The CENTRE for EDUCATION
in MATHEMATICS and COMPUTING
cemc.uwaterloo.ca

Pascal Contest
(Grade 9)
Tuesday, February 26, 2019
(in North America and South America)
Wednesday, February 27, 2019
(outside of North America and South America)

Time: 60 minutes
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Part A: Each correct answer is worth 5.

1. The expression $2 \times 3 + 2 \times 3$ equals
   (A) 10  (B) 20  (C) 36  (D) 12  (E) 16

2. The perimeter of a square is 28. What is the side length of this square?
   (A) 9  (B) 6  (C) 8  (D) 4  (E) 7

3. In the diagram, some of the hexagons are shaded. What fraction of all of the hexagons are shaded?
   (A) $\frac{1}{2}$  (B) $\frac{5}{5}$  (C) $\frac{4}{5}$
   (D) $\frac{1}{3}$  (E) $\frac{5}{6}$

4. Yesterday, each student at Pascal C.I. was given a snack. Each student received either a muffin, yogurt, fruit, or a granola bar. No student received more than one of these snacks. The percentages of the students who received each snack are shown in the circle graph. What percentage of students did not receive a muffin?
   (A) 27%  (B) 38%  (C) 52%
   (D) 62%  (E) 78%

5. What is the smallest integer that can be placed in the box so that $1 \frac{1}{2} < \square \frac{9}{9}$?
   (A) 7  (B) 3  (C) 4  (D) 5  (E) 6

6. If $4x + 14 = 8x - 48$, what is the value of $2x$?
   (A) 17  (B) 31  (C) 35  (D) 24  (E) 36

7. In the diagram, point $P$ is on the number line at 3 and $V$ is at 33. The number line between 3 and 33 is divided into six equal parts by the points $Q, R, S, T, U$.

   What is the sum of the lengths of $PS$ and $TV$?
   (A) 25  (B) 23  (C) 24  (D) 21  (E) 27

8. The median of the numbers in the list $19^{20}, \frac{20}{19}, 20^{19}, 2019, 20 \times 19$ is
   (A) $19^{20}$  (B) $\frac{20}{19}$  (C) $20^{19}$  (D) 2019  (E) $20 \times 19$
9. In the diagram, each partially shaded circle has a radius of 1 cm and has a right angle marked at its centre. In cm², what is the total shaded area?
(A) $4\pi^2$  (B) $9\pi^2$  (C) $4\pi$
(D) $9\pi$  (E) $3\pi$

10. Three $1 \times 1 \times 1$ cubes are joined face to face in a single row and placed on a table, as shown. The cubes have a total of 11 exposed $1 \times 1$ faces. If sixty $1 \times 1 \times 1$ cubes are joined face to face in a single row and placed on a table, how many $1 \times 1$ faces are exposed?
(A) 125  (B) 220  (C) 182
(D) 239  (E) 200

Part B: Each correct answer is worth 6.

11. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the value of $x$ is
(A) 3.8  (B) 3.6  (C) 3.1
(D) 2.9  (E) 2.2

12. In the diagram, $PR$ and $QS$ meet at $X$. Also, $\triangle PQX$ is right-angled at $Q$ with $\angle QPX = 62^\circ$ and $\triangle RXS$ is isosceles with $RX = SX$ and $\angle XSR = y^\circ$. The value of $y$ is
(A) 54  (B) 71  (C) 76
(D) 59  (E) 60

13. The list $p,q,r,s$ consists of four consecutive integers listed in increasing order. If $p + s = 109$, the value of $q + r$ is
(A) 108  (B) 109  (C) 110  (D) 117  (E) 111

14. Many of the students in M. Gamache’s class brought a skateboard or a bicycle to school yesterday. The ratio of the number of skateboards to the number of bicycles was $7 : 4$. There were 12 more skateboards than bicycles. How many skateboards and bicycles were there in total?
(A) 44  (B) 33  (C) 11  (D) 22  (E) 55
15. Sophie has written three tests. Her marks were 73%, 82% and 85%. She still has two tests to write. All tests are equally weighted. Her goal is an average of 80% or higher. With which of the following pairs of marks on the remaining tests will Sophie not reach her goal?

(A) 79% and 82%  
(B) 70% and 91%  
(C) 76% and 86%  
(D) 73% and 83%  
(E) 61% and 99%

16. If $x$ is a number less than $-2$, which of the following expressions has the least value?

(A) $x$  
(B) $x + 2$  
(C) $\frac{1}{2}x$  
(D) $x - 2$  
(E) $2x$

17. Hagrid has 100 animals. Among these animals,
- each is either striped or spotted but not both,
- each has either wings or horns but not both,
- there are 28 striped animals with wings,
- there are 62 spotted animals, and
- there are 36 animals with horns.

How many of Hagrid’s spotted animals have horns?

(A) 8  
(B) 10  
(C) 2  
(D) 38  
(E) 26

18. In the diagram, each of $\triangle QPT$, $\triangle QTS$ and $\triangle QSR$ is an isosceles, right-angled triangle, with $\angle QPT = \angle QTS = \angle QSR = 90^\circ$. The combined area of the three triangles is 56. If $QP = PT = k$, what is the value of $k$?

(A) $\sqrt{2}$  
(B) 1  
(C) 4  
(D) 2  
(E) $2\sqrt{2}$

19. There are six identical red balls and three identical green balls in a pail. Four of these balls are selected at random and then these four balls are arranged in a line in some order. How many different-looking arrangements are possible?

(A) 15  
(B) 16  
(C) 10  
(D) 11  
(E) 12

20. In the diagram, square $PQRS$ has side length 40. Points $J$, $K$, $L$, and $M$ are on the sides of $PQRS$, as shown, so that $JQ = KR = LS = MP = 10$. Line segments $JZ$, $KW$, $LX$, and $MY$ are drawn parallel to the diagonals of the square so that $W$ is on $JZ$, $X$ is on $KW$, $Y$ is on $LX$, and $Z$ is on $MY$. What is the area of quadrilateral $WXYZ$?

(A) 280  
(B) 200  
(C) 320  
(D) 240  
(E) 160
Part C: Each correct answer is worth 8.

21. What is the units (ones) digit of the integer equal to $5^{2019} - 3^{2019}$?
(A) 0  (B) 2  (C) 4  (D) 6  (E) 8

22. The integer 2019 can be formed by placing two consecutive two-digit positive integers, 19 and 20, in decreasing order. What is the sum of all four-digit positive integers greater than 2019 that can be formed in this way?
(A) 476 681  (B) 476 861  (C) 478 661  (D) 468 671  (E) 468 761

23. A path of length 14 m consists of 7 unshaded stripes, each of length 1 m, alternating with 7 shaded stripes, each of length 1 m. A circular wheel of radius 2 m is divided into four quarters which are alternately shaded and unshaded. The wheel rolls at a constant speed along the path from the starting position shown.

The wheel makes exactly 1 complete revolution. The percentage of time during which a shaded section of the wheel is touching a shaded part of the path is closest to
(A) 20%  (B) 18%  (C) 16%  (D) 24%  (E) 22%

24. If $p$, $q$, $r$, and $s$ are digits, how many of the 14-digit positive integers of the form 88 663 311 $pqrs48$ are divisible by 792?
(A) 48  (B) 56  (C) 40  (D) 60  (E) 50

25. In the diagram, $PR$ and $QS$ intersect at $V$. Also, $W$ is on $PV$, $U$ is on $PS$ and $T$ is on $PQ$ with $QU$ and $ST$ passing through $W$. For some real number $x$,
- the area of $\triangle PUW$ equals $4x + 4$,
- the area of $\triangle SUW$ equals $2x + 20$,
- the area of $\triangle SVW$ equals $5x + 20$,
- the area of $\triangle SVR$ equals $5x + 11$,
- the area of $\triangle QVR$ equals $8x + 32$, and
- the area of $\triangle QVW$ equals $8x + 50$.

The area of $\triangle PTW$ is closest to
(A) 35  (B) 34  (C) 33  (D) 32  (E) 31
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Part A: Each correct answer is worth 5.

1. Which of the following is the smallest number?
   (A) 1.4       (B) 1.2       (C) 2.0       (D) 1.5       (E) 2.1

2. The value of $\frac{2018 - 18 + 20}{2}$ is
   (A) 1010       (B) 2020       (C) 1008       (D) 2017       (E) 1011

3. July 3, 2030 is a Wednesday. What day of the week is July 14, 2030?
   (A) Wednesday   (B) Saturday   (C) Sunday
   (D) Monday      (E) Tuesday

4. An electric car is charged 3 times per week for 52 weeks. The cost to charge the car each time is $0.78. What is the total cost to charge the car over these 52 weeks?
   (A) $104.00     (B) $81.12     (C) $202.80    (D) $162.24    (E) $121.68

5. If $3 \times 3 \times 5 \times 5 \times 7 \times 9 = 3 \times 3 \times 7 \times n \times n$, what is a possible value of $n$?
   (A) 15         (B) 25         (C) 45         (D) 35         (E) 5

6. In the diagram, 18 identical 1×2 rectangles are put together to form a 6×6 square. Part of the square is shaded, as shown. What percentage of the area of the 6×6 square is shaded?
   (A) 50%        (B) 67%        (C) 75%
   (D) 33%        (E) 25%

7. A box contains 5 black ties, 7 gold ties, and 8 pink ties. Stephen randomly chooses a tie from the box. Each tie is equally likely to be chosen. The probability that Stephen chooses a pink tie is equivalent to
   (A) $\frac{1}{4}$    (B) $\frac{7}{20}$    (C) $\frac{2}{5}$    (D) $\frac{3}{5}$    (E) $\frac{3}{4}$

8. In the diagram, the number line between 0 and 5 is divided into 20 equal parts. The numbers $S$ and $T$ are marked on the line. What is the value of $S + T$?

   (A) 5.25       (B) 5.5       (C) 4.5       (D) 4.75       (E) 5

9. The symbols $\Bigheartsuit$ and $\nabla$ represent different positive integers less than 20. If $\Bigheartsuit \times \Bigheartsuit \times \Bigheartsuit = \nabla$, what is the value of $\nabla \times \nabla$?
   (A) 12         (B) 16         (C) 36         (D) 64         (E) 81
10. Which of the following points lies on the line that passes through \((-2, 1)\) and \((2, 5)\)?

(A) \((0, 0)\)    (B) \((0, 2)\)    (C) \((0, 3)\)    (D) \((0, 4)\)    (E) \((0, 5)\)

Part B: Each correct answer is worth 6.

11. In the diagram, the circle graph shows how a baby polar bear spent 24 hours. How many hours did it spend playing?

(A) 6    (B) 7    (C) 8    (D) 9    (E) 10

12. Glenda, Helga, Ioana, Julia, Karl, and Liu participated in the 2017 Canadian Team Mathematics Contest. On their team uniforms, each had a different number chosen from the list 11, 12, 13, 14, 15, 16. Helga’s and Julia’s numbers were even. Karl’s and Liu’s numbers were prime numbers. Glenda’s number was a perfect square. What was Ioana’s number?

(A) 11    (B) 13    (C) 14    (D) 15    (E) 12

13. A rectangle with height \(x\) and width \(2x\) has the same perimeter as an equilateral triangle with side length 10. What is the area of the rectangle?

\[
\text{Area} = \frac{2 \times 10 \times x}{2}
\]

(A) 18    (B) 50    (C) 25    (D) 200    (E) 100

14. In the list 7, 9, 10, 11, 18, which number is the average (mean) of the other four numbers?

(A) 9    (B) 18    (C) 7    (D) 11    (E) 10

15. A digital clock shows the time 4:56. How many minutes will pass until the clock next shows a time in which all of the digits are consecutive and are in increasing order?

(A) 458    (B) 587    (C) 376    (D) 315    (E) 518

16. Reading from left to right, a sequence consists of 6 X’s, followed by 24 Y’s, followed by 96 X’s. After the first \(n\) letters, reading from left to right, one letter has occurred twice as many times as the other letter. The sum of the four possible values of \(n\) is

(A) 72    (B) 54    (C) 135    (D) 81    (E) 111
17. Suppose that $p$ and $q$ are two different prime numbers and that $n = p^2q^2$. The number of possible values of $n$ with $n < 1000$ is

(A) 5  (B) 6  (C) 4  (D) 8  (E) 7

18. In the diagram, $\triangle PQR$ has $\angle PQR = 120^\circ$. Also, $\angle QPS = \angle RPS$ and $\angle QRS = \angle PRS$. (In other words, $SP$ and $SR$ bisect $\angle QPR$ and $\angle QRP$, respectively.) What is the measure of $\angle PSR$?

(A) $130^\circ$  (B) $120^\circ$  (C) $140^\circ$  (D) $160^\circ$  (E) $150^\circ$

19. On Monday, Mukesh travelled $x$ km at a constant speed of 90 km/h. On Tuesday, he travelled on the same route at a constant speed of 120 km/h. His trip on Tuesday took 16 minutes less than his trip on Monday. The value of $x$ is

(A) 90  (B) 112  (C) 100  (D) 96  (E) 92

20. In the diagram, $PQRST$ is a pentagon with $PQ = 8$, $QR = 2$, $RS = 13$, $ST = 13$, and $TP = 8$. Also, $\angle TPQ = \angle PQR = 90^\circ$. What is the area of pentagon $PQRST$?

(A) 76  (B) 84  (C) 92  (D) 100  (E) 108

Part C: Each correct answer is worth 8.

21. A coin travels along a path that starts in an unshaded square in the top row of the figure, that uses only diagonal moves, and that ends in an unshaded square in the bottom row. A diagonal move takes the coin either one square down and one square left, or one square down and one square right. How many different paths from the top row to the bottom row are possible?

(A) 16  (B) 20  (C) 32  (D) 24  (E) 28
22. A Miniou circuit contains nodes and wires and obeys the following rules:
   - Each wire connects two different nodes.
   - There is at most one wire between each pair of nodes.
   - Exactly three wires are connected to each node.
An example of a Miniou circuit is shown. If a Miniou circuit has 13 788 wires, how many nodes does it have?
(A) 9190     (B) 9192     (C) 9188
(D) 9186     (E) 9184

23. In the diagram, two larger circles with radius 1 have centres P and Q. Also, the smaller circle has diameter PQ. The region inside the two larger circles and outside the smaller circle is shaded.

