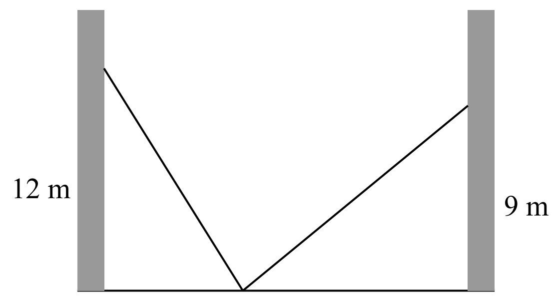
The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
*cemc.uwaterloo.ca*

# 2015 Canadian Team Mathematics Contest

## Individual Problems

1. What is the smallest integer $n$ for which $5 + 3n$ is larger than 300?

2. Kim places two very long (and very heavy) ladders, each 15 m long, on a flat floor between two vertical and parallel walls. Each ladder leans against one of the walls. The two ladders touch the floor at exactly the same place. One ladder reaches 12 m up one wall and the other ladder reaches 9 m up the other wall. In metres, how far apart are the walls?




3. In a group of 20 friends, 11 like to ski, 13 like to snowboard, and 3 do not like to do either. How many of the friends like to both ski and snowboard?

4. The pair $(x, y) = (2, 5)$ is the solution of the system of equations

$$\begin{aligned} ax + 2y &= 16 \\ 3x - y &= c \end{aligned}$$

Determine the value of $\frac{a}{c}$.

5. What is the smallest two-digit positive integer $k$ for which the product $45k$ is a perfect square?

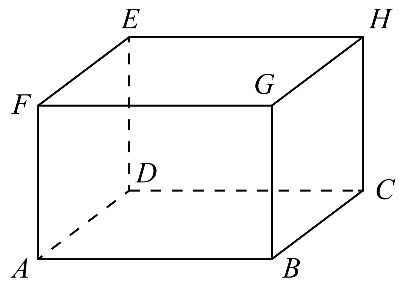
6. Clara leaves home by bike at 1:00 p.m. for a meeting scheduled with Quinn later that afternoon. If Clara travels at an average of 20 km/h, she would arrive half an hour before their scheduled meeting time. If Clara travels at an average of 12 km/h, she would arrive half an hour after their scheduled meeting time. At what average speed, in km/h, should Clara travel to meet Quinn at the scheduled time?

7. Each entry in the list below is a positive integer:

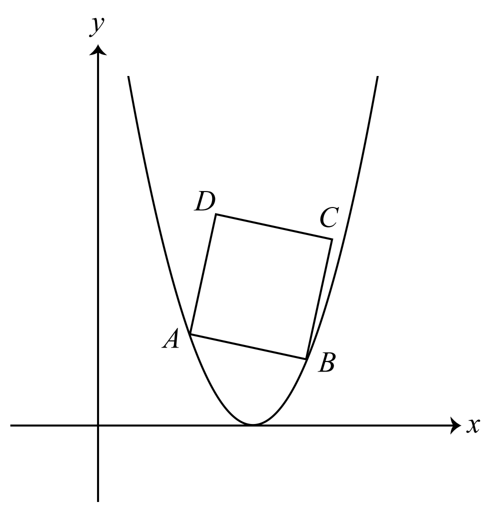
$$a, 8, b, c, d, e, f, g, 2$$

If the sum of any four consecutive terms in the list is 17, what is the value of $c + f$?

8. The sum of the lengths of all of the edges of rectangular prism $ABCDEFGH$ is 24. If the total surface area of the prism is 11, determine the length of the diagonal $AH$.




9. In the diagram, $ABCD$ is a square. Points $A(1, 4)$ and $B$ are on a parabola that is tangent to the $x$-axis. If $C$ has coordinates $\left(\frac{39}{4}, \frac{37}{4}\right)$, determine the equation of the parabola.




10. Determine the sum of all positive integers $N < 1000$ for which $N + 2^{2015}$ is divisible by 257.

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## 2015 Canadian Team Mathematics Contest

## Team Problems

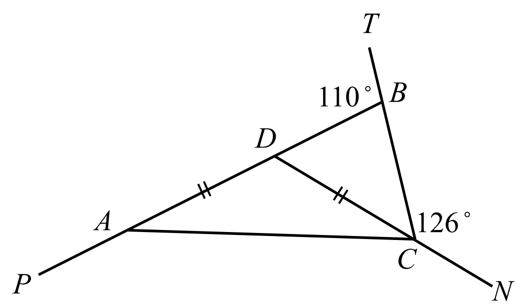
1. What is the value of $1 + 3 + 5 + 7 + 9 + 11 + 13 + 15 + 17 + 19$ ?

