ and n^2 increases, so $(n-1)^2+n^2$ increases, and so there is only one n that works.)

Solution 2
Suppose that the pyramid has n layers.
Each of the four triangular faces consists of n rows of golfballs. There is 1 golfball in the top row, 2 golfballs in the next row, and so on, with n golfballs in the bottom row.
Thus, each of the four triangular faces includes $1+2+3+\cdots+n=\frac{1}{2}n(n+1)$ golfballs.
The very top golfball is included in each of the four faces.
Each of the other golfballs along the edges of each triangular face is included in two faces. There are $n-1$ golfballs in each such edge.
To determine the total number of golfballs included in the four triangular faces in terms of n, we

- begin with 4 times the number of golfballs in one face,
- subtract 3, to account for the fact that the very top golfball is included 4 times, thus is counted 3 extra times, and
- subtract $4\times(n-1)$, to account for the fact that each golfball below the top along each of the 4 edges is included twice in the total instead of once.

Thus, the total number of golfballs included in the four triangular faces is

$$4\left(\tfrac{1}{2}n(n+1)\right)-3-4(n-1)=2n(n+1)-3-4n+4=2n^2+2n-3-4n+4=2n^2-2n+1$$

For the total number of golfballs included in the four triangular faces to be 145, we have

$$\begin{aligned}2n^2-2n+1 &= 145\\ 2n^2-2n-144 &= 0\\ n^2-n-72 &= 0\\ (n-9)(n+8) &= 0\end{aligned}$$

Therefore, $n=9$ or $n=-8$.
Since $n>0$, then $n=9$, so there are 9 layers.

Solution 3

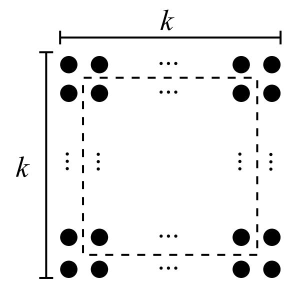
We count the number of golfballs, by level, that are included in the four triangular faces.

The top level consists of $1 \times 1 = 1$ golfball. This golfball is included in the triangular faces.

The next level consists of $2 \times 2 = 4$ golfballs. All 4 golfballs are included in the triangular faces.

The next level consists of $3 \times 3 = 9$ golfballs. Only the middle golfball of this level is not included in the triangular faces, so 8 golfballs are included in the triangular faces.

In general, for a level that is $k \times k$ with $k > 2$, how many of the golfballs are included in the triangular faces?



The golfballs on the perimeter of this square are included in the triangular faces. How many such balls are there?

We start by counting each of the balls on two opposite edges. This gives $k + k = 2k$ golfballs.

On the two remaining edges, the balls at each end are already included, leaving $k - 2$ golfballs on each of these edges to be counted.

In total, this gives $k + k + (k - 2) + (k - 2) = 4k - 4$ golfballs from this level that are included in the four triangular faces.

We make a table of the level number, the number of golfballs included in the four triangular faces at this level, and the running total of such golfballs. We stop when we get to a total of 145:

Level	Exterior golfballs	Running total
1	1	1
2	4	5
3	8	13
4	12	25
5	16	41
6	20	61
7	24	85
8	28	113
9	32	145

Therefore, 9 layers are needed to obtain a total of 145 golfballs included in the four triangular faces.

ANSWER: 9 layers

6. Let N be the largest positive integer with these properties.
   We show that $N = 619737131179$ through a number of steps.

   (i) Each digit of N after the first is 1, 3, 7 or 9:

   Each pair of consecutive digits of N must form a two-digit prime.
   Every two-digit prime number is odd and does not end in 5, since if a two-digit positive integer is even or ends in 5, it would be divisible by 2 or 5.
   We note that each pair of consecutive digits of N forms a two-digit prime, each of these primes ends in 1, 3, 7 or 9, and each digit of N after the first digit is the second digit of a two-digit prime.
   Thus, each digit of N after the first is 1, 3, 7 or 9.

