



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

2021 Canadian Team Mathematics Contest

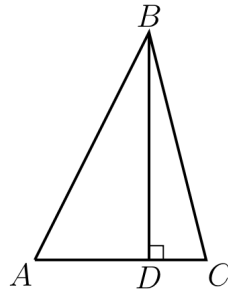
Individual Problems (45 minutes)

IMPORTANT NOTES:

- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

1. Determine the largest 6-digit positive integer that is divisible by 5.
2. In the diagram, $\triangle ABC$ has an area of 84 and $AC = 12$. Point D is on AC so that BD is perpendicular to AC . What is the length of BD ?

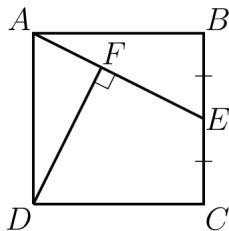


3. Below are five facts about the ages of five students, Adyant, Bernice, Cici, Dara, and Ellis.
 - Adyant is older than Bernice.
 - Dara is the youngest.
 - Bernice is older than Ellis.
 - Bernice is younger than Cici.
 - Cici is not the oldest.

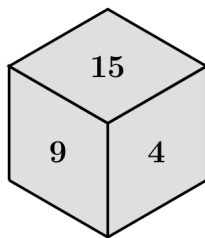
Determine which of the five students is the third oldest.

4. For non-zero integers x and y we define $x \nabla y = x^y + 4x$. For example, $2 \nabla 3 = 2^3 + 4(2) = 16$. Determine all real numbers x such that $x \nabla 2 = 12$.
5. The equations $x - 2y - 3 = 0$ and $18x - k^2y - 9k = 0$ represent two lines. For some real number k , these two lines are distinct and parallel. Determine the value of k .

6. Square $ABCD$ has sides of length 2. The midpoint of BC is E . Point F is on AE so that DF is perpendicular to AE . Determine the length of DF .

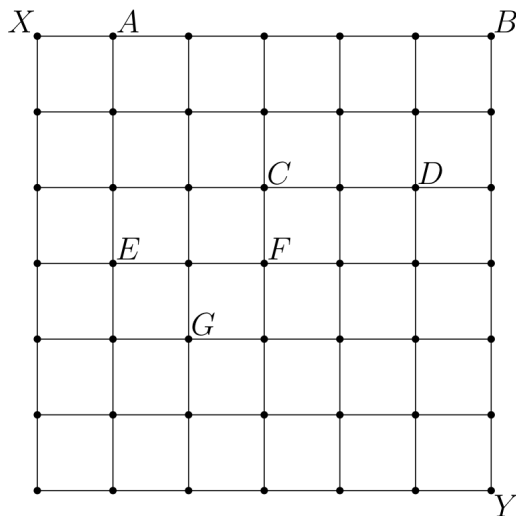


7. Determine the number of ordered pairs of positive integers (a, b) for which $20a + 21b = 2021$.
8. Amanda has two identical cubes. Each cube has one integer on each face so that the following statements are all true:
- Three adjacent faces of each cube have the numbers 15, 9 and 4 as shown.
 - The numbers on all pairs of opposite faces have the same sum s .
 - When both cubes are rolled and the numbers on the top faces are added, the probability that the sum equals 24 is $\frac{1}{12}$.



Determine the sum of all possible values of s .

9. Celine traces paths on the grid below starting at point X and ending at point Y . Each path must follow the lines connecting the dots and only ever move horizontally to the right or vertically down. It may be useful to know that there are a total of 924 such paths. Consider the 7 points labelled A , B , C , D , E , F , and G . List these points in decreasing order of the number of paths passing through that point. For example, B is on exactly one path, so B should be the last point in your list.



10. $ABCDE$ is a pyramid with square base $ABCD$. Point E is directly above A with $AE = 1024$ and $AB = 640$. The pyramid is cut into two pieces by a horizontal plane parallel to $ABCD$. This horizontal plane is a distance h above the base $ABCD$. The portion of $ABCDE$ that is above the plane is a new pyramid. For how many integers h is the volume of the new pyramid an integer?