The area of the shaded region is closest to
(A) 0.36     (B) 0.38     (C) 0.40     (D) 0.42     (E) 0.44

24. In Mrs. Warner’s class, there are 30 students. Strangely, 15 of the students have a height of 1.60 m and 15 of the students have a height of 1.22 m. Mrs. Warner lines up n students so that the average height of any four consecutive students is greater than 1.50 m and the average height of any seven consecutive students is less than 1.50 m. What is the largest possible value of n?
(A) 8       (B) 12       (C) 11       (D) 9       (E) 10

25. P.J. starts with \( m = 500 \) and chooses a positive integer \( n \) with \( 1 \leq n \leq 499 \). He applies the following algorithm to \( m \) and \( n \):
   - P.J. sets \( r \) equal to the remainder when \( m \) is divided by \( n \).
   - If \( r = 0 \), P.J. sets \( s = 0 \).
     If \( r > 0 \), P.J. sets \( s \) equal to the remainder when \( n \) is divided by \( r \).
   - If \( s = 0 \), P.J. sets \( t = 0 \).
     If \( s > 0 \), P.J. sets \( t \) equal to the remainder when \( r \) is divided by \( s \).
For example, when \( n = 8 \), P.J. obtains \( r = 4 \), \( s = 0 \), and \( t = 0 \). For how many of the positive integers \( n \) with \( 1 \leq n \leq 499 \) does P.J.’s algorithm give \( 1 \leq r \leq 15 \) and \( 2 \leq s \leq 9 \) and \( t = 0 \)?
(A) 14       (B) 12       (C) 16       (D) 15       (E) 13
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1. The value of $\frac{4 \times 3}{2 + 1}$ is
   (A) 4   (B) 7   (C) 3   (D) 6   (E) 5

2. In the diagram, how many $1 \times 1$ squares are shaded in the $6 \times 6$ grid?
   (A) 29   (B) 30   (C) 31
   (D) 32   (E) 33

3. In the diagram, the ratio of the number of shaded triangles to the number of unshaded triangles is
   (A) 5 : 2   (B) 5 : 3   (C) 8 : 5
   (D) 5 : 8   (E) 2 : 5

4. Which of the following is closest in value to 7?
   (A) $\sqrt{70}$   (B) $\sqrt{60}$   (C) $\sqrt{50}$   (D) $\sqrt{40}$   (E) $\sqrt{80}$

5. Kamal turned his computer on at 2 p.m. on Friday. He left his computer on for exactly 30 consecutive hours. At what time did he turn his computer off?
   (A) 4 p.m. on Saturday
   (B) 6 p.m. on Saturday
   (C) 8 p.m. on Sunday
   (D) 6 p.m. on Sunday
   (E) 8 p.m. on Saturday

6. At six different times on Canada Day in 2016, the number of people at the Pascal Zoo were counted. The graph to the right shows these results. During which of the following periods did the number of people at the zoo have the largest increase?
   (A) 9:00 a.m. to 10:00 a.m.
   (B) 10:00 a.m. to 11:00 a.m.
   (C) 11:00 a.m. to 12:00 p.m.
   (D) 12:00 p.m. to 1:00 p.m.
   (E) 1:00 p.m. to 2:00 p.m.
7. If $2x - 3 = 10$, what is the value of $4x$?
(A) 23    (B) 24    (C) 28    (D) 26    (E) 20

8. Three integers from the list 1, 2, 4, 8, 16, 20 have a product of 80. What is the sum of these three integers?
(A) 21    (B) 22    (C) 25    (D) 29    (E) 26

9. Wally makes a whole pizza and shares it with three friends. Jovin takes $\frac{1}{3}$ of the pizza, Anna takes $\frac{1}{6}$ of the pizza, and Olivia takes $\frac{1}{4}$ of the pizza. What fraction of the pizza is left for Wally?
(A) $\frac{1}{6}$    (B) $\frac{1}{4}$    (C) $\frac{10}{13}$    (D) $\frac{1}{12}$    (E) $\frac{1}{3}$

10. Which of the following expressions is equal to an odd integer for every integer $n$?
(A) $2017 - 3n$    (B) $2017 + n$    (C) $2017n$    (D) $2017 + n^2$    (E) $2017 + 2n$

Part B: Each correct answer is worth 6.

11. Jeff and Ursula each run 30 km. Ursula runs at a constant speed of 10 km/h. Jeff also runs at a constant speed. If Jeff’s time to complete the 30 km is 1 hour less than Ursula’s time to complete the 30 km, at what speed does Jeff run?
(A) 6 km/h    (B) 11 km/h    (C) 12 km/h    (D) 15 km/h    (E) 22.5 km/h

12. A small square is drawn inside a larger square as shown.
The area of the shaded region and the area of the unshaded region are each 18 cm$^2$. What is the side length of the larger square?
(A) 3 cm    (B) 4 cm    (C) 6 cm
(D) 9 cm    (E) 12 cm

13. Janet picked a number, added 7 to the number, multiplied the sum by 2, and then subtracted 4. If the final result was 28, what number did Janet pick?
(A) 9    (B) 5    (C) 19    (D) 23    (E) 11

14. Tobias downloads $m$ apps. Each app costs $2.00 plus 10% tax. He spends $52.80 in total on these $m$ apps. What is the value of $m$?
(A) 20    (B) 22    (C) 18    (D) 24    (E) 26

15. In the diagram, the side lengths of four squares are shown.
The area of the fifth square is $k$. What is the value of $k$?
(A) 64    (B) 49    (C) 36
(D) 25    (E) 16
16. A circular spinner is divided into six regions, as shown. Four regions each have a central angle of $x^\circ$. The remaining regions have central angles of 20° and 140°. An arrow is attached to the centre of the circle. The arrow is spun once. What is the probability that the arrow stops on a shaded region?

(A) $\frac{2}{3}$  (B) $\frac{7}{8}$  (C) $\frac{1}{2}$  (D) $\frac{5}{12}$  (E) $\frac{7}{12}$

17. Igor is shorter than Jie. Faye is taller than Goa. Jie is taller than Faye. Han is shorter than Goa. Who is the tallest?

(A) Faye  (B) Goa  (C) Han  (D) Igor  (E) Jie

18. Given two different numbers on a number line, the number to the right is greater than the number to the left. The positions of $x$, $x^3$ and $x^2$ are marked on a number line. Which of the following is a possible value of $x$?

(A) $\frac{1}{5}$  (B) $\frac{3}{2}$  (C) $-\frac{2}{5}$  (D) $-\frac{4}{5}$  (E) 2

19. In the diagram, $M$ is the midpoint of $YZ$, $\angle XMZ = 30^\circ$, and $\angle XYZ = 15^\circ$. The measure of $\angle XZY$ is

(A) 75°  (B) 65°  (C) 60°  (D) 80°  (E) 85°

20. A solid cube is made of white plastic and has dimensions $n \times n \times n$, where $n$ is a positive integer larger than 1. The six faces of the cube are completely covered with gold paint. This cube is then cut into $n^3$ cubes, each of which has dimensions $1 \times 1 \times 1$. Each of these $1 \times 1 \times 1$ cubes has 0, 1, 2, or 3 gold faces. The number of $1 \times 1 \times 1$ cubes with 0 gold faces is strictly greater than the number of $1 \times 1 \times 1$ cubes with exactly 1 gold face. What is the smallest possible value of $n$?

(A) 7  (B) 8  (C) 9  (D) 10  (E) 4

---

**Part C:** Each correct answer is worth 8.

21. Each of the numbers 1, 5, 6, 7, 13, 14, 17, 22, 26 is placed in a different circle below. The numbers 13 and 17 are placed as shown.

Jen calculates the average of the numbers in the first three circles, the average of the numbers in the middle three circles, and the average of the numbers in the last three circles. These three averages are equal. What number is placed in the shaded circle?

(A) 1  (B) 5  (C) 6  (D) 7  (E) 14
22. In the diagram, $UVWX$ is a rectangle that lies flat on a horizontal floor. A vertical semi-circular wall with diameter $XW$ is constructed. Point $Z$ is the highest point on this wall. If $UV = 20$ and $VW = 30$, the perimeter of $\triangle UVZ$ is closest to

(A) 95  (B) 86  (C) 102
(D) 83  (E) 92

23. An Anderson number is a positive integer $k$ less than 10,000 with the property that $k^2$ ends with the digit or digits of $k$. For example, 25 is an Anderson number because 625 ends with 25, but 75 is not an Anderson number because 5625 does not end with 75. If $S$ is the sum of all even Anderson numbers, what is the sum of the digits of $S$?

(A) 17  (B) 18  (C) 11  (D) 33  (E) 24

24. A town has 2017 houses. Of these 2017 houses, 1820 have a dog, 1651 have a cat, and 1182 have a turtle. If $x$ is the largest possible number of houses that have a dog, a cat, and a turtle, and $y$ is the smallest possible number of houses that have a dog, a cat, and a turtle, then $x - y$ is

(A) 1182  (B) 638  (C) 563  (D) 619  (E) 466

25. Sam thinks of a 5-digit number. Sam’s friend Sally tries to guess his number. Sam writes the number of matching digits beside each of Sally’s guesses. A digit is considered “matching” when it is the correct digit in the correct position.

<table>
<thead>
<tr>
<th>Guess</th>
<th>Number of Matching Digits</th>
</tr>
</thead>
<tbody>
<tr>
<td>51545</td>
<td>2</td>
</tr>
<tr>
<td>21531</td>
<td>1</td>
</tr>
<tr>
<td>71794</td>
<td>0</td>
</tr>
<tr>
<td>59135</td>
<td>1</td>
</tr>
<tr>
<td>58342</td>
<td>2</td>
</tr>
<tr>
<td>37348</td>
<td>2</td>
</tr>
<tr>
<td>71744</td>
<td>1</td>
</tr>
</tbody>
</table>

What is the sum of all of the possibilities for Sam’s number?

(A) 525 768  (B) 527 658  (C) 527 568  (D) 526 578  (E) 526 758
For students...

Thank you for writing the 2017 Pascal Contest! Each year, more than 235,000 students from more than 75 countries register to write the CEMC's Contests.

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Pascal Contest
(Grade 9)
Wednesday, February 24, 2016
(in North America and South America)
Thursday, February 25, 2016
(outside of North America and South America)

University of Waterloo

Time: 60 minutes

Calculators are allowed, with the following restriction: you may not use a device that has internet access, that can communicate with other devices, or that contains previously stored information. For example, you may not use a smartphone or a tablet.

Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name and city/town in the box in the upper right corner.
5. Be certain that you code your name, age, grade, and the Contest you are writing in the response form. Only those who do so can be counted as eligible students.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.
10. You may not write more than one of the Pascal, Cayley or Fermat Contest in any given year.

Do not discuss the problems or solutions from this contest online for the next 48 hours.

The name, grade, school and location, and score range of some top-scoring students will be published on our website, cemc.uwaterloo.ca. In addition, the name, grade, school and location, and score of some top-scoring students may be shared with other mathematical organizations for other recognition opportunities.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The result of the addition shown is
   (A) 15021 (B) 12231 (C) 12051 (D) 13231 (E) 12321
   
   \[
   \begin{array}{c}
   300 \\
   2020 \\
   + 10001 \\
   \end{array}
   \]

2. Which of the following has the largest value?
   (A) \(4^2\) (B) \(4 \times 2\) (C) \(4 - 2\) (D) \(\frac{4}{2}\) (E) \(4 + 2\)

3. In the diagram, the 5 × 6 grid is made out of thirty 1 × 1 squares. What is the total length of the six solid line segments shown?
   (A) 6 (B) 12 (C) 16 (D) 18 (E) 20

4. In the diagram, each of the five squares is 1 × 1. What percentage of the total area of the five squares is shaded?
   (A) 25% (B) 30% (C) 35% (D) 40% (E) 45%

5. Numbers \(m\) and \(n\) are on the number line, as shown. The value of \(n - m\) is
   (A) 66 (B) 35 (C) 55 (D) 60 (E) 54

6. If the symbol \(\frac{p}{r} | \frac{q}{s}\) is defined by \(p \times s - q \times r\), then the value of \(\frac{4}{2} | \frac{5}{3}\) is
   (A) -3 (B) -2 (C) 2 (D) 3 (E) 14

7. Which of the following is equal to 2 m plus 3 cm plus 5 mm?
   (A) 2.035 m (B) 2.35 m (C) 2.0305 m (D) 2.53 m (E) 2.053 m

8. If \(x = 3\), \(y = 2x\), and \(z = 3y\), then the average of \(x\), \(y\) and \(z\) is
   (A) 6 (B) 7 (C) 8 (D) 9 (E) 10
9. A soccer team played three games. Each game ended in a win, loss, or tie. (If a game finishes with both teams having scored the same number of goals, the game ends in a tie.) In total, the team scored more goals than were scored against them. Which of the following combinations of outcomes is not possible for this team?

(A) 2 wins, 0 losses, 1 tie
(B) 1 win, 2 losses, 0 ties
(C) 0 wins, 1 loss, 2 ties
(D) 1 win, 1 loss, 1 tie
(E) 1 win, 0 losses, 2 ties

10. Exactly three faces of a $2 \times 2 \times 2$ cube are partially shaded, as shown. (Each of the three faces not shown in the diagram is not shaded.) What fraction of the total surface area of the cube is shaded?

(A) $\frac{1}{3}$  
(B) $\frac{1}{4}$  
(C) $\frac{1}{6}$  
(D) $\frac{3}{8}$  
(E) $\frac{2}{3}$

Part B: Each correct answer is worth 6.

11. An oblong number is the number of dots in a rectangular grid with one more row than column. The first four oblong numbers are 2, 6, 12, and 20, and are represented below:

What is the 7th oblong number?

(A) 42  
(B) 49  
(C) 56  
(D) 64  
(E) 72

12. In the diagram, the area of square $QRST$ is 36. Also, the length of $PQ$ is one-half of the length of $QR$. What is the perimeter of rectangle $PRSU$?

(A) 24  
(B) 30  
(C) 90  
(D) 45  
(E) 48

13. Multiplying $x$ by 10 gives the same result as adding 20 to $x$. The value of $x$ is

(A) $\frac{9}{20}$  
(B) $\frac{20}{9}$  
(C) $\frac{11}{20}$  
(D) $\frac{20}{11}$  
(E) 2

14. In the diagram, $PQ$ is perpendicular to $QR$, $QR$ is perpendicular to $RS$, and $RS$ is perpendicular to $ST$. If $PQ = 4$, $QR = 8$, $RS = 8$, and $ST = 3$, then the distance from $P$ to $T$ is

(A) 16  
(B) 12  
(C) 17  
(D) 15  
(E) 13
15. When two positive integers \( p \) and \( q \) are multiplied together, their product is 75. The sum of all of the possible values of \( p \) is

(A) 96  (B) 48  (C) 109  (D) 115  (E) 124

16. An integer from 10 to 99 inclusive is randomly chosen so that each such integer is equally likely to be chosen. The probability that at least one digit of the chosen integer is a 6 is

(A) \( \frac{1}{5} \)  (B) \( \frac{1}{10} \)  (C) \( \frac{1}{9} \)  (D) \( \frac{19}{90} \)  (E) \( \frac{19}{89} \)

17. What is the tens digit of the smallest six-digit positive integer that is divisible by each of 10, 11, 12, 13, 14, and 15?

(A) 0  (B) 6  (C) 2  (D) 8  (E) 4

18. Each integer from 1 to 12 is to be placed around the outside of a circle so that the positive difference between any two integers next to each other is at most 2. The integers 3, 4, \( x \), and \( y \) are placed as shown. What is the value of \( x + y \)?

(A) 17  (B) 18  (C) 19  (D) 20  (E) 21

19. Chris received a mark of 50% on a recent test. Chris answered 13 of the first 20 questions correctly. Chris also answered 25% of the remaining questions on the test correctly. If each question on the test was worth one mark, how many questions in total were on the test?

