The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Wednesday, February 22, 2023
(in North America and South America)

Thursday, February 23, 2023
(outside of North America and South America)

UNIVERSITY OF WATERLOO

Time: 60 minutes

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as "7") must be coded with a leading zero ("07").
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is *no penalty* for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring:	There is *no penalty* for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $0.3 + 0.03$ is

   (A) 0.303 **(B)** 0.6 **(C)** 3.3 **(D)** 0.33 **(E)** 0.06

2. If $3 + x = 5$ and $-3 + y = 5$, then $x + y$ equals

   (A) 4 **(B)** 16 **(C)** 6 **(D)** 12 **(E)** 10

3. When $x = 2$, the expression $2x^2 + 3x^2$ equals

   (A) 14 **(B)** 10 **(C)** 12 **(D)** 22 **(E)** 20

4. The number of minutes in a week is closest to

   (A) 100 **(B)** 1000 **(C)** 10 000 **(D)** 100 000 **(E)** 1 000 000

5. Ava's machine takes four-digit positive integers as input. When the four-digit integer $ABCD$ is input, the machine outputs the integer $A \times B + C \times D$. For example, when the input is 1234, the output is $1 \times 2 + 3 \times 4 = 2 + 12 = 14$. When the input is 2023, the output is

   (A) 0 **(B)** 2 **(C)** 3 **(D)** 6 **(E)** 8

6. Vivek is painting three doors. The doors are numbered 1, 2 and 3. Each door is to be painted one colour: either black or gold. One possibility is that door 1 is painted black, door 2 is painted gold, and door 3 is painted gold. In total, how many different ways can the three doors be painted?

   (A) 8 **(B)** 7 **(C)** 5 **(D)** 4 **(E)** 3

7. Snacks are purchased for 17 soccer players. Juice boxes come in packs of 3 and cost \$2.00 per pack. Apples come in bags of 5 and cost \$4.00 per bag. Danny buys packs of juice boxes and bags of apples so that every player gets a juice box and an apple. What is the minimum amount of money that Danny spends?

   (A) \$26.00 **(B)** \$28.00 **(C)** \$24.00 **(D)** \$30.00 **(E)** \$36.00

8. A bicycle trip is 30 km long. Ari rides at an average speed of 20 km/h. Bri rides at an average speed of 15 km/h. If Ari and Bri begin at the same time, how many minutes after Ari finishes the trip will Bri finish?

   (A) 50 min **(B)** 40 min **(C)** 30 min **(D)** 20 min **(E)** 10 min

9. Three tanks contain water. The number of litres in each is shown in the table below:

Tank A	Tank B	Tank C
3600 L	1600 L	3800 L

Water is moved from each of Tank A and Tank C into Tank B so that each tank contains the same volume of water. How many litres of water are moved from Tank A to Tank B?

(A) 500 L **(B)** 600 L **(C)** 700 L **(D)** 800 L **(E)** 900 L

10. Points A, B, C, and D lie along a line, in that order. If $AB : AC = 1 : 5$, and $BC : CD = 2 : 1$, then the ratio $AB : CD$ is equal to

(A) $1 : 1$ **(B)** $1 : 2$ **(C)** $1 : 3$ **(D)** $2 : 5$ **(E)** $3 : 5$

Part B: Each correct answer is worth 6.

11. At the start of this month, Mathilde and Salah each had 100 coins. For Mathilde, this was 25% more coins than she had at the start of last month. For Salah, this was 20% fewer coins than he had at the start of last month. The total number of coins that they had at the start of last month was

(A) 180 **(B)** 185 **(C)** 190 **(D)** 200 **(E)** 205

12. A rectangle has length 8 cm and width π cm. A semi-circle has the same area as the rectangle. What is its radius?

(A) $4\sqrt{2}$ cm **(B)** 4 cm **(C)** 16 cm **(D)** 8 cm **(E)** 2 cm

13. If $a(x + 2) + b(x + 2) = 60$ and $a + b = 12$, then x is equal to

(A) 3 **(B)** 5 **(C)** 1 **(D)** 7 **(E)** 48

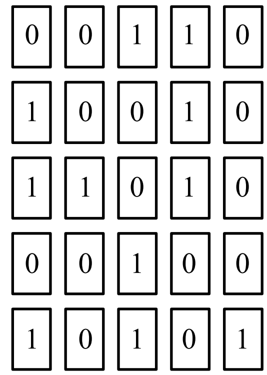
14. A line with a slope of 2 and a line with a slope of -4 each have a y-intercept of 6. The distance between the x-intercepts of these lines is