   (ii) When the two-digit primes formed by pairs of consecutive digits of N are listed, each prime but the first has each digit equal to 1, 3, 7 or 9:

   This is because all digits of N after the first are odd.
   We have not yet discussed the first prime generated by N, so we cannot make this conclusion about this first prime.

   (iii) There are 10 two-digit prime numbers with each digit equal to 1, 3, 7 or 9:

   These are $11, 13, 17, 19, 31, 37, 71, 73, 79, 97$.

   (iv) N has at most 12 digits:

   The number of digits in N is one more than the number of two-digit primes in the list generated by N, since these primes are formed starting with each digit of N except the last.
   The number of primes in this list is at most one more than the number of primes with each digit equal to 1, 3, 7 or 9, since all but the first prime in the list has this property.
   There are 10 primes with this property, so at most 11 primes in the list, so there are at most 12 digits in N.

   (v) For N to have 12 digits, all 10 primes with each digit equal to 1, 3, 7 or 9 must be part of the list generated by N:

   If this were not the case, then by (iv), N would have fewer than 12 digits.

   (vi) For all 10 primes with each digit equal to 1, 3, 7 or 9 to be part of the list generated by N, then the second digit of N equals 1 and the last digit of N equals 9:

   These 10 primes form the entire list of primes, except for the first in the list.
   Therefore, these primes are generated by all of the digits of N except for the first digit.
   We need to try to put these 10 primes in an order so that the second digit of one prime equals the first digit of the next, so that these primes can be generated by a sequence of 11 digits.
   Of the primes in the list,

   - 4 begin with 1 and 3 end with 1
   - 2 begin with 3 and 2 end with 3
   - 3 begin with 7 and 3 end with 7
   - 1 begins with 9 and 2 end with 9

   Thus, the first prime in the list of 10 begins with 1 (since there is one first digit of 1 that cannot be paired with a second digit of 1) and end with 9 (since there

is one second digit of 9 that cannot be paired with a first digit).
(Note that the number of beginning and ending 3s and 7s match.)
Therefore, N is $\_1\_\_\_\_\_\_\_\_\_9$.

(vii) If N has 12 digits, then N has the form $61\_\_\_\_\_\_\_\_\_9$:

For N to be the largest possible 12 digit number, it is necessary for its first digit to be as large as possible.
Since the first two digits of N, $\_1$, need to form a two-digit prime, then these digits could be 11, 31, 41, 61, or 71.
Since 71 will be used already in the list of 10 primes, then this initial prime cannot be 71 and so must be 61 to make N has large as possible.
Thus, N has the form $61\_\_\_\_\_\_\_\_\_9$.

(viii) $N = 619737131179$

To maximize N, we put the largest possible third digit 9 (using the prime 19) and then the largest possible fourth digit 7 (using the prime 97).
This gives $6197\_\_\_\_\_\_\_9$.
There is only one other prime ending with 9 (namely, 79) so the second last digit is 7, giving $6197\_\_\_\_\_\_79$.
The maximum possible digit for the fifth position is now 3 (using the prime 73 since 79 is already used), giving $61973\_\_\_\_\_79$.
The maximum possible digit for the sixth position is now 7 (using the prime 37), giving $619737\_\_\_\_79$.
The maximum possible digit for the seventh position is now 1 (using the prime 71), giving $6197371\_\_\_79$.
We have used the primes 19, 97, 73, 37, 71, 79, which leaves the primes 11, 13, 17, 31.
The only remaining prime ending in 7 is 17, so the third last digit must be 1, giving $6197371\_\_179$.
The maximum possible digit for the eighth position is now 3 (using the prime 13 since 17 and 19 are already used), giving 61973713_179.
The only possible digit for the ninth position is now 1 (using the primes 31 and 11), giving 619737131179.

Because we have constructed this integer so that it has as many digits as possible and so that the largest digits possible are in the positions with largest place value, then this is N, the largest positive integer satisfying the required properties.
Therefore, $N = 619737131179$.

ANSWER: $N = 619737131179$

Part B

1. (a) The average of the 6 integers given is $\dfrac{22+23+23+25+26+31}{6} = \dfrac{150}{6} = 25$.