2. A cube has an edge length of 10. The length of each edge is increased by 10%. What is the volume of the resulting cube?

3. A cylinder has height 4. Each of its circular faces has a circumference of $10\pi$. What is the volume of the cylinder?

4. In how many different ways can 22 be written as the sum of 3 different prime numbers? That is, determine the number of triples $(a, b, c)$ of prime numbers with $1 < a < b < c$ and $a+b+c = 22$.

5. In the diagram, $PADB$, $TBC$ and $NCD$ are straight line segments. If $\angle TBD = 110°$, $\angle BCN = 126°$, and $DC = DA$, determine the measure of $\angle PAC$.



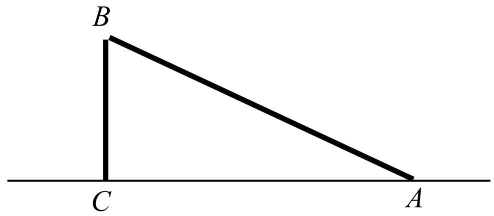

6. For how many one-digit positive integers $k$ is the product $k \cdot 234$ divisible by 12?

7. The points $A(5, -8)$, $B(9, -30)$ and $C(n, n)$ are collinear (that is, lie on the same straight line). What is the value of $n$?

8. What is the difference between the largest possible three-digit positive integer with no repeated digits and the smallest possible three-digit positive integer with no repeated digits?

9. Determine the number of pairs $(x, y)$ of positive integers for which $0 < x < y$ and $2x+3y = 80$.

10. A telephone pole that is 10 m tall was struck by lightning and broken into two pieces. The top piece, $AB$, has fallen down. The top of the pole is resting on the ground, but it is still connected to the main pole at $B$. The pole is still perpendicular to the ground at $C$. If the angle between $AB$ and the flat ground is $30°$, how high above the ground is the break (that is, what is the length of $BC$)?



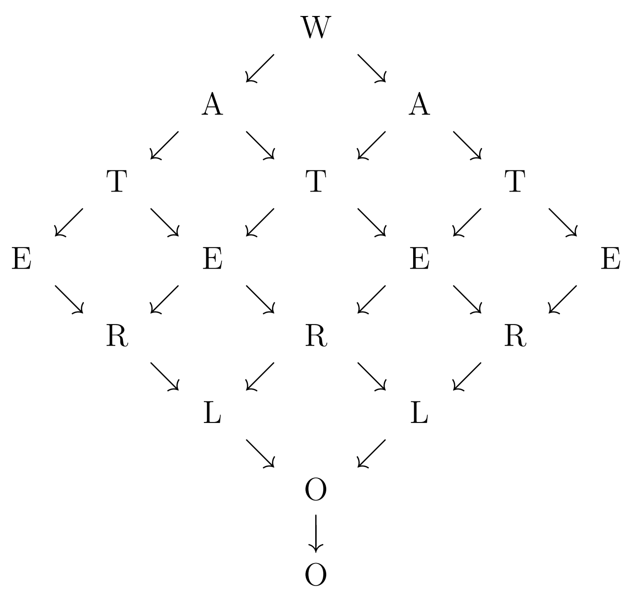

11. If $a = 2^3$ and $b = 3^2$ evaluate $\dfrac{(a-b)^{2015} + 1^{2015}}{(a-b)^{2015} - 1^{2015}}$ .

12. What is the largest perfect square that can be written as the product of three different one-digit positive integers?

13. A moving sidewalk runs from Point $A$ to Point $B$. When the sidewalk is turned off (that is, is not moving) it takes Mario 90 seconds to walk from Point $A$ to Point $B$. It takes Mario 45 seconds to be carried from Point $A$ to Point $B$ by the moving sidewalk when he is not walking. If his walking speed and the speed of the moving sidewalk are constant, how long does it take him to walk from Point $A$ to Point $B$ along the moving sidewalk when it is moving?

14. A square has side length $s$ and a diagonal of length $s + 1$. Write the area of the square in the form $a + b\sqrt{c}$ where $a$, $b$ and $c$ are positive integers.

15. In the diagram, determine the number of paths that follow the arrows and spell the word "WATERLOO".




16. What is the measure, in degrees, of the smallest positive angle $x$ for which $4^{\sin^2 x} \cdot 2^{\cos^2 x} = 2\sqrt[4]{8}$ ?

