The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fryer Contest

(Grade 9)

Wednesday, April 5, 2023
(in North America and South America)

Thursday, April 6, 2023
(outside of North America and South America)

UNIVERSITY OF
WATERLOO

Time: 75 minutes

Do not open this booklet until instructed to do so.

Number of questions: 4 **Each question is worth 10 marks**

Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Parts of each question can be of two types:

1. **SHORT ANSWER** parts indicated by
   - worth 2 or 3 marks each
   - full marks given for a correct answer which is placed in the box
   - **part marks awarded only if relevant work** is shown in the space provided
2. **FULL SOLUTION** parts indicated by
   - worth the remainder of the 10 marks for the question
   - **must be written in the appropriate location** in the answer booklet
   - marks awarded for completeness, clarity, and style of presentation
   - a correct solution poorly presented will not earn full marks

WRITE ALL ANSWERS IN THE ANSWER BOOKLET PROVIDED.

- Extra paper for your finished solutions must be supplied by your supervising teacher and inserted into your answer booklet. Write your name, school name, and question number on any inserted pages.
- Express answers as simplified exact numbers except where otherwise indicated. For example, $\pi + 1$ and $1 - \sqrt{2}$ are simplified exact numbers.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

NOTE:

1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked , place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked , provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the x-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.
7. No student may write more than one of the Fryer, Galois and Hypatia Contests in the same year.

1. At Monday's practice, Lonnie sprints 200 m a total of 24 times. At Tuesday's practice, he sprints 240 m a total of 20 times. On both days, he rests for 30 s between each consecutive pair of sprints. Lonnie sprints at a constant speed of 8 m/s.

   (a) On Monday, how many times does Lonnie take the 30 s rest between consecutive pairs of sprints?

   (b) On Monday, determine Lonnie's total practice time. That is, determine the total number of seconds between the start of his first sprint and the end of his last sprint, including the rests.

   (c) Determine how many fewer seconds Tuesday's practice will take compared to Monday's practice.

2. Consider the following arrangement of positive integers.

1			
2	4		
5	7	9	
10	12	14	16
	⋮		

The 1st row includes the odd integer 1, and the 2nd row includes the two even integers 2 and 4. For $k \geq 2$, the kth row

- begins with the integer that is one more than the last integer in the previous row,
- includes, in increasing order, k consecutive odd integers when k is odd, and
- includes, in increasing order, k consecutive even integers when k is even.

A useful fact about this arrangement is that the integer in the kth row and kth position (that is, the last position in the kth row) is k^2. For example, $4^2 = 16$ and 16 is the integer in the 4th position of the 4th row.

(a) What is the average of the integers in the 5th row?

(b) Which row has the integer 145 in the 1st position?

(c) Determine the row and the position in which the integer 1598 appears.

(d) The average of the integers in row r is 241. Determine the value of r.

3. A positive integer is divisible by 3 exactly when the sum of its digits is divisible by 3. A positive integer is divisible by 4 exactly when the positive integer formed by its last two digits is divisible by 4. For example:

- 3816 is divisible by 3, since $3 + 8 + 1 + 6 = 18$ and 18 is divisible by 3;
- 3817 is not divisible by 3, since $3 + 8 + 1 + 7 = 19$ and 19 is not divisible by 3;
- 3816 is divisible by 4, since 16 is divisible by 4;
- 3817 is not divisible by 4, since 17 is not divisible by 4.

In each part that follows, A, B and C are non-zero digits (1, 2, 3, 4, 5, 6, 7, 8, or 9), and not necessarily distinct.

(a) The five-digit positive integer $4B5B2$ is divisible by 3. What are the possible values of the non-zero digit B?

(b) The five-digit positive integer $ABABA$ is divisible by 4 and not divisible by 3. Determine the number of different pairs of non-zero digits A and B that are possible.

(c) A positive integer, t, is equal to the product of the four-digit positive integer $ACA2$ and the three-digit positive integer BAC; that is, $t = ACA2 \times BAC$. If t is divisible by 15 and not divisible by 12, determine the number of different triples of non-zero digits A, B, C that are possible.

4. A lab has 50 computers numbered 1 through 50. Each pair of computers is connected to each other by a cord. The cords are coloured according to the following rules.

- If the numbers of the two computers are both even or both odd, then the cord connecting them is red.
- Otherwise, the cord connecting them is blue.

A *route* is a sequence of cords along which data can travel to get from one computer to another computer within the lab. For example, data could travel the route from Computer 5 to Computer 12 directly, or the route from Computer 5 to Computer 15 to Computer 12.

(a) There is a route connecting Computer 1 to Computer n using only red cords. If $n \neq 1$, how many possible values are there for n?

(b) Show that for every pair of distinct computers, Computer A and Computer B, there is always a route between them that uses only blue cords.

(c) Each red cord and blue cord is removed and randomly replaced with either a green cord or a yellow cord. Dani notices that there is no route that connects Computer 1 to Computer 50 that uses only green cords. Show that there is always a route between Computer 13 and Computer 14 that uses only yellow cords.

2023 Fryer Contest (English)

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For students...

Thank you for writing the 2023 Fryer Contest! Each year, more than 260 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Canadian Intermediate Mathematics Contest or the Canadian Senior Mathematics Contest, which will be written in November 2023.

Visit our website `cemc.uwaterloo.ca` to find

- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website `cemc.uwaterloo.ca` to

- Obtain information about our 2023/2024 contests
- Register your students for the Canadian Senior and Intermediate Mathematics Contests which will be written in November
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