(A) 23  (B) 38  (C) 32  (D) 24  (E) 40

20. In the diagram, points \( Q \) and \( R \) lie on \( PS \) and \( \angle QWR = 38^\circ \). If \( \angle TQP = \angle TQW = x^\circ \), \( \angle VRS = \angle VRW = y^\circ \), and \( U \) is the point of intersection of \( TQ \) extended and \( VR \) extended, then the measure of \( \angle QUR \) is

(A) 71°  (B) 45°  (C) 76°  (D) 81°  (E) 60°

Part C: Each correct answer is worth 8.

21. Grid lines drawn on three faces of a rectangular prism, as shown. A squirrel walks from \( P \) to \( Q \) along the edges and grid lines in such a way that she is always getting closer to \( Q \) and farther away from \( P \). How many different paths from \( P \) to \( Q \) can the squirrel take?

(A) 14  (B) 10  (C) 20  (D) 12  (E) 16
22. There are \( n \) students in the math club at Scoins Secondary School. When Mrs. Fryer tries to put the \( n \) students in groups of 4, there is one group with fewer than 4 students, but all of the other groups are complete. When she tries to put the \( n \) students in groups of 3, there are 3 more complete groups than there were with groups of 4, and there is again exactly one group that is not complete. When she tries to put the \( n \) students in groups of 2, there are 5 more complete groups than there were with groups of 3, and there is again exactly one group that is not complete. The sum of the digits of the integer equal to \( n^2 - n \) is

(A) 11  (B) 12  (C) 20  (D) 13  (E) 10

23. In the diagram, \( \triangle PQR \) is isosceles with \( PQ = PR = 39 \) and \( \triangle SQR \) is equilateral with side length 30. The area of \( \triangle PQS \) is closest to

(A) 68  (B) 75  (C) 50  (D) 180  (E) 135

24. Ten very small rubber balls begin equally spaced inside a 55 m long tube. They instantly begin to roll inside the tube at a constant velocity of 1 m/s. When a ball reaches an end of the tube, it falls out of the tube. When two balls bump into each other, they both instantly reverse directions but continue to roll at 1 m/s. Five configurations giving the initial direction of movement of each ball are shown. All gaps indicated in the diagram are the same length and are equal in length to the distance from the ends of the tube to the nearest ball. For which configuration will it take the least time for more than half of the balls to fall out of the tube?

(A)  (B)  (C)  (D)  (E)

25. A 0 or 1 is to be placed in each of the nine \( 1 \times 1 \) squares in the \( 3 \times 3 \) grid shown so that each row contains at least one 0 and at least one 1, and each column contains at least one 0 and at least one 1. The number of ways in which this can be done is

(A) 126  (B) 120  (C) 138  (D) 102  (E) 96
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- Subscribe to our free Problem of the Week
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Pascal Contest
(Grade 9)
Tuesday, February 24, 2015
(in North America and South America)
Wednesday, February 25, 2015
(outside of North America and South America)

Time: 60 minutes
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Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of \( \frac{20 + 15}{30 - 25} \) is
   - (A) 1
   - (B) 5
   - (C) 2
   - (D) 7
   - (E) 0

2. Which of the following figures is obtained when the shaded figure shown is reflected about the line segment PQ?
   - (A)
   - (B)
   - (C)
   - (D)
   - (E)

3. If \( 8 + 6 = n + 8 \), then \( n \) equals
   - (A) 14
   - (B) 22
   - (C) 6
   - (D) −2
   - (E) 9

4. Which of the following numbers is greater than 0.7?
   - (A) 0.07
   - (B) −0.41
   - (C) 0.8
   - (D) 0.35
   - (E) −0.9

5. The expression \( 4 + \frac{3}{10} + \frac{9}{1000} \) is equal to
   - (A) 4.12
   - (B) 4.309
   - (C) 4.039
   - (D) 4.012
   - (E) 4.39

6. The average age of Andras, Frances and Gerta is 22 years. What is Gerta’s age?
   - (A) 19
   - (B) 20
   - (C) 21
   - (D) 22
   - (E) 23

7. If \( n = 7 \), which of the following expressions is equal to an even integer?
   - (A) 9n
   - (B) \( n + 8 \)
   - (C) \( n^2 \)
   - (D) \( n(n - 2) \)
   - (E) 8n

8. Jitka hiked a trail. After hiking 60% of the length of the trail, she had 8 km left to go. What is the length of the trail?
   - (A) 28 km
   - (B) 12.8 km
   - (C) 11.2 km
   - (D) 13 \( \frac{1}{2} \) km
   - (E) 20 km

9. In the diagram, line segments PQ and RS intersect at T. The value of \( x \) is
   - (A) 30
   - (B) 20
   - (C) 40
   - (D) 50
   - (E) 35
10. The value of $\sqrt{16} \times \sqrt{16}$ is
   (A) $2^1$  (B) $2^2$  (C) $2^3$  (D) $2^4$  (E) $2^5$

Part B: Each correct answer is worth 6.

11. Jim wrote the sequence of symbols $\heartsuit \spadesuit \spadesuit \clubsuit \clubsuit \diamondsuit \diamondsuit \diamondsuit \diamondsuit$ a total of 50 times. How many more $\heartsuit$ symbols than $\spadesuit$ symbols did he write?
   (A) 50  (B) 150  (C) 200  (D) 250  (E) 275

12. What is the smallest positive integer that is a multiple of each of 3, 5, 7, and 9?
   (A) 35  (B) 105  (C) 210  (D) 315  (E) 630

13. Sixteen squares are arranged to form a region, as shown. Each square has an area of 400 m$^2$. Anna walks along the path formed by the outer edges of the region exactly once. Aaron walks along the path formed by the inner edges of the region exactly once. In total, how far did Anna and Aaron walk?
   (A) 160 m  (B) 240 m  (C) 320 m  
   (D) 400 m  (E) 640 m

14. The operation $\otimes$ is defined by $a \otimes b = \frac{a}{b} + \frac{b}{a}$. What is the value of $4 \otimes 8$?
   (A) $\frac{1}{2}$  (B) 1  (C) $\frac{5}{4}$  (D) 2  (E) $\frac{5}{2}$

15. At the end of the year 2000, Steve had $100 and Wayne had $10,000. At the end of each following year, Steve had twice as much money as he did at the end of the previous year and Wayne had half as much money as he did at the end of the previous year. At the end of which year did Steve have more money than Wayne for the first time?
   (A) 2002  (B) 2003  (C) 2004  (D) 2005  (E) 2006

16. Anca and Bruce left Mathville at the same time. They drove along a straight highway towards Staton. Bruce drove at 50 km/h. Anca drove at 60 km/h, but stopped along the way to rest. They both arrived at Staton at the same time. For how long did Anca stop to rest?
   (A) 40 minutes  (B) 10 minutes  (C) 67 minutes  
   (D) 33 minutes  (E) 27 minutes

17. In the diagram, six identical circles just touch the edges of rectangle $PQRS$ and each circle just touches the adjacent circles. The centres $T, V, W, Y$ of four of these circles form a smaller rectangle $TVWY$, as shown. The centres $U$ and $X$ lie on this rectangle. If the perimeter of $TVWY$ is 60, what is the area of $PQRS$?
   (A) 600  (B) 900  (C) 400  
   (D) 1200  (E) 1000
18. In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum. In the magic square shown, the sum of $a + b + c$ equals

(A) 49  (B) 54  (C) 47  
(D) 50  (E) 46

19. Krystyna has some raisins. She gives one-third of her raisins to Mike. She then eats 4 raisins, after which she gives one-half of her remaining raisins to Anna. If Krystyna then has 16 raisins left, how many raisins did she have to begin?

(A) 42  (B) 54  (C) 60  (D) 84  (E) 108

20. André has an unlimited supply of $\$1$ coins, $\$2$ coins, and $\$5$ bills. Using only these coins and bills and not necessarily using some of each kind, in how many different ways can he form exactly $\$10$?

(A) 10  (B) 9  (C) 8  (D) 7  (E) 6

**Part C: Each correct answer is worth 8.**

21. Each diagram shows a triangle, labelled with its area.

What is the correct ordering of the areas of these triangles?

(A) $m < n < p$  
(D) $n < p < m$  
(B) $p < n < m$  
(E) $p < m < n$

22. The chart shown gives the cost of installing carpet in four rectangular rooms of various sizes. The cost per square metre of installing carpet is always the same.

<table>
<thead>
<tr>
<th>Length (metres)</th>
<th>Width (metres)</th>
<th>10</th>
<th>$y$</th>
<th>$x$</th>
<th>$z$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15</td>
<td>10</td>
<td>$$397.50$</td>
<td>$$675.75$</td>
<td>$$742.00$</td>
<td>$$z$</td>
</tr>
</tbody>
</table>

What is the value of $z$?

(A) 331.25  (B) 463.75  (C) 1815.25  (D) 476.00  (E) 1261.40
23. How many triples \((a, b, c)\) of positive integers satisfy the conditions \(6ab = c^2\) and \(a < b < c \leq 35\)?

(A) 10    (B) 8    (C) 6    (D) 7    (E) 9

24. Paula, Quinn, Rufus, and Sarah are suspects in a crime. The police found links between exactly four pairs of suspects: Paula and Quinn, Quinn and Rufus, Rufus and Paula, and Quinn and Sarah. These links can be shown in a diagram by drawing a point to represent each suspect and a line or curve joining two points whenever the two corresponding suspects are linked. An example of a drawing that represents this information is:

![Diagram of suspects](image)

Ali, Bob, Cai, Dee, Eve, and Fay are suspects in a second crime. The police found links between exactly eight pairs of suspects: Ali and Bob, Bob and Cai, Cai and Dee, Dee and Eve, Eve and Fay, Fay and Ali, Ali and Dee, and Bob and Eve. For how many of the following drawings can the six dots be labelled with the names of the six suspects so that each of the eight links given is represented by a line or curve in that drawing?

(A) 4    (B) 2    (C) 1    (D) 3    (E) 5

25. The first four rows of a table with columns \(V\), \(W\), \(X\), \(Y\), and \(Z\) are shown. For each row, whenever integer \(n\) appears in column \(V\), column \(W\) contains the integer \(2n + 1\), column \(X\) contains \(3n + 1\), column \(Y\) contains \(5n + 1\), and column \(Z\) contains \(7n + 1\). For every row after the first, the number in column \(V\) is the smallest positive integer that does not yet appear in any previous row. The integer 2731 appears in column \(W\). The complete list of columns in which 2731 appears is

(A) \(W\)
(B) \(W, X, Y,\) and \(Z\)
(C) \(W, X\) and \(Z\)
(D) \(W, Y\) and \(Z\)
(E) \(W\) and \(Z\)
For students...

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Pascal Contest
(Grade 9)
Thursday, February 20, 2014
(in North America and South America)
Friday, February 21, 2014
(outside of North America and South America)

Time: 60 minutes
Calculators are permitted

Instructions
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   There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
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Part A: Each correct answer is worth 5.

1. The value of \((8 \times 6) - (4 \div 2)\) is
   - (A) 6  
   - (B) 8  
   - (C) 46  
   - (D) 32  
   - (E) 22

2. In the diagram, what is the value of \(x\)?
   - (A) 65  
   - (B) 75  
   - (C) 85  
   - (D) 95  
   - (E) 105

3. 30% of 200 equals
   - (A) 0.06  
   - (B) 0.6  
   - (C) 6  
   - (D) 60  
   - (E) 600

4. If \(x = 3\), what is the perimeter of the figure shown?
   - (A) 23  
   - (B) 20  
   - (C) 21  
   - (D) 22  
   - (E) 19

5. A sports team earns 2 points for each win, 0 points for each loss, and 1 point for each tie. How many points does the team earn for 9 wins, 3 losses and 4 ties?
   - (A) 26  
   - (B) 16  
   - (C) 19  
   - (D) 21  
   - (E) 22

6. At 2 p.m., Sanjay measures the temperature to be 3°C. He measures the temperature every hour after this until 10 p.m. He plots the temperatures that he measures on the graph shown. At what time after 2 p.m. does he again measure a temperature of 3°C?
   - (A) 9 p.m.  
   - (B) 5 p.m.  
   - (C) 8 p.m.  
   - (D) 10 p.m.  
   - (E) 7 p.m.

7. If \(2 \times 2 \times 3 \times 3 \times 5 \times 6 = 5 \times 6 \times n \times n\), then \(n\) could equal
   - (A) 2  
   - (B) 3  
   - (C) 4  
   - (D) 5  
   - (E) 6
8. In the diagram, a figure is formed dividing a square into eight identical pieces using its two diagonals and the two lines joining the midpoints of opposite sides, and then drawing a circle in the square as shown. This figure is reflected in line $L$. Which of the following shows the final position of the figure?

(A) \[\text{Figure 1}\]
(B) \[\text{Figure 2}\]
(C) \[\text{Figure 3}\]
(D) \[\text{Figure 4}\]
(E) \[\text{Figure 5}\]

9. The value of $2^4 - 2^3$ is
(A) $2^1$  
(B) $2^2$  
(C) $2^3$  
(D) $2^4$  
(E) $2^5$

10. What number should go in the □ to make the equation $\frac{3}{4} + \frac{4}{□} = 1$ true?
(A) 1  
(B) 3  
(C) 5  
(D) 13  
(E) 16

\[\text{Part B: Each correct answer is worth 6.}\]

11. Two cubes are stacked as shown. The faces of each cube are labelled with 1, 2, 3, 4, 5, and 6 dots. A total of five faces are shown. What is the total number of dots on the other seven faces of these two cubes?
(A) 13  
(B) 14  
(C) 18  
(D) 21  
(E) 24

12. Strips are made up of identical copies of \[\text{Strip 1}\]. Each \[\text{Strip 1}\] has length $\frac{2}{3}$. Which strip has length 4?
(A) \[\text{Strip 2}\]  
(B) \[\text{Strip 3}\]  
(C) \[\text{Strip 4}\]
(D) \[\text{Strip 5}\]  
(E) \[\text{Strip 6}\]

13. In the subtraction shown, $X$ and $Y$ are digits. What is the value of $X + Y$?
(A) 15  
(B) 12  
(C) 10  
(D) 13  
(E) 9

14. If $x = 2y$ and $y \neq 0$, then $(x + 2y) - (2x + y)$ equals
(A) $-2y$  
(B) $-y$  
(C) 0  
(D) $y$  
(E) $2y$
15. In \( \triangle PQR \), \( \angle RPQ = 90^\circ \) and \( S \) is on \( PQ \). If \( SQ = 14 \), \( SP = 18 \), and \( SR = 30 \), then the area of \( \triangle QRS \) is

(A) 84  (B) 168  (C) 210  
(D) 336  (E) 384

16. In the 4 \( \times \) 4 grid shown, each of the four symbols has a different value. The sum of the values of the symbols in each row is given to the right of that row. What is the value of ♦?

