



The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Fermat Contest

(Grade 11)

Wednesday, February 28, 2024
(in North America and South America)

Thursday, February 29, 2024
(outside of North America and South America)



UNIVERSITY OF
WATERLOO

Time: 60 minutes

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Calculating devices are allowed, provided that they do not have any of the following features: (i) internet access, (ii) the ability to communicate with other devices, (iii) information previously stored by students (such as formulas, programs, notes, etc.), (iv) a computer algebra system, (v) dynamic geometry software.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. **Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.**
6. Part A and Part B of this contest are multiple choice. Each of the questions in these parts is followed by five possible answers marked **A**, **B**, **C**, **D**, and **E**. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. The correct answer to each question in Part C is an integer from 0 to 99, inclusive. After deciding on your answer, fill in the appropriate two circles on the response form. A one-digit answer (such as “7”) must be coded with a leading zero (“07”).
8. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
9. Diagrams are *not* drawn to scale. They are intended as aids only.
10. When your supervisor tells you to begin, you will have 60 minutes of working time.
11. You may not write more than one of the Pascal, Cayley and Fermat Contests in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

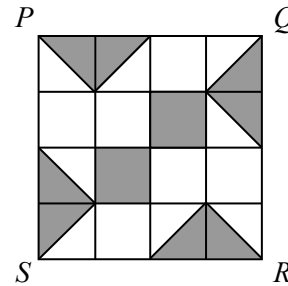
The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.

Scoring: There is *no penalty* for an incorrect answer.
 Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

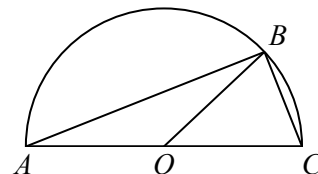
1. The value of $3\left(\frac{5}{3} - \frac{1}{3}\right)$ is
 (A) 1 (B) 2 (C) 3 (D) 4 (E) 6
2. If $x = 2$, the value of $4x^2 - 3x^2$ is
 (A) 1 (B) 3 (C) 4 (D) 2 (E) 5
3. How many solid $1 \times 1 \times 1$ cubes are required to make a solid $2 \times 2 \times 2$ cube?
 (A) 4 (B) 7 (C) 6 (D) 5 (E) 8
4. Shuxin begins with 10 red candies, 7 yellow candies, and 3 blue candies. After eating some of the candies, there are equal numbers of red, yellow, and blue candies remaining. What is the smallest possible number of candies that Shuxin ate?
 (A) 11 (B) 7 (C) 17 (D) 20 (E) 14

5. Square $PQRS$ is divided into 16 smaller congruent squares, as shown. What fraction of $PQRS$ is shaded?
 (A) $\frac{1}{3}$ (B) $\frac{1}{6}$ (C) $\frac{1}{2}$
 (D) $\frac{1}{4}$ (E) $\frac{3}{8}$



6. How many integers are greater than $\sqrt{15}$ and less than $\sqrt{50}$?
 (A) 3 (B) 4 (C) 6 (D) 5 (E) 2
7. The line with equation $y = 3x + 6$ is reflected in the y -axis. What is the x -intercept of the new line?
 (A) 2 (B) -2 (C) 6 (D) -6 (E) $\frac{1}{2}$
8. If $10^n = 1000^{20}$, what is the value of n ?
 (A) 1000 (B) 60 (C) 2000 (D) 300 (E) 102

9. In the diagram, a semi-circle has centre O and diameter AC . Also, B is a point on the circumference such that $\angle BAC = 25^\circ$. The measure of $\angle BOC$ is
 (A) 60° (B) 55° (C) 45°
 (D) 50° (E) 65°



10. In a photograph, Aristotle, David, Flora, Munirah, and Pedro are seated in a random order in a row of 5 chairs. If David is seated in the middle of the row, what is the probability that Pedro is seated beside him?

(A) $\frac{1}{5}$ (B) $\frac{1}{4}$ (C) $\frac{1}{2}$ (D) $\frac{2}{5}$ (E) $\frac{3}{5}$

Part B: Each correct answer is worth 6.

11. Figure 1 shows an arrangement of 3 lines with 1 intersection point, and Figure 2 shows an arrangement of 3 lines with 3 intersection points.

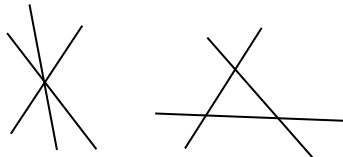


Figure 1

Figure 2

What is the maximum number of intersection points that can appear in an arrangement of 4 lines?

(A) 5 (B) 4 (C) 8 (D) 6 (E) 7

12. The numbers 5, 6, 10, 17, and 21 are rearranged so that the sum of the first three numbers is equal to the sum of the last three numbers. Which number is in the middle of this rearrangement?