   (b) Since the average of the three numbers $y+7$, $2y-9$ and $8y+6$ is 27, then the sum of the three numbers is $3(27) = 81$.
   Therefore, $(y+7)+(2y-9)+(8y+6) = 81$ or $11y+4 = 81$, and so $11y = 77$ or $y = 7$.

   (c) Since the average of four integers is 94, then their sum is $4(94) = 376$.
   Since the sum of the integers is constant, then for one of the integers to be as small as possible, the other three integers must be as large as possible.
   To see this algebraically, we can call the four integers a, b, c, d with a the smallest.
   Since $a+b+c+d = 376$, then $a = 376-b-c-d$.
   To make a as small as possible, we want to subtract as much as possible from 376.
   Since each of the four integers is smaller than 100, then the largest that each can be is 99.
   Therefore, to make one of the four integers as small as possible, we set the other three integers equal to 99.
   This means that the fourth integer equals $376-3(99) = 376-297 = 79$.

2. (a) Since $\triangle PQR$ is right-angled at R, we can apply the Pythagorean Theorem to say that $PR^2+RQ^2 = PQ^2$.
   Since $RQ = 24$ and $PQ = 25$, then $PR^2 = 25^2-24^2 = 625-576 = 49$.
   Since $PR > 0$, then $PR = 7$.
   The perimeter of $\triangle PQR$ is $PR+RQ+PQ = 7+24+25 = 56$.
   Since $\triangle PQR$ is right-angled at R, its area equals $\frac{1}{2}(PR)(RQ) = \frac{1}{2}(7)(24) = \frac{1}{2}(168) = 84$.

   (b) Since the perimeter of $\triangle ABC$ is 144 and its side lengths are a, b and c, then $a+b+c = 144$.
   Since $\triangle ABC$ is right-angled at C and its area is 504, then $\frac{1}{2}(CB)(AC) = 504$ or $\frac{1}{2}ab = 504$ or $ab = 1008$.
   Since $a+b+c = 144$, then

$$\begin{aligned}
a+b &= 144-c \\
(a+b)^2 &= (144-c)^2 \qquad \text{(squaring both sides)} \\
a^2+2ab+b^2 &= c^2-288c+144^2 \qquad \text{(expanding)} \\
(a^2+b^2)+2ab &= c^2-288c+144^2 \\
2ab &= -288c+144^2 \qquad \text{(since } a^2+b^2=c^2\text{)} \\
2(1008) &= -288c+144^2 \qquad \text{(since } ab=1008\text{)} \\
288c &= 144^2-2(1008) \\
144(2c) &= 144^2-2(7)(144) \\
2c &= 144-2(7) \\
c &= 65
\end{aligned}$$

   Therefore, $c = 65$.
   (We note that when $c = 65$, we obtain $a+b = 144-65 = 79$ and $ab = 1008$ which can be solved to obtain $a = 16$ and $b = 63$ or $a = 63$ and $b = 16$. We can verify that $16^2+63^2 = 65^2$ so we do obtain a right-angled triangle.)

3. (a) The initial list includes 1 digit equal to 0, 0 digits equal to 1, 1 digit equal to 2, and 2 digits equal to 3.
Therefore, the list produced by the machine is $(1, 0, 1, 2)$.

(b) Suppose Vicky inputs the list (a, b, c, d) and the machine outputs the identical list (a, b, c, d).
The fact that the output is (a, b, c, d) tells us that the input consists a 0s, b 1s, c 2s, and d 3s.
Since the input consists of 4 digits, each equal to 0, 1, 2, or 3, then the sum of the number of 0s (a), the number of 1s (b), the number of 2s (c), and the number of 3s (d) in the input is 4, and so $a + b + c + d = 4$.
Since the input actually equals (a, b, c, d), then the sum of the digits in the input is 4.
Now, we add up the digits of the input in a second way.
Since the output equals (a, b, c, d), the input consists of a 0s, b 1s, c 2s, and d 3s.
The a 0s contribute $0a$ to the sum of the digits, the b 1s contribute $1b$, the c 2s contribute $2c$, and the d 3s contribute $3d$, and so the sum of the digits in the input also equals $0a + 1b + 2c + 3d$.
Since the sum of the digits in the input is 4, then $0a + 1b + 2c + 3d = 4$.
Therefore, $b + 2c + 3d = 4$ for any input (a, b, c, d) which produces the identical output (a, b, c, d).