(A) 5  (B) 6  (C) 7  
(D) 8  (E) 9

17. A cube has an edge length of 30. A rectangular solid has edge lengths 20, 30 and \( L \). If the cube and the rectangular solid have equal surface areas, what is the value of \( L \)?

(A) 15  (B) 21  (C) 42  
(D) 40  (E) 96

18. How many pairs of positive integers \((x, y)\) have the property that the ratio \( x : 4 \) equals the ratio \( 9 : y \)?

(A) 6  (B) 7  (C) 8  (D) 9  (E) 10

19. On each spin of the spinner shown, the arrow is equally likely to stop on any one of the four numbers. Deanna spins the arrow on the spinner twice. She multiplies together the two numbers on which the arrow stops. Which product is most likely to occur?

(A) 2  (B) 4  (C) 6  
(D) 8  (E) 12

20. In the diagram, line segment \( PS \) has length 4. Points \( Q \) and \( R \) are on line segment \( PS \). Four semi-circles are drawn on the same side of \( PS \). The diameters of these semi-circles are \( PS \), \( PQ \), \( QR \), and \( RS \). The region inside the largest semi-circle and outside the three smaller semi-circles is shaded. What is the area of a square whose perimeter equals the perimeter of the shaded region?

(A) 4  (B) \( \pi \)  (C) \( \pi^2 \)  
(D) \( 2\pi^2 \)  (E) \( \frac{\pi^2}{4} \)
Part C: Each correct answer is worth 8.

21. Twenty-four identical $1 \times 1$ squares form a $4 \times 6$ rectangle, as shown. A lattice point is a point where a horizontal grid line intersects a vertical grid line. A diagonal of this rectangle passes through the three lattice points $P$, $Q$, and $R$. When a $30 \times 45$ rectangle is constructed using identical $1 \times 1$ squares, how many lattice points will a diagonal of this rectangle pass through?  
(A) 19  (B) 16  (C) 15  
(D) 18  (E) 12

22. A rectangular flag is divided into four triangles, labelled Left, Right, Top, and Bottom, as shown. Each triangle is to be coloured one of red, white, blue, green, and purple so that no two triangles that share an edge are the same colour. How many different flags can be made?  
(A) 180  (B) 200  (C) 220  
(D) 240  (E) 260

23. In the diagram, the shape consists of 48 identical cubes with edge length $\sqrt{n}$. Entire faces of the cubes are attached to one another, as shown. What is the smallest positive integer $n$ so that the distance from $P$ to $Q$ is an integer?  
(A) 17  (B) 68  (C) 7  
(D) 28  (E) 3

24. Nadia walks along a straight path that goes directly from her house ($N$) to her Grandmother’s house ($G$). Some of this path is on flat ground, and some is downhill or uphill. Nadia walks on flat ground at 5 km/h, walks uphill at 4 km/h, and walks downhill at 6 km/h. It takes Nadia 1 hour and 36 minutes to walk from $N$ to $G$ and 1 hour and 39 minutes to walk from $G$ to $N$. If 2.5 km of the path between $N$ and $G$ is on flat ground, the total distance from $N$ to $G$ is closest to  
(A) 8.0 km  (B) 8.2 km  (C) 8.1 km  (D) 8.3 km  (E) 7.9 km

25. Suppose that $\frac{2009}{2014} + \frac{2019}{n} = \frac{a}{b}$, where $a$, $b$, and $n$ are positive integers with $\frac{a}{b}$ in lowest terms. What is the sum of the digits of the smallest positive integer $n$ for which $a$ is a multiple of 1004?  
(A) 16  (B) 17  (C) 14  (D) 20  (E) 21
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Pascal Contest
(Grade 9)

Thursday, February 21, 2013
(in North America and South America)

Friday, February 22, 2013
(outside of North America and South America)

Time: 60 minutes

Calculators are permitted

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Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

**Part A: Each correct answer is worth 5.**

1. The value of \((4 + 44 + 444) \div 4\) is
   - (A) 111
   - (B) 123
   - (C) 459
   - (D) 489
   - (E) 456

2. Jing purchased eight identical items. If the total cost was $26, then the cost per item, in dollars, was
   - (A) 26 ÷ 8
   - (B) 8 ÷ 26
   - (C) 26 − 8
   - (D) 8 × 26
   - (E) 8 + 26

3. The diagram shows a square divided into eight pieces. Which shape is not one of those pieces?
   - (A) 
   - (B) 
   - (C) 
   - (D) 
   - (E) 

4. The graph shows the mass, in kilograms, of Jeff’s pet Atlantic cod, given its age in years. What is the age of the cod when its mass is 15 kg?

5. What is the value of \(1^3 + 2^3 + 3^3 + 4^3\)?
   - (A) \(10^1\)
   - (B) \(10^3\)
   - (C) \(10^2\)
   - (D) \(10^5\)
   - (E) \(10^4\)

6. Erin walks \(\frac{3}{8}\) of the way home in 30 minutes. If she continues to walk at the same rate, how many minutes will it take her to walk the rest of the way home?
   - (A) 24
   - (B) 20
   - (C) 6
   - (D) 18
   - (E) 12

7. The expression \((\sqrt{100} + \sqrt{9}) \times (\sqrt{100} - \sqrt{9})\) is equal to
   - (A) 91
   - (B) 19
   - (C) 9991
   - (D) 9919
   - (E) 10991
8. In the diagram, rectangle $PQRS$ has $PS = 6$ and $SR = 3$. Point $U$ is on $QR$ with $QU = 2$. Point $T$ is on $PS$ with $\angle TUR = 90^\circ$. What is the length of $TR$?
\[
\begin{array}{c}
P \\
Q \quad 2 \quad U \quad R
\end{array}
\]
(A) 3 (B) 4 (C) 5 (D) 6 (E) 7

9. Owen spends $1.20 per litre on gasoline. He uses an average of 1 L of gasoline to drive 12.5 km. How much will Owen spend on gasoline to drive 50 km?
\[
\begin{array}{c}
(A) \$4.80 \quad (B) \$1.50 \quad (C) \$4.50 \quad (D) \$6.00 \quad (E) \$7.50
\end{array}
\]

10. The time on a cell phone is 3:52. How many minutes will pass before the phone next shows a time using each of the digits 2, 3 and 5 exactly once?
\[
\begin{array}{c}
(A) 27 \quad (B) 59 \quad (C) 77 \quad (D) 91 \quad (E) 171
\end{array}
\]

Part B: Each correct answer is worth 6.

11. The same sequence of four symbols repeats to form the following pattern:

\[
\begin{align*}
\heartsuit \spadesuit \spadesuit \heartsuit \\
\spadesuit \spadesuit \heartsuit \heartsuit \spadesuit \spadesuit \\
\heartsuit \spadesuit \spadesuit \heartsuit \spadesuit \heartsuit \heartsuit
\end{align*}
\]

How many times does the symbol $\heartsuit$ occur within the first 53 symbols of the pattern?
\[
\begin{array}{c}
(A) 25 \quad (B) 26 \quad (C) 27 \quad (D) 28 \quad (E) 29
\end{array}
\]

12. If $x = 11$, $y = -8$, and $2x - 3z = 5y$, what is the value of $z$?
\[
\begin{array}{c}
(A) -6 \quad (B) 13 \quad (C) 54 \quad (D) \frac{62}{3} \quad (E) -\frac{71}{3}
\end{array}
\]

13. Which number from the set \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\} must be removed so that the mean (average) of the numbers remaining in the set is 6.1?
\[
\begin{array}{c}
(A) 4 \quad (B) 5 \quad (C) 6 \quad (D) 7 \quad (E) 8
\end{array}
\]

14. In the diagram, $PQR$ is a straight line segment and $QS = QT$. Also, $\angle PQS = x^\circ$ and $\angle TQR = 3x^\circ$. If $\angle QTS = 76^\circ$, the value of $x$ is
\[
\begin{array}{c}
(A) 28 \quad (B) 38 \quad (C) 26 \quad (D) 152 \quad (E) 45
\end{array}
\]

15. If $4^n = 64^2$, then $n$ equals
\[
\begin{array}{c}
(A) 3 \quad (B) 5 \quad (C) 6 \quad (D) 8 \quad (E) 12
\end{array}
\]
16. An integer \( x \) is chosen so that \( 3x + 1 \) is an even integer. Which of the following must be an odd integer?

(A) \( x + 3 \)  
(B) \( x - 3 \)  
(C) \( 2x \)  
(D) \( 7x + 4 \)  
(E) \( 5x + 3 \)

17. The graph shows styles of music on a playlist. Country music songs are added to the playlist so that now 40% of the songs are Country. If the ratio of Hip Hop songs to Pop songs remains the same, what percentage of the total number of songs are now Hip Hop?

(A) 7  
(B) 15  
(C) 21  
(D) 35  
(E) 39

18. In the diagram, \( PQRS \) is a square with side length 2. Each of \( P, Q, R, \) and \( S \) is the centre of a circle with radius 1. What is the area of the shaded region?

(A) \( 16 - \pi^2 \)  
(B) \( 16 - 4\pi \)  
(C) \( 4 - 4\pi \)  
(D) \( 4 - 4\pi^2 \)  
(E) \( 4 - \pi \)

19. The rectangular flag shown is divided into seven stripes of equal height. The height of the flag is \( h \) and the length of the flag is twice its height. The total area of the four shaded regions is 1400 cm\(^2\). What is the height of the flag?

(A) 70 cm  
(B) 200 cm  
(C) 35 cm  
(D) 1225 cm  
(E) 14 cm

20. Sam rolls a fair four-sided die containing the numbers 1, 2, 3, and 4. Tyler rolls a fair six-sided die containing the numbers 1, 2, 3, 4, 5, and 6. What is the probability that Sam rolls a larger number than Tyler?

(A) \( \frac{1}{5} \)  
(B) \( \frac{5}{12} \)  
(C) \( \frac{3}{5} \)  
(D) \( \frac{3}{4} \)  
(E) \( \frac{1}{4} \)
Part C: Each correct answer is worth 8.

21. The integer 636 405 may be written as the product of three 2-digit positive integers. The sum of these three integers is
   (A) 259    (B) 132    (C) 74    (D) 140    (E) 192

22. A water tower in the shape of a cylinder has radius 10 m and height 30 m. A spiral staircase, with constant slope, circles once around the outside of the water tower. A vertical ladder of height 5 m then extends to the top of the tower. Which of the following is closest to the total distance along the staircase and up the ladder to the top of the tower?
   (A) 72.6 m    (B) 320.2 m    (C) 74.6 m
   (D) 67.6 m    (E) 45.1 m

23. Joshua chooses five distinct numbers. In how many different ways can he assign these numbers to the variables \( p, q, r, s, \) and \( t \) so that \( p < s, q < s, r < t, \) and \( s < t? \)
   (A) 4    (B) 5    (C) 6    (D) 8    (E) 15

24. Pascal High School organized three different trips. Fifty percent of the students went on the first trip, 80% went on the second trip, and 90% went on the third trip. A total of 160 students went on all three trips, and all of the other students went on exactly two trips. How many students are at Pascal High School?
   (A) 1400    (B) 600    (C) 1200    (D) 800    (E) 1600

25. The \( GEB \) sequence 1, 3, 7, 12, \ldots is defined by the following properties:
   (i) the \( GEB \) sequence is increasing (that is, each term is larger than the previous term),
   (ii) the sequence formed using the differences between each pair of consecutive terms in the \( GEB \) sequence (namely, the sequence 2, 4, 5, \ldots) is increasing, and
   (iii) each positive integer that does not occur in the \( GEB \) sequence occurs exactly once in the sequence of differences in (ii).
   What is the 100th term of the \( GEB \) sequence?
   (A) 5751    (B) 5724    (C) 5711    (D) 5777    (E) 5764
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Part A: Each correct answer is worth 5.

1. The value of \(\frac{1 + (3 \times 5)}{2}\) is
   (A) 2   (B) 3   (C) 6   (D) 8   (E) 16

2. The circle graph shows the results of asking 200 students to choose pizza, Thai food, or Greek food. How many students chose Greek food?
   (A) 20   (B) 40   (C) 60
   (D) 80   (E) 100

3. Which of the following is not equal to a whole number?
   (A) \(\frac{60}{12}\)   (B) \(\frac{60}{8}\)   (C) \(\frac{60}{5}\)
   (D) \(\frac{60}{4}\)   (E) \(\frac{60}{3}\)

4. If 7:30 a.m. was 16 minutes ago, in how many minutes will it be 8:00 a.m.?
   (A) 12   (B) 14   (C) 16   (D) 24   (E) 46

5. The expression \(8 \times 10^5 + 4 \times 10^3 + 9 \times 10 + 5\) is equal to
   (A) 804,095   (B) 804,905   (C) 804,950   (D) 840,095   (E) 840,950

6. What is the difference between the largest and smallest of the numbers in the list 0.023, 0.302, 0.203, 0.320, 0.032?
   (A) 0.090   (B) 0.270   (C) 0.343   (D) 0.288   (E) 0.297

7. Anna walked at a constant rate. The graph shows that she walked 600 metres in 4 minutes. If she continued walking at the same rate, how far did she walk in 6 minutes?
   (A) 700 m   (B) 750 m   (C) 800 m
   (D) 900 m   (E) 1000 m
8. According to the ruler shown, what is the length of PQ?
   (A) 2.25  (B) 2.5  (C) 2.0
   (D) 1.5  (E) 1.75

9. If \( y = 1 \) and \( 4x - 2y + 3 = 3x + 3y \), what is the value of \( x \)?
   (A) \(-2\)  (B) 0  (C) 2  (D) 4  (E) 8

10. At the Lacsap Hospital, Emily is a doctor and Robert is a nurse. Not including Emily, there are five doctors and three nurses at the hospital. Not including Robert, there are \( d \) doctors and \( n \) nurses at the hospital. The product of \( d \) and \( n \) is
    (A) 8  (B) 12  (C) 15  (D) 16  (E) 20

Part B: Each correct answer is worth 6.