17. If $\log_{3n} 675\sqrt{3} = \log_n 75$, determine the value of $n^5$.

18. The roots of $x^2 + bx + c = 0$ are the squares of the roots of $x^2 - 5x + 2 = 0$. What is the value of $\frac{c}{b}$?

19. Zach has twelve identical-looking chocolate eggs. Exactly three of the eggs contain a special prize inside. Zach randomly gives three of the twelve eggs to each of Vince, Wendy, Xin, and Yolanda. What is the probability that only one child will receive an egg that contains a special prize (that is, that all three special prizes go to the same child)?

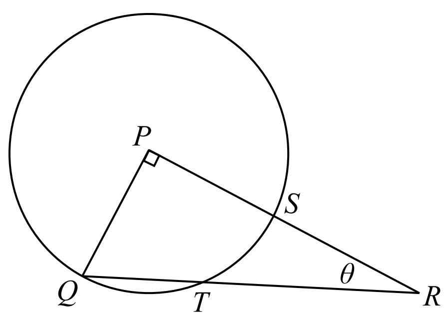
20. Define $f(x) = \dfrac{x^2}{1+x^2}$ and suppose that

$$\begin{aligned} A &= f(1) + f(2) + f(3) + \cdots + f(2015) \\ B &= f(1) + f\left(\tfrac{1}{2}\right) + f\left(\tfrac{1}{3}\right) + \cdots + f\left(\tfrac{1}{2015}\right) \end{aligned}$$

(Each sum contains 2015 terms.) Determine the value of $A + B$.

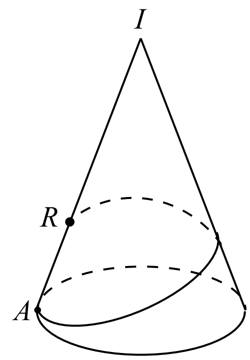
21. For each positive integer $n$, define the point $P_n$ to have coordinates $((n-1)^2, n(n-1))$ and the point $Q_n$ to have coordinates $((n-1)^2, 0)$. For how many integers $n$ with $2 \le n \le 99$ is the area of trapezoid $Q_nP_nP_{n+1}Q_{n+1}$ a perfect square?

22. In the diagram, $\triangle PQR$ is right-angled at $P$ and $\angle PRQ = \theta$. A circle with centre $P$ is drawn passing through $Q$. The circle intersects $PR$ at $S$ and $QR$ at $T$. If $QT = 8$ and $TR = 10$, determine the value of $\cos\theta$.




23. Suppose that $n$ is a positive integer and that the set $S$ contains exactly $n$ distinct positive integers. If the mean of the elements of $S$ is equal to $\frac{2}{5}$ of the largest element of $S$ and is also equal to $\frac{7}{4}$ of the smallest element of $S$, determine the minimum possible value of $n$.

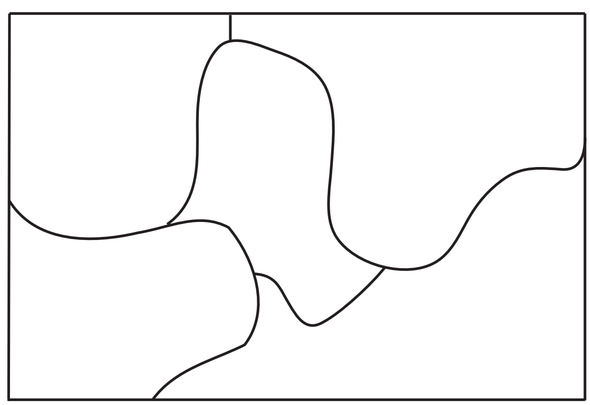
24. A circular cone has vertex $I$, a base with radius 1, and a slant height of 4. Point $A$ is on the circumference of the base and point $R$ is on the line segment $IA$ with $IR = 3$. Shahid draws the shortest possible path starting at $R$, travelling once around the cone, and ending at $A$. If $P$ is the point on this path that is closest to $I$, what is the length $IP$?




25. Each of the five regions in the figure below is to be labelled with a unique integer taken from the set $\{1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 14\}$. The labelling is to be done so that if two regions share a boundary and these regions are labelled with the integers $a$ and $b$, then

- $a$ and $b$ are not both multiples of 2,
- $a$ and $b$ are not both multiples of 3, and
- $a$ and $b$ are not both multiples of 5.

In how many ways can the regions be labelled?



0 (a). Evaluate $2 + 0 + 1 + 5$.