(c) We determine all possible lists (a, b, c, d) that Vicky could input to produce the identical output (a, b, c, d).
From (b), we must have $a + b + c + d = 4$ and $b + 2c + 3d = 4$.
Note that each of a, b, c, d is a non-negative integer.
Since $b + 2c + 3d = 4$, then $d = 0$ or $d = 1$. (If $d \geq 2$, the left side would be too large.)
If $d = 1$, then $b + 2c + 3 = 4$ or $b + 2c = 1$. Since b and c are non-negative integers, then we must have $b = 1$ and $c = 0$.
From the first equation, $a + b + c + d = 4$ and so $a + 1 + 0 + 1 = 4$ or $a = 2$.
This would give $(a, b, c, d) = (2, 1, 0, 1)$, but this input produces the output $(1, 2, 1, 0)$, and so does not satisfy the requirements.
Thus, we cannot have $d = 1$.
(We note here that if (a, b, c, d) is a list with the desired property, then $a+b+c+d = 4$ and $b+2c+3d = 4$, but it is not necessarily true that if $a+b+c+d = 4$ and $b+2c+3d = 4$, then the list (a, b, c, d) has the required property.)
Since d cannot be 1, then $d = 0$, giving $a + b + c = 4$ and $b + 2c = 4$.
Using the second equation, $c = 0$ or $c = 1$ or $c = 2$.
If $c = 0$, then $b = 4$. However, none of the digits can be larger than 3, so this is not possible.
If $c = 1$, then $b = 2$. In this case, $a+2+1 = 4$ and so $a = 1$, giving $(a, b, c, d) = (1, 2, 1, 0)$.
Using this list as input gives output $(1, 2, 1, 0)$, as required.
If $c = 2$, then $b = 0$. In this case, $a+0+2 = 4$ and so $a = 2$, giving $(a, b, c, d) = (2, 0, 2, 0)$.
Using this list as input gives output $(2, 0, 2, 0)$, as required.
We have exhausted all possibilities, so the lists that Vicky could input that produce identical output are $(1, 2, 1, 0)$ and $(2, 0, 2, 0)$.

(d) We show that there is exactly one such list L.
First, we model the work from (b) to determine similar equations that will help with our work.
Suppose Vicky enters the list $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ and the machine produces the identical list $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$.
(Here, each of $a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ is a digit between 0 and 9 inclusive. The smaller numbers set below the main line of text are called *subscripts* and are sometimes used to give more meaningful names to unknown quantities.)
The fact that the output is $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ tells us that the input includes a_0 0s and a_1 1s and a_2 2s, and so on. Since the input consists of a total of 10 digits, then $a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 10$.
This means that the sum of the digits in the input is 10.
Again, the output $(a_0, a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9)$ tells us that the input consists of a_0 0s, a_1 1s, a_2 2s, and so on.
We can add up the digits of the input in a second way by saying that the a_0 0s contribute $0a_0$ to the sum of its digits, the a_1 1s contribute $1a_1$, the a_2 2s contribute $2a_2$, and so on.
Thus, the sum of the digits of the input equals

$$0a_0 + 1a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7 + 8a_8 + 9a_9$$

Since we already know that the sum of the digits in the input is 10, then

$$0a_0 + 1a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7 + 8a_8 + 9a_9 = 10$$