11. Points with coordinates (1, 1), (5, 1) and (1, 7) are three vertices of a rectangle. What are the coordinates of the fourth vertex of the rectangle?
    (A) (1, 5)  (B) (5, 5)  (C) (5, 7)
    (D) (7, 1)  (E) (7, 5)

12. Seven students shared the cost of a $26.00 pizza. Each student paid either $3.71 or $3.72. How many students paid $3.72?
    (A) 1  (B) 3  (C) 5  (D) 4  (E) 2

13. The operation \( \nabla \) is defined by \( g \nabla h = g^2 - h^2 \). For example, \( 2 \nabla 1 = 2^2 - 1^2 = 3 \). If \( g > 0 \) and \( g \nabla 6 = 45 \), the value of \( g \) is
    (A) 39  (B) 6  (C) 81  (D) 3  (E) 9

14. In the diagram, the horizontal distance between adjacent dots in the same row is 1. Also, the vertical distance between adjacent dots in the same column is 1. What is the perimeter of quadrilateral \( PQRS \)?
    (A) 12  (B) 13  (C) 14
    (D) 15  (E) 16

15. A hockey team has 6 more red helmets than blue helmets. The ratio of red helmets to blue helmets is 5 : 3. The total number of red helmets and blue helmets is
    (A) 16  (B) 18  (C) 24  (D) 30  (E) 32
16. The diagram shows a square quilt that is made up of identical squares and two sizes of right-angled isosceles triangles. What percentage of the quilt is shaded?
(A) 36%  (B) 40%  (C) 44%
(D) 48%  (E) 50%

17. In the diagram, points $R$ and $S$ lie on $QT$. Also, $\angle PTQ = 62^\circ$, $\angle RPS = 34^\circ$, and $\angle QPR = x^\circ$. What is the value of $x$?
(A) 11  (B) 28  (C) 17
(D) 31  (E) 34

18. The entire exterior of a solid $6 \times 6 \times 3$ rectangular prism is painted. Then, the prism is cut into $1 \times 1 \times 1$ cubes. How many of these cubes have no painted faces?
(A) 16  (B) 32  (C) 36  (D) 50  (E) 54

19. In the diagram, rectangle $PRTV$ is divided into four rectangles. The area of rectangle $PQXW$ is 9. The area of rectangle $QRSX$ is 10. The area of rectangle $XSTU$ is 15. What is the area of rectangle $WXUV$?
(A) 6  (B) $\frac{27}{2}$  (C) 14
(D) $\frac{50}{3}$  (E) $\frac{95}{2}$

20. When the three-digit positive integer $N$ is divided by 10, 11 or 12, the remainder is 7. What is the sum of the digits of $N$?
(A) 15  (B) 17  (C) 23  (D) 11  (E) 19

Part C: Each correct answer is worth 8.

21. A string has been cut into 4 pieces, all of different lengths. The length of each piece is 2 times the length of the next smaller piece. What fraction of the original string is the longest piece?
(A) $\frac{8}{15}$  (B) $\frac{2}{5}$  (C) $\frac{1}{2}$  (D) $\frac{6}{13}$  (E) $\frac{1}{4}$
22. Two circles with equal radii intersect as shown. The area of the shaded region equals the sum of the areas of the two unshaded regions. If the area of the shaded region is \(216 \pi\), what is the circumference of each circle?

(A) \(18 \pi\)  
(B) \(27 \pi\)  
(C) \(36 \pi\)  
(D) \(108 \pi\)  
(E) \(324 \pi\)

23. Mike has two containers. One container is a rectangular prism with width 2 cm, length 4 cm, and height 10 cm. The other is a right cylinder with radius 1 cm and height 10 cm. Both containers sit on a flat surface. Water has been poured into the two containers so that the height of the water in both containers is the same. If the combined volume of the water in the two containers is \(80 \text{ cm}^3\), then the height of the water in each container is closest to

(A) 6.8 cm  
(B) 7.2 cm  
(C) 7.8 cm  
(D) 8.2 cm  
(E) 8.6 cm

24. The smallest of nine consecutive integers is 2012. These nine integers are placed in the circles to the right. The sum of the three integers along each of the four lines is the same. If this sum is as small as possible, what is the value of \(u\)?

(A) 2012  
(B) 2013  
(C) 2014  
(D) 2015  
(E) 2016

25. There are four people in a room. For every two people, there is a 50\% chance that they are friends. Two people are connected if:

- they are friends, or
- a third person is friends with both of them, or
- they have different friends who are friends of each other.

What is the probability that every pair of people in this room is connected?

(A) \(\frac{18}{32}\)  
(B) \(\frac{20}{32}\)  
(C) \(\frac{22}{32}\)  
(D) \(\frac{19}{32}\)  
(E) \(\frac{21}{32}\)
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Pascal Contest
(Grade 9)
Thursday, February 24, 2011

Time: 60 minutes
Calculators are permitted

Instructions
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.

The names of some top-scoring students will be published in the PCF Results on our Web site, http://www.cemc.uwaterloo.ca.
Part A: Each correct answer is worth 5.

1. What is the value of $6 \times (5 - 2) + 4$?
   - (A) 18
   - (B) 22
   - (C) 24
   - (D) 32
   - (E) 42

2. Nine hundred forty-three minus eighty-seven equals
   - (A) $-1030$
   - (B) $-856$
   - (C) 770
   - (D) 1030
   - (E) 856

3. Which list of numbers is written in increasing order?
   - (A) $2011, \sqrt{2011}, 2011^2$
   - (B) $2011, 2011^2, \sqrt{2011}$
   - (C) $\sqrt{2011}, 2011, 2011^2$
   - (D) $\sqrt{2011}, 2011^2, 2011$
   - (E) $2011^2, \sqrt{2011}, 2011$

4. The graph shows the nutritional contents of a Pascal Burger. Which ratio compares the mass of fats to the mass of carbohydrates?
   - (A) $3 : 2$
   - (B) $2 : 3$
   - (C) $2 : 1$
   - (D) $4 : 3$
   - (E) $3 : 4$

5. When $x = -2$, the value of $(x + 1)^3$ is
   - (A) $-1$
   - (B) $-8$
   - (C) $-5$
   - (D) 1
   - (E) $-3$

6. Peyton puts 30 L of oil and 15 L of vinegar into a large empty can. He then adds 15 L of oil to create a new mixture. What percentage of the new mixture is oil?
   - (A) 75
   - (B) 25
   - (C) 45
   - (D) 50
   - (E) 60

7. Three 1 by 1 by 1 cubes are joined side by side, as shown. What is the surface area of the resulting prism?
   - (A) 13
   - (B) 14
   - (C) 15
   - (D) 16
   - (E) 17

8. The 17th day of a month is Saturday. The first day of that month was
   - (A) Monday
   - (B) Tuesday
   - (C) Wednesday
   - (D) Thursday
   - (E) Friday
9. Two rectangles $PQUV$ and $WSTV$ overlap as shown. What is the area of $PQRSTV$?

(A) 35  (B) 24  (C) 25  
(D) 17  (E) 23

10. John lists the integers from 1 to 20 in increasing order. He then erases the first half of the integers in the list and rewrites them in order at the end of the second half of the list. Which integer in the new list has exactly 12 integers to its left?

(A) 1  (B) 2  (C) 3  (D) 12  (E) 13

**Part B: Each correct answer is worth 6.**

11. Which of the following numbers is closest to 1?

(A) $\frac{11}{10}$  (B) $\frac{111}{100}$  (C) 1.101  (D) $\frac{1111}{1000}$  (E) 1.011

12. The number of odd integers between $\frac{17}{4}$ and $\frac{35}{2}$ is

(A) 4  (B) 5  (C) 6  (D) 7  (E) 8

13. The first four terms of a sequence are 1, 4, 2, and 3. Beginning with the fifth term in the sequence, each term is the sum of the previous four terms. Therefore, the fifth term is 10. What is the eighth term?

(A) 66  (B) 65  (C) 69  (D) 134  (E) 129

14. In the diagram, a garden is enclosed by six straight fences. If the area of the garden is 97 m$^2$, what is the length of the fence around the garden?

(A) 48 m  (B) 47 m  (C) 40 m  
(D) 38 m  (E) 37 m

15. Six friends ate at a restaurant and agreed to share the bill equally. Because Luxmi forgot her money, each of her five friends paid an extra $3 to cover her portion of the total bill. What was the total bill?

(A) $90  (B) $84  (C) $75  (D) $108  (E) $60

16. The set $S = \{1, 2, 3, \ldots, 49, 50\}$ contains the first 50 positive integers. After the multiples of 2 and the multiples of 3 are removed, how many integers remain in the set $S$?

(A) 8  (B) 9  (C) 16  (D) 17  (E) 18
17. In the subtraction shown, $K$, $L$, $M$, and $N$ are digits. What is the value of $K + L + M + N$?
(A) 17  (B) 18  (C) 19
(D) 23  (E) 27

18. On the number line, points $M$ and $N$ divide $LP$ into three equal parts. What is the value at $M$?
(A) $\frac{1}{7}$  (B) $\frac{1}{8}$  (C) $\frac{1}{9}$
(D) $\frac{1}{10}$  (E) $\frac{1}{11}$

19. Two circles are centred at the origin, as shown. The point $P(8,6)$ is on the larger circle and the point $S(0,k)$ is on the smaller circle. If $QR = 3$, what is the value of $k$?
(A) 3.5  (B) 4  (C) 6
(D) 6.5  (E) 7

20. In the diagram, $PR$, $PS$, $QS$, $QT$, and $RT$ are straight line segments. $QT$ intersects $PR$ and $PS$ at $U$ and $V$, respectively. If $PU = PV$, $\angle UPV = 24^\circ$, $\angle PSQ = x^\circ$, and $\angle TQS = y^\circ$, what is the value of $x + y$?
(A) 48  (B) 66  (C) 72
(D) 78  (E) 156

Part C: Each correct answer is worth 8.

21. In the diagram, there are 26 levels, labelled $A, B, C, \ldots, Z$. There is one dot on level $A$. Each of levels $B, D, F, H, J, \ldots$, and $Z$ contains twice as many dots as the level immediately above. Each of levels $C, E, G, I, K, \ldots$, and $Y$ contains the same number of dots as the level immediately above. How many dots does level $Z$ contain?
(A) 1024  (B) 2048  (C) 4096
(D) 8192  (E) 16384
22. Each of the integers 1 to 7 is to be written, one in each circle in the diagram. The sum of the three integers in any straight line is to be the same. In how many different ways can the centre circle be filled?

(A) 1  (B) 2  (C) 3  
(D) 4  (E) 5

23. An ordered list of four numbers is called a *quadruple*.
A quadruple \((p, q, r, s)\) of integers with \(p, q, r, s \geq 0\) is chosen at random such that

\[
2p + q + r + s = 4
\]

What is the probability that \(p + q + r + s = 3\)?

(A) \(\frac{3}{22}\)  (B) \(\frac{3}{11}\)  (C) \(\frac{3}{19}\)  (D) \(\frac{6}{19}\)  (E) \(\frac{2}{7}\)

24. Let \(n\) be the largest integer for which \(14^n\) has exactly 100 digits. Counting from right to left, what is the 68th digit of \(n\)?

(A) 1  (B) 2  (C) 4  (D) 5  (E) 8

25. Dolly, Molly and Polly each can walk at 6 km/h. Their one motorcycle, which travels at 90 km/h, can accommodate at most two of them at once (and cannot drive by itself!). Let \(t\) hours be the time taken for all three of them to reach a point 135 km away. Ignoring the time required to start, stop or change directions, what is true about the smallest possible value of \(t\)?

(A) \(t < 3.9\)  (B) \(3.9 \leq t < 4.1\)  (C) \(4.1 \leq t < 4.3\)  
(D) \(4.3 \leq t < 4.5\)  (E) \(t \geq 4.5\)
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www.cemc.uwaterloo.ca
Pascal Contest  (Grade 9)  
Thursday, February 25, 2010

Time:  60 minutes  

Calculators are permitted

Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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9. When your supervisor tells you to begin, you will have sixty minutes of working time.

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Part A: Each correct answer is worth 5.

1. Which of the following is closest in value to $1.00?  
   (A) $0.50 (B) $0.90 (C) $0.95 (D) $1.01 (E) $1.15

2. The value of $\frac{(20 - 16) \times (12 + 8)}{4}$ is  
   (A) 5 (B) 9 (C) 20 (D) 44 (E) 56

3. To make pizza dough, Luca mixes 50 mL of milk for every 250 mL of flour. How much milk does he mix with 750 mL of flour?  
   (A) 100 mL (B) 125 mL (C) 150 mL (D) 200 mL (E) 250 mL

4. One of the following 8 figures is randomly chosen. What is the probability that the chosen figure is a triangle?  
   ![Diagram of 8 figures]
   (A) $\frac{3}{8}$ (B) $\frac{3}{4}$ (C) $\frac{7}{8}$ (D) $\frac{1}{2}$ (E) $\frac{1}{3}$

5. If $\frac{1}{9} + \frac{1}{18} = \frac{1}{\Box}$, then the number that replaces the $\Box$ to make the equation true is  
   (A) 2 (B) 3 (C) 6 (D) 9 (E) 18

6. Squares of side length 1 are arranged to form the figure shown. What is the perimeter of the figure?  
   (A) 12 (B) 16 (C) 20 (D) 24 (E) 26

7. The value of $\sqrt{3^3 + 3^3 + 3^3}$ is  
   (A) 3 (B) 9 (C) 27 (D) 81 (E) 243

8. In the diagram, the points are equally spaced on the number line. What number is represented by point $P$?  
   (A) 7.48 (B) 7.49 (C) 7.50 (D) 7.51 (E) 7.52

9. The nine interior intersection points on a 4 by 4 grid of squares are shown. How many interior intersection points are there on a 12 by 12 grid of squares?  
   (A) 100 (B) 121 (C) 132 (D) 144 (E) 169
10. The diagram shows a circle graph which shows the amount of homework done each day by Mr. Auckland’s Grade 9 class. Based on the circle graph, what percentage of students do at least one hour of homework per day?

(A) 25%  (B) 33%  (C) 50%  (D) 67%  (E) 75%

Part B: Each correct answer is worth 6.

11. Several three-legged tables and four-legged tables have a total of 23 legs. If there is more than one table of each type, what is the number of three-legged tables?

(A) 6  (B) 7  (C) 3  (D) 4  (E) 5

12. Twelve 1 by 1 squares form a rectangle, as shown. What is the total area of the shaded regions?

(A) 8  (B) 9  (C) 10  (D) 11  (E) 12

13. There are 400 students at Cayley H.S., where the ratio of boys to girls is 3 : 2. There are 600 students at Fermat C.I., where the ratio of boys to girls is 2 : 3. When considering all the students from both schools, what is the ratio of boys to girls?

(A) 2 : 3  (B) 12 : 13  (C) 1 : 1  (D) 6 : 5  (E) 3 : 2

14. The numbered net shown is folded to form a cube. What is the product of the numbers on the four faces sharing an edge with the face numbered 1?

(A) 120  (B) 144  (C) 180  (D) 240  (E) 360

15. If 10% of $s$ is $t$, then $s$ equals

(A) 0.1$t$  (B) 0.9$t$  (C) 9$t$  (D) 10$t$  (E) 90$t$

16. Four identical squares are cut from the corners of the rectangular sheet of cardboard shown. This sheet is then folded along the dotted lines and taped to make a box with an open top. The base of the box measures 5 cm by 4 cm. The volume of the box is 60 cm$^3$. What was the area of the original sheet of cardboard?

(A) 56 cm$^2$  (B) 110 cm$^2$  (C) 156 cm$^2$  (D) 180 cm$^2$  (E) 210 cm$^2$
17. In the diagram, \( PW \) is parallel to \( QX \), \( S \) and \( T \) lie on \( QX \), and \( U \) and \( V \) are the points of intersection of \( PW \) with \( SR \) and \( TR \), respectively. If \( \angle SUV = 120^\circ \) and \( \angle VTX = 112^\circ \), what is the measure of \( \angle URV \)?