(A) 2 **(B)** 6 **(C)** $\frac{3}{2}$ **(D)** $\frac{5}{2}$ **(E)** $\frac{9}{2}$

15. A sequence has 101 terms, each of which is a positive integer. If a term, n, is even, the next term is equal to $\frac{1}{2}n + 1$. If a term, n, is odd, the next term is equal to $\frac{1}{2}(n + 1)$. For example, if the first term is 7, then the second term is 4 and the third term is 3. If the first term is 16, the 101st term is

(A) 1 **(B)** 2 **(C)** 3 **(D)** 4 **(E)** 5

16. Twenty-five cards are randomly arranged in a grid, as shown. Five of these cards have a 0 on one side and a 1 on the other side. The remaining twenty cards either have a 0 on both sides or a 1 on both sides. Loron chooses one row or one column and flips over each of the five cards in that row or column, leaving the rest of the cards untouched. After this operation, Loron determines the ratio of 0s to 1s facing upwards. No matter which row or column Loron chooses, it is *not possible* for this ratio to be



(A) 12 : 13 **(B)** 2 : 3 **(C)** 9 : 16 **(D)** 3 : 2 **(E)** 16 : 9

17. The positive divisors of 6 are 1, 2, 3, 6. The sum of the positive divisors of 1184 is

(A) 2394 **(B)** 2396 **(C)** 2398 **(D)** 2400 **(E)** 2402

18. A robotic grasshopper jumps 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After every fourth jump, the grasshopper restarts the sequence of jumps: 1 cm to the east, then 2 cm to the north, then 3 cm to the west, then 4 cm to the south. After a total of n jumps, the position of the grasshopper is 162 cm to the west and 158 cm to the south of its original position. The sum of the squares of the digits of n is

(A) 22 **(B)** 29 **(C)** 17 **(D)** 14 **(E)** 13

19. If x and y are integers with $2x^2 + 8y = 26$, a possible value of $x - y$ is

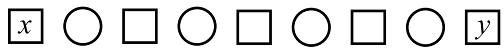
(A) -8 **(B)** 26 **(C)** -16 **(D)** 22 **(E)** 30

20. If n is a positive integer, the symbol $n!$ (which is read "n factorial") represents the product of the integers from 1 to n, inclusive. For example, $5! = (1)(2)(3)(4)(5)$ or $5! = 120$, which ends with exactly 1 zero. For how many integers m, with $1 \leq m \leq 30$, is it possible to find a value of n so that $n!$ ends with exactly m zeros?

(A) 30 **(B)** 27 **(C)** 28 **(D)** 24 **(E)** 25

Part C: Each correct answer is worth 8.
Each correct answer is an integer from 0 to 99, inclusive.
A one-digit answer (such as "7") must be coded with a leading zero ("07").
Note: The integer formed by the rightmost two digits of 12 345 is 45.
The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. The integers 1, 2, 4, 5, 6, 9, 10, 11, 13 are to be placed in the circles and squares below with one number in each shape.



Each integer must be used exactly once and the integer in each circle must be equal to the sum of the integers in the two neighbouring squares. If the integer x is placed in the leftmost square and the integer y is placed in the rightmost square, what is the largest possible value of $x + y$?

22. If x and y are positive real numbers with $\dfrac{1}{x+y} = \dfrac{1}{x} - \dfrac{1}{y}$, what is the value of $\left(\dfrac{x}{y} + \dfrac{y}{x}\right)^2$?

23. For each positive integer n, define $s(n)$ to equal the sum of the digits of n. For example, $s(2023) = 2 + 0 + 2 + 3$. The number of integers n with $100 \le n \le 999$ and $7 \le s(n) \le 11$ is S. What is the integer formed by the rightmost two digits of S?

24. Quadrilateral $ABCD$ has $\angle BCD = \angle DAB = 90°$. The perimeter of $ABCD$ is 224 and its area is 2205. One side of $ABCD$ has length 7. The remaining three sides have integer lengths. The sum of the squares of the side lengths of $ABCD$ is S. What is the integer formed by the rightmost two digits of S?

25. A cube has edge length 4 m. One end of a rope of length 5 m is anchored to the centre of the top face of the cube. The area of the surface of the cube that can be reached by the other end of the rope is A m^2. What is the integer formed by the rightmost two digits of the integer closest to $100A$?

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For students...

Thank you for writing the 2023 Fermat Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website `cemc.uwaterloo.ca` to find

- More information about the Hypatia Contest
- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
- Learn about our face-to-face workshops and our web resources
- Subscribe to our free Problem of the Week
- Investigate our online Master of Mathematics for Teachers
- Find your school's contest results