Now, we determine the possible lists L that satisfy the desired condition.
Suppose that $a_0 = k$. This is the first digit in the input and the first digit in the output.
Since the output L includes $a_0 = k$, then by the definition of a_0, the input L must include k digits equal to 0.
Note that $k \neq 0$, since if $k = 0$, there would be at least 1 digit equal to 0 in the input, so the k in the output would be at least 1, which would contradict $k = 0$.
Since $1a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7 + 8a_8 + 9a_9 = 10$, then at most 4 of the digits $a_1, a_2, a_3, a_4, a_5, a_6, a_7, a_8, a_9$ can be non-zero. This is true because if 5 of these digits were non-zero, the five non-zero terms in the sum on the left side would be at least $1 + 2 + 3 + 4 + 5$ which is larger than 10.
Therefore, at most 4 of the digits in L after $a_0 = k$ are non-zero, so at least 5 of the digits in L after $a_0 = k$ equal 0, which tells us that $k \geq 5$.

If $k = 5$, then the last 9 digits of L include exactly 5 zeros, which means that the last 9 digits include exactly 4 non-zero digits.
Since $1a_1+2a_2+3a_3+4a_4+5a_5+6a_6+7a_7+8a_8+9a_9 = 10$ and the sum of the 4 non-zero terms is at least $1+2+3+4 = 10$, then the sum of the 4 non-zero terms must be exactly $1 + 2 + 3 + 4 = 10$.
This means that $a_1 = a_2 = a_3 = a_4 = 1$ and $a_5 = a_6 = a_7 = a_8 = a_9 = 0$.
But an input $(5, 1, 1, 1, 1, 0, 0, 0, 0, 0)$ gives output $(5, 4, 0, 0, 0, 1, 0, 0, 0, 0)$ which is not equal to the input.
Thus, $k \neq 5$, and so $k \geq 6$.

Since $a_0 = k$ and $k \geq 6$, then the input L includes at least one digit equal to k, so the output L tells us that $a_k \neq 0$.
Since $1a_1 + 2a_2 + 3a_3 + 4a_4 + 5a_5 + 6a_6 + 7a_7 + 8a_8 + 9a_9 = 10$, then $ka_k \leq 10$.
Since $k \geq 6$, then $a_k \leq 1$ (otherwise, $ka_k > 10$). Since $a_k \neq 0$, then $a_k = 1$.

Consider a_1 next.
Since $a_k = 1$, this fact from the input L tells us that $a_1 \geq 1$ in the output L.
If $a_1 = 1$, then the input L would include at least two 1s (a_1 and a_k), so in the output version of L, $a_1 \geq 2$, which disagrees with the input.
Therefore, $a_1 \neq 1$, and so $a_1 \geq 2$.

Since we know that $a_0 = k$ and $k \geq 6$ and $a_0+a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9 = 10$, then $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 \leq 4$.
Since $a_k = 1$ already (and $k \geq 6$), then we must have $a_1 \leq 3$.
If $a_1 = 3$, then the a_1 in the output would tell us that the input L includes at least three 1s, which would give $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 \geq 3 + 1 + 1 + 1 = 6$, which is not possible.
Since $2 \leq a_1 \leq 3$ and $a_1 \neq 3$, then $a_1 = 2$.

Therefore, the input L includes at least one 2, and so the output L has $a_2 \geq 1$.
We know that $a_1 = 2$ and $a_2 \geq 1$ and $a_k = 1$ and $a_1+a_2+a_3+a_4+a_5+a_6+a_7+a_8+a_9 \leq 4$.
Thus, it must be the case that $a_2 = 1$, otherwise $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9$ would be too large.
Since there are exactly 4 non-zero digits, we can also conclude that all of the entries other than a_0, a_1, a_2, and a_k must equal 0.
Since $a_0 + a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9 = 10$, then

$$k = a_0 = 10 - (a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7 + a_8 + a_9) = 10 - (2 + 1 + 1) = 6$$

Thus, the only possible input L that produces the same output L is $(6, 2, 1, 0, 0, 0, 1, 0, 0, 0)$.
This list includes six 0s, two 1s, one 2, and one 6, so the output matches the input.
Therefore, there is exactly one list L that can be input to produce the same output. This list is $(6, 2, 1, 0, 0, 0, 1, 0, 0, 0)$.