(A) 52°  (B) 56°  (C) 60°  
(D) 64°  (E) 68°

18. The gas tank in Catherine’s car is \( \frac{1}{8} \) full. When 30 litres of gas are added, the tank becomes \( \frac{3}{4} \) full. If the gas costs Catherine \$1.38 per litre, how much will it cost her to fill the remaining quarter of the tank?

(A) $8.80  (B) $13.80  (C) $16.56  (D) $24.84  (E) $41.40

19. In the diagram, points \( U \), \( V \), \( W \), \( X \), \( Y \), and \( Z \) lie on a straight line with \( UV = VW = WX = XY = YZ = 5 \). Semicircles with diameters \( UZ \), \( UV \), \( VW \), \( WX \), \( XY \), and \( YZ \) create the shape shown. What is the area of the shaded region?

(A) \( \frac{325\pi}{4} \)  (B) \( \frac{375\pi}{4} \)  (C) \( \frac{325\pi}{2} \)  
(D) \( \frac{625\pi}{4} \)  (E) \( \frac{625\pi}{2} \)

20. The odd integers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9, and 17 are placed as shown, what is the value of \( x \)?

(A) 7  (B) 11  (C) 13  
(D) 15  (E) 19

### Part C: Each correct answer is worth 8.

21. In the diagram, each of the five boxes is to contain a number. Each number in a shaded box must be the average of the number in the box to the left of it and the number in the box to the right of it. What is the value of \( x \)?

(A) 28  (B) 30  (C) 31  
(D) 32  (E) 34
22. Rhombus $PQRS$ is inscribed in rectangle $JKLM$, as shown. (A rhombus is a quadrilateral with four equal side lengths.) Segments $PZ$ and $XR$ are parallel to $JM$. Segments $QW$ and $YS$ are parallel to $JK$. If $JP = 39$, $JS = 52$, and $KQ = 25$, what is the perimeter of rectangle $WXYZ$?

(A) 48   (B) 58   (C) 84
(D) 96   (E) 108

23. The product of $N$ consecutive four-digit positive integers is divisible by $2010^2$. What is the least possible value of $N$?

(A) 5   (B) 12   (C) 10   (D) 6   (E) 7

24. A sequence consists of 2010 terms. Each term after the first is 1 larger than the previous term. The sum of the 2010 terms is 5307. When every second term is added up, starting with the first term and ending with the second last term, the sum is

(A) 2155   (B) 2153   (C) 2151   (D) 2149   (E) 2147

25. Six soccer teams are competing in a tournament in Waterloo. Every team is to play three games, each against a different team. (Note that not every pair of teams plays a game together.) Judene is in charge of pairing up the teams to create a schedule of games that will be played. Ignoring the order and times of the games, how many different schedules are possible?

(A) 90   (B) 100   (C) 80   (D) 60   (E) 70
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Pascal Contest (Grade 9)
Wednesday, February 18, 2009

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Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $2 \times 9 - \sqrt{36} + 1$?
   (A) 7   (B) 11   (C) 8   (D) 13   (E) 4

2. The graph shows the number of hours Deepit worked over a three day period. What is the total number of hours that he worked on Saturday and Sunday?
   (A) 2   (B) 4   (C) 6   (D) 8   (E) 10

3. The cost of 1 piece of gum is 1 cent. What is the cost of 1000 pieces of gum?
   (A) $0.01   (B) $0.10   (C) $1.00   (D) $10.00   (E) $100.00

4. There are 18 classes at Webster Middle School. Each class has 28 students. On Monday, 496 students were at school. How many students were absent?
   (A) 8   (B) 11   (C) 18   (D) 26   (E) 29

5. In the diagram, the value of $x$ is
   (A) 15   (B) 20   (C) 24   (D) 30   (E) 36

6. What is the value of $(-1)^5 - (-1)^4$?
   (A) -2   (B) -1   (C) 0   (D) 1   (E) 2

7. In the diagram, $\triangle PQR$ is right-angled at $Q$, $PQ$ is horizontal and $QR$ is vertical. What are the coordinates of $Q$?
   (A) (5, 2)   (B) (5, 0)   (C) (5, 1)   (D) (4, 1)   (E) (1, 5)

8. If $y = 3$, the value of $\frac{y^3 + y}{y^2 - y}$ is
   (A) 2   (B) 3   (C) 4   (D) 5   (E) 6
9. In the diagram, any ♣ may be moved to any unoccupied space. What is the smallest number of ♣’s that must be moved so that each row and each column contains three ♣’s?
   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

10. If \( z = 4 \), \( x + y = 7 \), and \( x + z = 8 \), what is the value of \( x + y + z \)?
   (A) 9   (B) 17   (C) 11   (D) 19   (E) 13

Part B: Each correct answer is worth 6.

11. When the numbers 5.076, 5.076, 5.07, 5.076, 5.076 are arranged in increasing order, the number in the middle is
   (A) 5.076   (B) 5.076   (C) 5.07   (D) 5.076   (E) 5.076

12. If Francis spends \( \frac{1}{3} \) of his day sleeping, \( \frac{1}{4} \) of his day studying and \( \frac{1}{8} \) of his day eating, how many hours in the day does he have left?
   (A) 4   (B) 6   (C) 5   (D) 7   (E) 9

13. In the diagram, \( QRS \) is a straight line. What is the measure of \( \angle RPS \)?
   (A) 27°   (B) 47°   (C) 48°   (D) 65°   (E) 67°

14. In the diagram, \( O \) is the centre of a circle with radii \( OP = OQ = 5 \). The perimeter of the shaded region, including the two radii, is closest to
   (A) 34   (B) 41   (C) 52   (D) 59   (E) 68

15. The increasing list of five different integers \{3, 4, 5, 8, 9\} has a sum of 29. How many increasing lists of five different single-digit positive integers have a sum of 33?
   (A) 1   (B) 2   (C) 3   (D) 4   (E) 5

16. In the diagram, a \( 4 \times 9 \) grid \( PQTV \) is formed from thirty-six \( 1 \times 1 \) squares. Lines \( PR \) and \( US \) are drawn with \( R \) and \( S \) on \( QT \). What is the ratio of the shaded area to the unshaded area?
   (A) 5 : 9   (B) 9 : 8   (C) 4 : 5   (D) 9 : 5   (E) 5 : 4
17. Nerissa writes five mathematics tests, each worth the same amount, and obtains an average of 73%. After her teacher deletes one of her test marks, Nerissa’s new average is 76%. What was the mark on the test that the teacher deleted?

(A) 60%  (B) 61%  (C) 62%  (D) 63%  (E) 64%

18. Every 4 years, the population of the town of Arloe doubles. On December 31, 2008, the population of Arloe was 3456. What was the population on December 31, 1988?

(A) 54  (B) 576  (C) 216  (D) 108  (E) 864

19. The distance from Coe Hill to Calabogie is 150 kilometres. Pat leaves Coe Hill at 1:00 p.m. and drives at a speed of 80 km/h for the first 60 km. How fast must he travel for the remainder of the trip to reach Calabogie at 3:00 p.m.?

(A) 65 km/h  (B) 70 km/h  (C) 72 km/h  (D) 75 km/h  (E) 90 km/h

20. Different positive integers can be written in the eight empty circles so that the product of any three integers in a straight line is 3240. What is the largest possible sum of the eight numbers surrounding 45?

(A) 139  (B) 211  (C) 156  (D) 159  (E) 160

Part C: Each correct answer is worth 8.

21. Alice rolls a standard 6-sided die. Bob rolls a second standard 6-sided die. Alice wins if the values shown differ by 1. What is the probability that Alice wins?

(A) $\frac{1}{3}$  (B) $\frac{2}{5}$  (C) $\frac{5}{18}$  (D) $\frac{1}{6}$  (E) $\frac{5}{36}$

22. In the diagram, $PQ$ and $RS$ are diameters of a circle with radius 4. If $PQ$ and $RS$ are perpendicular, what is the area of the shaded region?

(A) $16 + 4\pi$  (B) $8 + 8\pi$  (C) $8 + 4\pi$

(D) $16 + 16\pi$  (E) $16 + 8\pi$

23. A one-dollar coin should have a mass of 7.0 g. Each individual coin may be lighter or heavier by as much as 2.14%. Joshua has a number of these coins and determines that they have a total mass of 1 kg. What is the difference between the greatest possible number and the least possible number of these coins that he could have?

(A) 4  (B) 5  (C) 6  (D) 7  (E) 8
24. Eight identical spheres, each of diameter 20, fit tightly into a cube of side length 40 so that each sphere just touches three of the faces of the cube. The radius of the largest sphere that will fit in the central space, just touching all eight spheres, is closest to

(A) 7.0   (B) 7.3   (C) 7.6   (D) 7.9   (E) 8.2

25. Starting with the input \((m, n)\), Machine A gives the output \((n, m)\).
Starting with the input \((m, n)\), Machine B gives the output \((m + 3n, n)\).
Starting with the input \((m, n)\), Machine C gives the output \((m - 2n, n)\).
Natalie starts with the pair \((0, 1)\) and inputs it into one of the machines. She takes the output and inputs it into any one of the machines. She continues to take the output that she receives and inputs it into any one of the machines. (For example, starting with \((0, 1)\), she could use machines B, B, A, C, B in that order to obtain the output \((7, 6)\).) Which of the following pairs is impossible for her to obtain after repeating this process any number of times?

(A) \((2009, 1016)\)   (B) \((2009, 1004)\)   (C) \((2009, 1002)\)
(D) \((2009, 1008)\)   (E) \((2009, 1032)\)
Canadian Mathematics Competition

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Pascal Contest (Grade 9)
Tuesday, February 19, 2008

Time: 60 minutes
Calculators are permitted
Instructions

1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.

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Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. The value of $\frac{2 + 3 + 4}{2 \times 3 \times 4}$ is

   (A) 1      (B) $\frac{5}{6}$     (C) $\frac{7}{12}$     (D) 3     (E) $\frac{3}{8}$

2. If $3x - 9 = 12$, then the value of $6x$ is

   (A) 42    (B) 24    (C) 6    (D) 32    (E) 52

3. $\sqrt{5^2 - 4^2}$ is equal to

   (A) 1    (B) 3    (C) 5    (D) 4    (E) 2

4. In the diagram, $JLMR$ and $JKQR$ are rectangles. Also, $JR = 2$, $RQ = 3$ and $JL = 8$. What is the area of rectangle $KLMQ$?

   (A) 6    (B) 16    (C) 10    (D) 15    (E) 24

5. If $x = 12$ and $y = -6$, then the value of $\frac{3x + y}{x - y}$ is

   (A) 3    (B) 7    (C) $\frac{5}{3}$    (D) 5    (E) $\frac{7}{3}$

6. In the diagram, $PQR$ is a straight line. The value of $x$ is

   (A) 72    (B) 44    (C) 58    (D) 64    (E) 52

7. A bag contains 5 red, 6 green, 7 yellow, and 8 blue jelly beans. A jelly bean is selected at random. What is the probability that it is blue?

   (A) $\frac{5}{26}$    (B) $\frac{3}{13}$    (C) $\frac{7}{26}$    (D) $\frac{4}{13}$    (E) $\frac{6}{13}$

8. Olave sold 108 apples at a constant rate over 6 hours. If she continues to sell apples at the same rate, how many apples will she sell in the next 1 hour and 30 minutes?

   (A) 27    (B) 33    (C) 45    (D) 36    (E) 21

9. In the diagram, the rectangular wire grid contains 15 identical squares. The length of the rectangular grid is 10. What is the length of wire needed to construct the grid?

   (A) 60    (B) 70    (C) 120    (D) 66    (E) 76
10. On the number line, $S$ is three-quarters of the way from $P$ to $Q$. Also, $T$ is one-third of the way from $P$ to $Q$. What is the distance along the number line from $T$ to $S$?

(A) 20  (B) 15  (C) 6  
(D) 25  (E) 31

Part B: Each correct answer is worth 6.

11. At Mathville Junior High School, 30 boys and 20 girls wrote the Pascal Contest. Certificates were awarded to 30% of the boys and 40% of the girls. What percentage of all of the participating students received certificates?

(A) 34  (B) 35  (C) 36  (D) 17  (E) 70

12. In the diagram, the perimeter of the rectangle is 56. What is its area?

(A) 247  (B) 187  (C) 169  
(D) 135  (E) 775

13. $2^3 \times 2^2 \times 3^3 \times 3^2$ is equal to

(A) $6^5$  (B) $6^6$  (C) $6^{10}$  (D) $36^{10}$  (E) $36^{36}$

14. Two 3-digit numbers, $abc$ and $def$, have the following property:

\[
\begin{array}{ccc}
    a & b & c \\
    + & d & e & f \\
    \hline
    1 & 0 & 0 & 0
\end{array}
\]

None of $a$, $b$, $c$, $d$, $e$, or $f$ is 0. What is $a + b + c + d + e + f$?

(A) 10  (B) 19  (C) 21  (D) 28  (E) 30

15. In the diagram, what is the perimeter of $\triangle PQR$?

(A) 63  (B) 60  (C) 55  
(D) 85  (E) 70

16. A circle has an area of $M$ cm$^2$ and a circumference of $N$ cm. If $\frac{M}{N} = 20$, what is the radius of the circle, in cm?

(A) 10  (B) 20  (C) 40  (D) $\frac{1}{10}$  (E) $\frac{1}{20}$

17. The surface area of a large cube is 5400 cm$^2$. This cube is cut into a number of identical smaller cubes. Each smaller cube has a volume of 216 cm$^3$. How many smaller cubes are there?

(A) 25  (B) 125  (C) 164  (D) 180  (E) 216
18. Alex has $2.65. He has only dimes (worth $0.10 each) and quarters (worth $0.25 each). He has more dimes than quarters. What is the smallest number of coins that Alex could have?
(A) 25  (B) 16  (C) 13  (D) 19  (E) 22

19. An integer is defined to be upright if the sum of its first two digits equals its third digit. For example, 145 is an upright integer since $1 + 4 = 5$. How many positive 3-digit integers are upright?
(A) 28  (B) 39  (C) 36  (D) 45  (E) 50

20. Four of the six numbers 1867, 1993, 2019, 2025, 2109, and 2121 have a mean (average) of 2008. What is the mean (average) of the other two numbers?
(A) 1994  (B) 2006  (C) 2022  (D) 2051  (E) 2064

Part C: Each correct answer is worth 8.