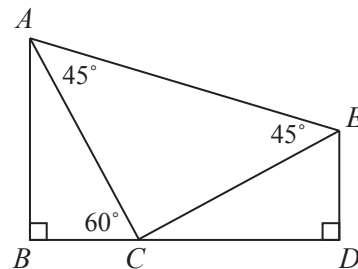
(A) 5 (B) 6 (C) 10 (D) 17 (E) 21

13. For some integers m and n , the expression $(x + m)(x + n)$ is equal to a quadratic expression in x with a constant term of -12 . Which of the following cannot be a value of m ?

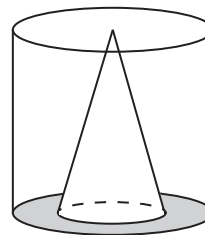
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

14. In diagram, point C is on side BD of quadrilateral $ABDE$. Also, AB and ED are perpendicular to BD , $\angle ACB = 60^\circ$, $\angle CAE = 45^\circ$, and $\angle AEC = 45^\circ$. If $AB = \sqrt{3}$, what is the perimeter of quadrilateral $ABDE$?

(A) $1 + \sqrt{2} + 2\sqrt{3}$
 (B) $2 + 2\sqrt{2} + \sqrt{3}$
 (C) $1 + 2\sqrt{2} + 2\sqrt{3}$
 (D) $2 + \sqrt{2} + 2\sqrt{3}$
 (E) $2 + 2\sqrt{2} + 2\sqrt{3}$



20. A cylinder contains some water. A solid cone with the same height and half the radius of the cylinder is submerged into the water until the circular face of the cone lies flat on the circular base of the cylinder, as shown. Once this is done, the depth of the water is half of the height of the cylinder. If the cone is then removed, the depth of the water will be what fraction of the height of the cylinder?



(The volume of a cylinder with radius r and height h is $\pi r^2 h$ and the volume of a cone with radius r and height h is $\frac{1}{3}\pi r^2 h$.)

- (A) $\frac{3}{16}$ (B) $\frac{41}{96}$ (C) $\frac{5}{16}$
 (D) $\frac{3}{8}$ (E) $\frac{7}{16}$

Part C: Each correct answer is worth 8.

Each correct answer is an integer from 0 to 99, inclusive.

A one-digit answer (such as “7”) must be coded with a leading zero (“07”).

Note: The integer formed by the rightmost two digits of 12 345 is 45.

The integer formed by the rightmost two digits of 6307 is 7, coded 07.

21. A 3×3 table starts with every entry equal to 0 and is modified using the following steps:

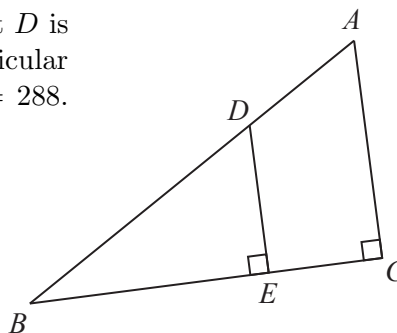
- (i) adding 1 to all three numbers in any row;
 (ii) adding 2 to all three numbers in any column.

After step (i) has been used a total of a times and step (ii) has been used a total of b times, the table appears as shown. What is the value of $a + b$?

7	1	5
9	3	7
8	2	6

22. Four distinct integers a , b , c , and d are chosen from the set $\{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$. What is the greatest possible value of $ac + bd - ad - bc$?

23. In the diagram, $\triangle ABC$ is right-angled at C . Point D is on AB and point E is on BC so that DE is perpendicular to BC , $BE = AC$, $BD = 120$, and $DE + BC = 288$. What is the length of DE ?



24. The integer N is the smallest positive integer that is a multiple of 2024, has more than 100 positive divisors (including 1 and N), and has fewer than 110 positive divisors (including 1 and N). What is the sum of the digits of N ?

25. A sequence of 11 positive real numbers, $a_1, a_2, a_3, \dots, a_{11}$, satisfies $a_1 = 4$ and $a_{11} = 1024$ and $a_n + a_{n-1} = \frac{5}{2}\sqrt{a_n \cdot a_{n-1}}$ for every integer n with $2 \leq n \leq 11$. For example when $n = 7$, $a_7 + a_6 = \frac{5}{2}\sqrt{a_7 \cdot a_6}$. There are S such sequences. What are the rightmost two digits of S ?



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For students...

Thank you for writing the 2024 Fermat Contest! Each year, more than 265 000 students from more than 80 countries register to write the CEMC's Contests.

Encourage your teacher to register you for the Hypatia Contest which will be written in April.

Visit our website cemc.uwaterloo.ca to find

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- Free copies of past contests
- Math Circles videos and handouts that will help you learn more mathematics and prepare for future contests
- Information about careers in and applications of mathematics and computer science

For teachers...

Visit our website cemc.uwaterloo.ca to

- Register your students for the Fryer, Galois and Hypatia Contests which will be written in April
- Look at our free online courseware
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- Find your school's contest results