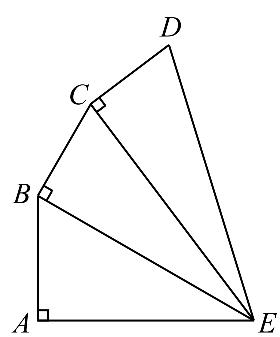
0 (b). Let $t$ be TNYWR.
The average of the five numbers $12, 15, 9, 14, 10$ is $m$.
The average of the four numbers $24, t, 8, 12$ is $n$.
What is the value of $n - m$?

0 (c). Let $t$ be TNYWR.
The lines with equations $y = 13$ and $y = 3x + t$ intersect at the point $(a, b)$. What is the value of $a$?

1 (a). If $2^{k+4} = 1024$, what is the value of $k$?

1 (b). Let $t$ be TNYWR.
If $2t + 2x - t - 3x + 4x + 2t = 30$, what is the value of $x$?

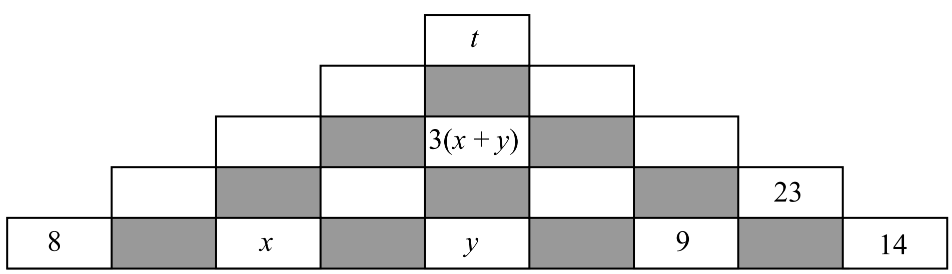
1 (c). Let $t$ be TNYWR.
In the diagram, $\angle BAE = \angle CBE = \angle DCE = 90°$. If $AE = \sqrt{5}$, $AB = \sqrt{4}$, $BC = \sqrt{3}$, and $CD = \sqrt{t}$, what is the length of $DE$?



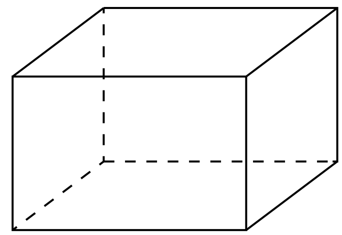
2 (a). $\triangle ABC$ has vertices $A(-1, 2)$, $B(5, 2)$ and $C(-4, -3)$. What is the area of $\triangle ABC$?

2 (b). Let $t$ be TNYWR.
In last night's 75 minute choir rehearsal, Canada's Totally Musical Choir spent 6 minutes warming up, 30 minutes learning notes, $t$ minutes learning words, and the rest of the rehearsal singing their pieces. If the choir spent $N\%$ of the rehearsal singing their pieces, what is the value of $N$?

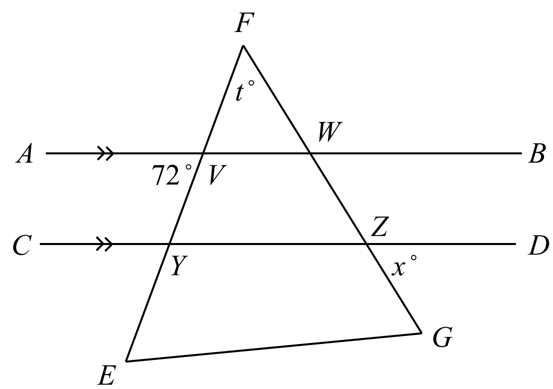
2 (c). Let $t$ be TNYWR.
In the diagram, the number that goes in each unshaded box above the bottom row is the sum of the numbers in the two unshaded boxes immediately below to the left and to the right. For example, $23 = 9 + 14$. What is the value of $x - y$?



3 (a). What is the surface area of a rectangular prism with edge lengths of 2, 3 and 4?



3 (b). Let $t$ be TNYWR.
In the diagram, line segments $AB$ and $CD$ are parallel. $AB$ intersects $EF$ at $V$ and $GF$ at $W$. $CD$ intersects $EF$ at $Y$ and $GF$ at $Z$. If $\angle AVE = 72°$, $\angle EFG = t°$, and $\angle GZD = x°$, what is the value of $x$?




3 (c). Let $t$ be TNYWR.
Determine the number of integers $b > 0$ for which $30t$ is divisible by $b!$ .

(If $n$ is a positive integer, the symbol $n!$ (read "$n$ factorial") represents the product of the integers from 1 to $n$. For example, $4! = (1)(2)(3)(4)$ or $4! = 24$.)