21. If $3 \leq p \leq 10$ and $12 \leq q \leq 21$, then the difference between the largest and smallest possible values of $\frac{p}{q}$ is
(A) $\frac{29}{42}$  (B) $\frac{29}{5}$  (C) $\frac{19}{70}$  (D) $\frac{19}{12}$  (E) $\frac{19}{84}$

22. Ginger walks at 4 km/h and runs at 6 km/h. She saves $3\frac{3}{4}$ minutes by running instead of walking from her home to her school. What is the distance, in kilometres, from her home to her school?
(A) 7$\frac{1}{2}$  (B) 3$\frac{3}{4}$  (C) 1$\frac{7}{8}$  (D) 1$\frac{1}{4}$  (E) $\frac{3}{4}$

23. Four pieces of lumber are placed in parallel positions, as shown, perpendicular to line $M$:
- Piece $W$ is 5 m long
- Piece $X$ is 3 m long and its left end is 3 m from line $M$
- Piece $Y$ is 5 m long and is 2 m from line $M$
- Piece $Z$ is 4 m long and is 1.5 m from from line $M$

A single cut, perpendicular to the pieces of lumber, is made along the dotted line $L$. The total length of lumber on each side of $L$ is the same. What is the length, in metres, of the part of piece $W$ to the left of the cut?
(A) 4.25  (B) 3.5  (C) 3.25  (D) 3.75  (E) 4.0

24. Five circles are drawn on a piece of paper and connected as shown. Each circle must be coloured red, blue or green. Two circles connected by a straight line may not be coloured the same. How many different ways are there to colour the circles?
(A) 24  (B) 60  (C) 72  (D) 36  (E) 48
25. In the diagram, \( \triangle PQR \) is right-angled at \( P \) and has \( PQ = 2 \) and \( PR = 2\sqrt{3} \). Altitude \( PL \) intersects median \( RM \) at \( F \). What is the length of \( PF \)?

(A) \( \frac{\sqrt{3}}{2} \)  
(B) \( \frac{3\sqrt{3}}{7} \)  
(C) \( \frac{4\sqrt{3}}{7} \)  
(D) \( \frac{5\sqrt{3}}{9} \)  
(E) \( \frac{3\sqrt{3}}{5} \)
Canadian Mathematics Competition

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Pascal Contest (Grade 9)
Tuesday, February 20, 2007

Time: 60 minutes
Calculators are permitted

Instructions
1. Do not open the Contest booklet until you are told to do so.
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3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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9. When your supervisor tells you to begin, you will have sixty minutes of working time.

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Part A: Each correct answer is worth 5.

1. The value of $3 \times (7 - 5) - 5$ is
   (A) 11       (B) 1       (C) -30     (D) 11       (E) -1

2. Which of the following is the best estimate for the value of $x$ shown on the number line?
   (A) 1.3       (B) -1.3     (C) -2.7
   (D) 0.7       (E) -0.7

3. What fraction of the area of rectangle $ABCD$ is the area of the shaded square?
   (A) $\frac{1}{15}$       (B) $\frac{1}{8}$       (C) $\frac{1}{10}$
   (D) $\frac{1}{4}$       (E) $\frac{1}{12}$

4. The value of $2^5 - 5^2$ is
   (A) 0       (B) -3       (C) -7       (D) 3       (E) 7

5. The table shows the pay Leona earned for two different shifts at the same fixed hourly rate. How much will she earn for a five hour shift at this rate?
<table>
<thead>
<tr>
<th>Shift</th>
<th>Total Pay</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 hours</td>
<td>$24.75</td>
</tr>
<tr>
<td>6 hours</td>
<td>$49.50</td>
</tr>
</tbody>
</table>
   (A) $43.75       (B) $46.25     (C) $38.75
   (D) $36.25       (E) $41.25

6. The value of $\frac{\sqrt{64} + \sqrt{36}}{\sqrt{64 + 36}}$ is
   (A) $\frac{7}{5}$       (B) $\frac{16}{5}$     (C) $\frac{1}{5}$
   (D) $\frac{24}{5}$     (E) $\frac{14}{5}$

7. Megan inherits $1\,000\,000$ and Dan inherits $10\,000$. Each donates 10% of his or her inheritance to charity. In total, they donate
   (A) $101\,000$       (B) $110\,000$     (C) $100\,000$
   (D) $11\,000$       (E) $10\,100$

8. In the diagram, what is the area of $\triangle ABC$?
   (A) 36       (B) 54       (C) 108
   (D) 72       (E) 48
9. The value of $\frac{5}{8} - \frac{1}{16}$ is
   (A) larger than $\frac{3}{4}$ \hspace{1cm} (B) larger than $\frac{3}{5}$ \hspace{1cm} (C) larger than $\frac{5}{9}$
   (D) less than $\frac{1}{2}$ \hspace{1cm} (E) less than $\frac{7}{16}$

10. If $M = 2007 ÷ 3$, $N = M ÷ 3$, and $X = M - N$, then the value of $X$ is
   (A) 669 \hspace{1cm} (B) 223 \hspace{1cm} (C) 1338 \hspace{1cm} (D) 892 \hspace{1cm} (E) 446

Part B: Each correct answer is worth 6.

11. The mean (average) of 6, 9 and 18 is equal to the mean (average) of 12 and $y$. What is the value of $y$?
   (A) 22 \hspace{1cm} (B) 21 \hspace{1cm} (C) 10 \hspace{1cm} (D) 11 \hspace{1cm} (E) 5

12. In the diagram, if $\angle PQR = 48^\circ$, what is the measure of $\angle PMN$?
   (A) 60° \hspace{1cm} (B) 42° \hspace{1cm} (C) 48°
   (D) 66° \hspace{1cm} (E) 84°

13. The sum of two different prime numbers is 10. The product of these two numbers is
   (A) 24 \hspace{1cm} (B) 21 \hspace{1cm} (C) 16 \hspace{1cm} (D) 9 \hspace{1cm} (E) 7

14. At Webster High School, the ratio of males to females writing the Pascal Contest is 3 : 7. If there are 21 males writing the Contest, what is the total number of students writing?
   (A) 30 \hspace{1cm} (B) 25 \hspace{1cm} (C) 49 \hspace{1cm} (D) 70 \hspace{1cm} (E) 79

15. Clara knocks over the two stacks of blocks shown in the diagram. She then uses the blocks to build a similar stack whose top layer has one block and each layer below has one more block than the layer above it. If she builds the largest possible stack, how many blocks will be left over?
   (A) 0 \hspace{1cm} (B) 1 \hspace{1cm} (C) 2
   (D) 3 \hspace{1cm} (E) 4

16. In the table, the sum of the numbers in each row, column and diagonal is the same. What is the value of $P + Q + R + S$?
   (A) 56 \hspace{1cm} (B) 60 \hspace{1cm} (C) 64
   (D) 68 \hspace{1cm} (E) 72

17. Norine can retire when her age and the number of years that she has worked add to 85. At present, she is 50 years old and has worked for 19 years. If she works continuously until she retires, how old will she be when she can retire?
   (A) 53 \hspace{1cm} (B) 54 \hspace{1cm} (C) 58 \hspace{1cm} (D) 66 \hspace{1cm} (E) 69
18. In the diagram, what is the perimeter of △PQS?
   (A) 74  (B) 55  (C) 80
   (D) 84  (E) 97

19. The reciprocal of \( \frac{3}{10} \) is \( \left( \frac{1}{x} + 1 \right) \). What is the value of \( x \)?
   (A) \( \frac{7}{3} \)  (B) \( \frac{3}{13} \)  (C) \( \frac{3}{7} \)  (D) \( \frac{5}{3} \)  (E) \( \frac{3}{5} \)

20. In the diagram, rectangle \( ABCD \) is divided into two regions, \( AEFCD \) and \( EBCF \), of equal area. If \( EB = 40 \), \( AD = 80 \) and \( EF = 30 \), what is the length of \( AE \)?
   (A) 20  (B) 24  (C) 10
   (D) 15  (E) 30

Part C: Each correct answer is worth 8.

21. \( P, Q, R, S, \) and \( T \) are five different integers between 2 and 19 inclusive.
   - \( P \) is a two-digit prime number whose digits add up to a prime number.
   - \( Q \) is a multiple of 5.
   - \( R \) is an odd number, but not a prime number.
   - \( S \) is the square of a prime number.
   - \( T \) is a prime number that is also the mean (average) of \( P \) and \( Q \).

Which number is the largest?
   (A) \( P \)  (B) \( Q \)  (C) \( R \)  (D) \( S \)  (E) \( T \)

22. Asafa ran at a speed of 21 km/h from \( P \) to \( Q \) to \( R \) to \( S \), as shown. Florence ran at a constant speed from \( P \) directly to \( R \) and then to \( S \). They left \( P \) at the same time and arrived at \( S \) at the same time. How many minutes after Florence did Asafa arrive at point \( R \)?
   (A) 0  (B) 8  (C) 6
   (D) 7  (E) 5
23. In the diagram, two circles, each with centre $D$, have radii of 1 and 2. The total area of the shaded regions is $\frac{5}{12}$ of the area of the larger circle. What is a possible measure of $\angle ADC$?

(A) 108°  (B) 120°  (C) 90°  
(D) 150°  (E) 135°

24. Starting with the “1” in the centre, the spiral of consecutive integers continues, as shown. What is the sum of the number that appears directly above 2007 and the number that appears directly below 2007?

(A) 4014  (B) 4016  (C) 4018  
(D) 4020  (E) 4022

25. How many four-digit positive integers $x$ are there with the property that $x$ and $3x$ have only even digits? (One such number is $x = 8002$, since $3x = 24006$ and each of $x$ and $3x$ has only even digits.)

(A) 82  (B) 84  (C) 86  (D) 88  (E) 90
Canadian Mathematics Competition

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• Register your students for the Fryer, Galois and Hypatia Contests which will be written on April 18, 2007
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Pascal Contest (Grade 9)
Wednesday, February 22, 2006

C.M.C. Sponsors: University of Waterloo, Deloitte & Touche Chartered Accountants, Great-West Life and London Life, Sybase iAnywhere Solutions

C.M.C. Supporter: Canadian Institute of Actuaries

Time: 60 minutes ©2005 Waterloo Mathematics Foundation

Calculators are permitted

Instructions

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
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9. When your supervisor tells you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of $\frac{550 + 50}{5^2 + 5}$?
   (A) 32  (B) 40  (C) 12  (D) 65  (E) 20

2. What is the value of $\sqrt{36 + 64} - \sqrt{25 - 16}$?
   (A) 5  (B) 7  (C) 13  (D) 11  (E) 9

3. How many positive whole numbers, including 1 and 18, divide exactly into 18?
   (A) 3  (B) 4  (C) 5  (D) 6  (E) 7

4. If $A + B = 5$, then the value of $B - 3 + A$ is
   (A) 2  (B) 8  (C) 7  (D) 15  (E) 13

5. In the diagram, the rectangular solid and the cube have equal volumes. The length of each edge of the cube is
   (A) 2  (B) 4  (C) 8  (D) 16  (E) 32

6. Ravindra and Hongshu made a pizza together. Ravindra ate $\frac{2}{5}$ of the pizza. Hongshu ate half as much as Ravindra. What percentage of the original pizza was left?
   (A) 20  (B) 30  (C) 40  (D) 50  (E) 60

7. In the diagram, two equal-armed balances are shown.

   How many $\bigcirc$ would it take to balance $\Box\Box\Box\Box$?
   (A) 2  (B) 1  (C) 4  (D) 5  (E) 3

8. The areas of three squares are 16, 49 and 169. What is the average (mean) of their side lengths?
   (A) 8  (B) 12  (C) 24  (D) 39  (E) 32

9. In the diagram, the rectangle has a width of $w$, a length of 8, and a perimeter of 24. What is the ratio of its width to its length?
   (A) 1 : 4  (B) 1 : 3  (C) 1 : 2  (D) 3 : 8  (E) 2 : 3
10. In the subtraction shown, \( M \) and \( N \) each represent a single digit. What is the value of \( M + N \)?

(A) 14  (B) 12  (C) 15  
(D) 13  (E) 11

\[
\begin{array}{cc}
M & 4 \\
- & 3 \\
\hline
\end{array}
\]

\[
\begin{array}{cc}
N & 1 \\
\hline
\end{array}
\]

Part B: Each correct answer is worth 6.

11. When \( x = 9 \), which of the following has the largest value?

(A) \( \sqrt{x} \)  (B) \( \frac{x}{2} \)  (C) \( x - 5 \)  (D) \( \frac{40}{x} \)  (E) \( \frac{x^2}{20} \)

12. The lengths of the three sides of a triangle are 7, \( x + 4 \) and \( 2x + 1 \). The perimeter of the triangle is 36. What is the length of the longest side of the triangle?

(A) 7  (B) 12  (C) 17  (D) 15  (E) 16

13. If Corina had added the numbers \( P \) and \( Q \) correctly, the answer would have been 16. By mistake, she subtracted \( Q \) from \( P \). Her answer was 4. What is the value of \( P \)?

(A) 4  (B) 5  (C) 8  (D) 10  (E) 16

14. If \( \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \frac{n}{12} = 2 \), the value of \( n \) is

(A) -4  (B) 13  (C) 18  (D) 4  (E) 1

15. From 7:45 p.m. to 9:30 p.m., Jim drove a distance of 84 km at a constant speed. What was this speed, in km/h?

(A) 60  (B) 80  (C) 112  (D) 63  (E) 48

16. An unusual die has the numbers 2, 2, 3, 3, 5, and 8 on its six faces. Two of these dice are rolled. The two numbers on the top faces are added. How many different sums are possible?

(A) 6  (B) 7  (C) 8  (D) 9  (E) 10

17. In the diagram, point \( E \) lies on line segment \( AB \), and triangles \( AED \) and \( BEC \) are isosceles. Also, \( \angle DEC \) is twice \( \angle ADE \). What is the size of \( \angle EBC \)?

(A) 75°  (B) 80°  (C) 60°  (D) 55°  (E) 45°

18. In the diagram, the grid is made up of squares. What is the area of the shaded region?

(A) 19  (B) 24  (C) 14  (D) 12  (E) 8
19. The sum of ten consecutive integers is $S$. Ten times the smallest of these integers is $T$. What is the value of $S - T$?

(A) 45  (B) 55  (C) 10  (D) 9  (E) 66

20. Five identical rectangles are arranged to form a larger rectangle $PQRS$, as shown. The area of $PQRS$ is 4000. The length, $x$, of each of the identical rectangles is closest to

(A) 35  (B) 39  (C) 41
(D) 37  (E) 33

Part C: Each correct answer is worth 8.

21. In each row of the table, the sum of the first two numbers equals the third number. Also, in each column of the table, the sum of the first two numbers equals the third number. What is the sum of the nine numbers in the table?

(A) 18  (B) 42  (C) −18
(D) −6  (E) 24

22. In the diagram, each of the three identical circles touch the other two. The circumference of each circle is 36. What is the perimeter of the shaded region?

(A) 18  (B) 6  (C) 36
(D) 12  (E) 24

23. Ben and Anna each have some CDs. If Anna gives six of her CDs to Ben, he would then have twice as many CDs as Anna. If, instead, Anna takes six CDs from Ben, then both would have the same number of the CDs. What is the total number of CDs that Ben and Anna have?

(A) 42  (B) 30  (C) 72  (D) 18  (E) 36

24. A bag contains eight yellow marbles, seven red marbles, and five black marbles. Without looking in the bag, Igor removes $N$ marbles all at once. If he is to be sure that, no matter which choice of $N$ marbles he removes, there are at least four marbles of one colour and at least three marbles of another colour left in the bag, what is the maximum possible value of $N$?

