The CENTRE for EDUCATION in MATHEMATICS and COMPUTING

cemc.uwaterloo.ca

2023 Canadian Team Mathematics Contest

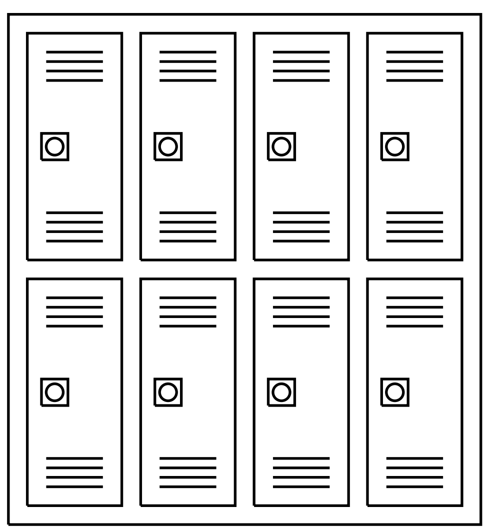
Individual Problems (45 minutes)

IMPORTANT NOTES:

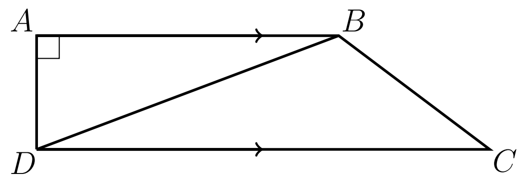
- Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) previously stored information such as formulas, programs, notes, etc., (iv) a computer algebra system, (v) dynamic geometry software.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

PROBLEMS:

1. Ingrid starts with n chocolates, while Jin and Brian each start with 0 chocolates. Ingrid gives one third of her chocolates to Jin. Jin gives 8 chocolates to Brian and then Jin eats half of her remaining chocolates. Jin now has 5 chocolates. What is the value of n?
2. For what value of k is $k\%$ of 25 equal to 20% of 30?
3. It is now 1:00 a.m. What time will it be 2023 minutes from now?
4. A group of eight students have lockers that are arranged as shown, in two rows of four lockers with one row directly on top of the other. The students are allowed to paint their lockers either blue or red according to two rules. The first rule is that there must be two blue lockers and two red lockers in each row. The second rule is that lockers in the same column must have different colours. How many ways are there for the students to paint their lockers according to the rules?

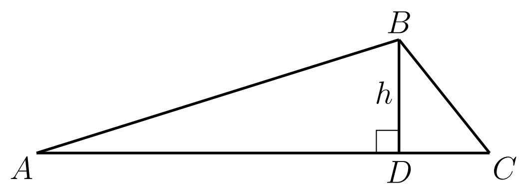


5. In trapezoid $ABCD$, $AB = 4$, $CD = 6$, $\angle DAB = 90°$, $\angle BCD = 45°$, and AB is parallel to CD. What is the length of BD?



6. A train is traveling from City A to City B. If the train travels at a speed of 80 km/h, it will arrive 24 minutes late. If it travels at a speed of 90 km/h, it will arrive 32 minutes early. At what speed in km/h should the train travel in order to arrive on time?

7. In $\triangle ABC$, $\tan \angle BCA = 1$ and $\tan \angle BAC = \frac{1}{7}$. The perimeter of $\triangle ABC$ is $24 + 18\sqrt{2}$. The altitude from B to AC has length h and intersects AC at D. What is the value of h?



8. A *Tim number* is a five-digit positive integer with the property that it is a multiple of 15, its hundreds digit is 3, and its tens digit is equal to the sum of its first (leftmost) three digits. How many Tim numbers are there?

9. The real numbers x, y and z satisfy both of the equations below:

$$4x + 7y + z = 11$$
$$3x + y + 5z = 15$$

Given that $x + y + z = \frac{p}{q}$ where p and q are positive integers and the fraction $\frac{p}{q}$ is in lowest terms, what is the value of $p - q$?

10. For every positive integer n, let $S_n = \{1, 2, 3, \ldots, n\}$; that is, S_n is the set of integers from 1 to n inclusive. There are 2^n subsets of S_n. If each subset has the same likelihood of being chosen, let $p(n)$ be the probability that a chosen subset does not contain two integers with a sum of $n + 1$.

For example, the subsets of S_2 are $\emptyset$ (the empty set), $\{1\}$, $\{2\}$, and $\{1, 2\}$. Of these four subsets, only $\{1, 2\}$ contains a pair of integers with a sum of $2 + 1 = 3$. The other three subsets do not contain such a pair, so $p(2) = \frac{3}{4}$.

What is the smallest *even* positive integer n for which $p(n) < \frac{1}{4}$?