(A) 6  (B) 7  (C) 8  (D) 9  (E) 10

25. John writes a number with 2187 digits on the blackboard, each digit being a 1 or a 2. Judith creates a new number from John’s number by reading his number from left to right and wherever she sees a 1 writing 112 and wherever she sees a 2 writing 111. (For example, if John’s number begins 2112, then Judith’s number would begin 111112112111.) After Judith finishes writing her number, she notices that the leftmost 2187 digits in her number and in John’s number are the same. How many times do five 1’s occur consecutively in John’s number?

(A) 182  (B) 183  (C) 184  (D) 185  (E) 186
Canadian Mathematics Competition

For students...

Thank you for writing the 2006 Pascal Contest! In 2005, more than 90,000 students around the world registered to write the Pascal, Cayley and Fermat Contests.

Encourage your teacher to register you for Fryer Contest which will be written on April 20, 2006. Visit our website www.cemc.uwaterloo.ca to find

- More information about the Fryer Contest
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Visit our website www.cemc.uwaterloo.ca to

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- Learn about workshops and resources we offer for teachers
- Find your school results
Time: 60 minutes
Calculators are permitted.

Instructions
1. Do not open the Contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper left corner.
5. Be certain that you code your name, age, sex, grade, and the Contest you are writing in the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. After making your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor tells you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. What is the value of \( \frac{200 + 10}{20 + 10} \)?
   (A) 2  (B) 10  (C) 1  (D) 11  (E) 7

2. The expression \( 6a - 5a + 4a - 3a + 2a - a \) is equal to
   (A) 3a  (B) \( 3a^6 \)  (C) 3  (D) \(-21a\)  (E) \(-21a^6\)

3. When \( x = 3 \), the value of \( x(x - 1)(x - 2)(x - 3)(x - 4) \) is
   (A) 6  (B) \(-6\)  (C) 0  (D) 24  (E) \(-24\)

4. Six balls, numbered 2, 3, 4, 5, 6, 7, are placed in a hat. Each ball is equally likely to be chosen. If one ball is chosen, what is the probability that the number on the selected ball is a prime number?
   (A) \( \frac{1}{6} \)  (B) \( \frac{1}{3} \)  (C) \( \frac{1}{2} \)  (D) \( \frac{2}{3} \)  (E) \( \frac{5}{6} \)

5. The value of \( \sqrt{36} \times \sqrt{16} \) is
   (A) 12  (B) 144  (C) 24  (D) 26  (E) 96

6. A glass filled with water has a mass of 1000 g. When half the water is removed from the glass, the mass of the glass and the remaining water is 700 g. What is the mass of the empty glass?
   (A) 600 g  (B) 500 g  (C) 350 g  (D) 400 g  (E) 300 g

7. If \( \frac{1}{3}x = 12 \), then \( \frac{1}{2}x \) equals
   (A) 1  (B) 16  (C) 9  (D) 144  (E) 64

8. Which of the numbers \(-5, \frac{3}{2}, 2, \frac{3}{5}, 8\) is larger than its square?
   (A) \(-5\)  (B) \( \frac{3}{2} \)  (C) 2  (D) \( \frac{3}{5} \)  (E) 8

9. In triangle \( ABC \), the value of \( x + y \) is
   (A) 104  (B) 76  (C) 180  
   (D) 90  (E) 166

10. In the sequence 32, 8, _____, _____, \( x \), each term after the second is the average of the two terms immediately before it. The value of \( x \) is
    (A) 17  (B) 20  (C) 44  (D) 24  (E) 14
Part B: Each correct answer is worth 6.

11. If \(a\), \(b\), and \(c\) are positive integers with \(a \times b = 13\), \(b \times c = 52\), and \(c \times a = 4\), the value of \(a \times b \times c\) is

(A) 2704   (B) 104   (C) 676   (D) 208   (E) 52

12. Point \(L\) lies on line segment \(KM\), as shown.

The value of \(w\) is

(A) 4   (B) 5   (C) 6

(D) 7   (E) 8

13. Eight unit cubes are used to form a larger 2 by 2 by 2 cube. The six faces of this larger cube are then painted red. When the paint is dry, the larger cube is taken apart. What fraction of the total surface area of the unit cubes is red?

(A) \(\frac{1}{6}\)   (B) \(\frac{2}{3}\)   (C) \(\frac{1}{2}\)

(D) \(\frac{1}{4}\)   (E) \(\frac{1}{3}\)

14. A positive integer whose digits are the same when read forwards or backwards is called a palindrome. For example, 4664 is a palindrome. How many integers between 2005 and 3000 are palindromes?

(A) 0   (B) 8   (C) 9   (D) 10   (E) more than 10

15. When 14 is divided by 5, the remainder is 4. When 14 is divided by a positive integer \(n\), the remainder is 2. For how many different values of \(n\) is this possible?

(A) 1   (B) 2   (C) 3   (D) 4   (E) 5

16. The digits 1, 2, 5, 6, and 9 are all used to form five-digit even numbers, in which no digit is repeated. The difference between the largest and smallest of these numbers is

(A) 83 916   (B) 79 524   (C) 83 952   (D) 79 236   (E) 83 016

17. In the diagram, rectangle \(PQRS\) is divided into three identical squares. If \(PQRS\) has perimeter 120 cm, what is its area, in cm\(^2\)?

(A) 225   (B) 675   (C) 360

(D) 432   (E) 144

18. When the expression \(2005^2 + 2005^0 + 2005^0 + 2005^5\) is evaluated, the final two digits are

(A) 52   (B) 25   (C) 20   (D) 50   (E) 05
19. A whole number is called *decreasing* if each digit of the number is less than the digit to its left. For example, 8540 is a decreasing four-digit number. How many decreasing numbers are there between 100 and 500?

(A) 11  (B) 10  (C) 9  (D) 8  (E) 7

20. Harry the Hamster is put in a maze, and he starts at point $S$. The paths are such that Harry can move forward only in the direction of the arrows. At any junction, he is equally likely to choose any of the forward paths. What is the probability that Harry ends up at $B$?

(A) $\frac{2}{3}$  (B) $\frac{13}{18}$  (C) $\frac{11}{18}$

(D) $\frac{1}{3}$  (E) $\frac{1}{4}$

---

**Part C: Each correct answer is worth 8.**

21. Integers $m$ and $n$ are each greater than 100. If $m + n = 300$, then $m : n$ could be equal to

(A) 9 : 1  (B) 17 : 8  (C) 5 : 3  (D) 4 : 1  (E) 3 : 2

22. In the diagram, two pairs of identical isosceles triangles are cut off of square $ABCD$, leaving rectangle $PQRS$. The total area cut off is 200 m². The length of $PR$, in metres, is

(A) $\sqrt{200}$  (B) 20  (C) $\sqrt{800}$

(D) 25  (E) 15

23. Starting with the 2, the number 2005 can be formed by moving either horizontally, vertically, or diagonally from square to square in the grid. How many different paths can be followed to form 2005?

(A) 96  (B) 72  (C) 80

(D) 64  (E) 88
24. A positive integer is called a *perfect power* if it can be written in the form $a^b$, where $a$ and $b$ are positive integers with $b \geq 2$. For example, 32 and 125 are perfect powers because $32 = 2^5$ and $125 = 5^3$.

The increasing sequence

$$2, 3, 5, 6, 7, 10, \ldots$$

consists of all positive integers which are not perfect powers. The sum of the squares of the digits of the 1000th number in this sequence is

(A) 42  (B) 26  (C) 33  (D) 18  (E) 21

25. In the diagram, right-angled triangles $AED$ and $BFC$ are constructed inside rectangle $ABCD$ so that $F$ lies on $DE$. If $AE = 21$, $ED = 72$ and $BF = 45$, what is the length of $AB$?

(A) 50  (B) 48  (C) 52

(D) 54  (E) 56
Canadian Mathematics Competition

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Pascal Contest (Grade 9)

Wednesday, February 18, 2004

Instructions
1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. To win a skateboard, the skill testing question is \(5 \times (10 - 6) \div 2\). The correct answer is
   (A) 10  (B) 35  (C) 32  (D) 22  (E) 40

2. The average of 2, \(x\) and 12 is 8. What is the value of \(x\)?
   (A) 8  (B) \(-2\)  (C) 12  (D) 24  (E) 10

3. The fractions \(\frac{1}{9}\), \(\frac{1}{4}\) and \(\frac{1}{18}\) are to be added. What is their lowest common denominator?
   (A) 648  (B) 162  (C) 72  (D) 36  (E) 18

4. In the diagram, the area of \(\Delta ABC\) is
   (A) 40  (B) 12  (C) 30  (D) 48  (E) 24

5. The value of \(\frac{5 - \sqrt{4}}{5 + \sqrt{4}}\) is
   (A) \(\frac{3}{7}\)  (B) \(\frac{1}{9}\)  (C) \(-\frac{11}{21}\)  (D) 0  (E) \(\frac{1}{3}\)

6. The value of \(4^1 + 3^2 - 2^3 + 1^4\) is
   (A) 4  (B) 8  (C) 6  (D) 5  (E) 9

7. When \(x = -3\), the value of \(3x^2 + 2x\) is
   (A) 81  (B) 75  (C) \(-33\)  (D) 21  (E) \(-24\)

8. If 18% of 42 is equal to 27% of \(x\), then the value of \(x\) is
   (A) 28  (B) 63  (C) 2  (D) 864  (E) 12

9. The surface area of a cube is 96 cm\(^2\). The volume of the cube, in cm\(^3\), is
   (A) 16  (B) 64  (C) 8  (D) 512  (E) 216

10. It is given that \(y = 3x - 5\) and \(z = 3x + 3\). If \(y = 1\), the value of \(z\) is
    (A) 8  (B) 6  (C) \(-3\)  (D) 3  (E) 9
11. In the diagram, square $ABCD$ has a side length of 4. What is the total area of the shaded regions?

(A) 4  (B) 8  (C) 9  
(D) 12  (E) 16

12. In the diagram, two equal-armed balances are shown. How many \( \square \) would it take to balance one \( \circ \)?

(A) 1  (B) 2  (C) 3  (D) 4  (E) 5

13. Nadia starts at $S$ and walks at a steady pace once around the perimeter of a square park. Which graph best represents her distance from $S$ as time passes?

14. How many unshaded squares are in the tenth figure of the pattern?

(A) 38  (B) 40  (C) 42  
(D) 44  (E) 46

15. In the Pascal family, each child has at least 2 brothers and at least 1 sister. What is the smallest possible number of children in this family?

(A) 3  (B) 4  (C) 5  (D) 6  (E) 7

16. If $a^2 + 3b = 33$, where $a$ and $b$ are positive integers, what is the value of $ab$?

(A) 11  (B) 24  (C) 16  (D) 32  (E) 27
17. The value of 0.1 + 0.12 + 0.123 is
(A) 0.343  (B) 0.355  (C) 0.3\(\bar{5}\)  (D) 0.355446  (E) 0.355445

18. The symbol \(\begin{array}{cc}
    a & b \\
    c & d
\end{array}\) equals \(ad - bc\). If \(\begin{array}{cc}
    x - 1 & 2 \\
    3 & -5
\end{array}\) = 9, the value of \(x\) is
(A) –4  (B) –3  (C) –2  (D) 2  (E) 4

19. Rafaello’s tree grows according to the following rule. After a branch has been growing for two weeks, it produces a new branch every week, while the original branch continues to grow. The tree has five branches after five weeks, as shown. How many branches, including the main branch, will the tree have at the end of eight weeks?
(A) 21  (B) 40  (C) 19
(D) 13  (E) 34

20. At the beginning of the game “Clock 7”, the arrow points to one of the seven numbers. On each turn, the arrow is rotated clockwise by the number of spaces indicated by the arrow at the beginning of the turn. For example, if “Clock 7” starts with the arrow pointing at 4, then on the first turn, the arrow is rotated clockwise 4 spaces so that it now points at 1. The arrow will then move 1 space on the next turn, and so on. If the arrow points at 6 after the 21st turn, at which number did the arrow point after the first turn?
(A) 3  (B) 6  (C) 5
(D) 2  (E) 7

Part C: Each correct answer is worth 8.

21. In the diagram, the number of different paths that spell “PASCAL” is
(A) 6  (B) 10  (C) 12
(D) 16  (E) 24

continued ...
22. A container in the shape of a cube has edge length 20 cm and contains some water. A solid gold cube, with edge length 15 cm, sinks to the bottom of this container, causing the water level to rise just to the top of the solid cube. Which of the following is closest to the original depth of the water?

(A) 6.56 cm  (B) 8.25 cm  (C) 10.50 cm  
(D) 5.31 cm  (E) 7.50 cm

23. A driver approaching a toll booth has exactly two quarters, two dimes and two nickels in his pocket. He reaches into his pocket and randomly selects two of these coins. What is the probability that the coins that he selects will be at least enough to pay the 30-cent toll?

(A) $\frac{3}{5}$  (B) $\frac{2}{5}$  (C) $\frac{1}{3}$  (D) $\frac{3}{10}$  (E) $\frac{2}{3}$

24. In the sequence of fractions $\frac{1}{1}$, $\frac{2}{1}$, $\frac{1}{2}$, $\frac{3}{1}$, $\frac{2}{2}$, $\frac{1}{3}$, $\frac{4}{1}$, $\frac{3}{2}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{5}{1}$, ..., fractions equivalent to any given fraction occur many times. For example, fractions equivalent to $\frac{1}{2}$ occur for the first two times in positions 3 and 14. In which position is the fifth occurrence of a fraction equivalent to $\frac{3}{7}$?

(A) 1207  (B) 1208  (C) 1209  (D) 1210  (E) 1211

25. In the diagram, $ABCD$ is a trapezoid with $AB$ parallel to $CD$ and with $AB = 2$ and $CD = 5$. Also, $AX$ is parallel to $BC$ and $BY$ is parallel to $AD$. If $AX$ and $BY$ intersect at $Z$, and $AC$ and $BY$ intersect at $W$, the ratio of the area of $\triangle AZW$ to the area of trapezoid $ABCD$ is

(A) $7 : 105$  (B) $8 : 105$  (C) $9 : 105$  
(D) $10 : 105$  (E) $12 : 105$
Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

**Copies of Previous Canadian Mathematics Competitions**

Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

**Problems Problems Problems Books**

Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. *(Available in English only.)*

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- 10 topics
- for students in Grades 9, 10, & 11
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- over 325 problems and full solutions
- 10 topics (different from Volume 1)
- for students in Grades 9, 10, & 11

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- 12 topics
- for senior high school students

**Volume 4**

- over 325 problems and full solutions
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- over 300 problems and full solutions
- 11 topics
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Pascal Contest (Grade 9)

Wednesday, February 19, 2003

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
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8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. $\sqrt{169} - \sqrt{25}$ equals
   \[ \text{(A) } 8 \quad \text{(B) } 12 \quad \text{(C) } 64 \quad \text{(D) } 72 \quad \text{(E) } 144 \]

2. The missing number in the geometric sequence 2, 6, 18, 54, ____, 486 is
   \[ \text{(A) } 72 \quad \text{(B) } 90 \quad \text{(C) } 108 \quad \text{(D) } 162 \quad \text{(E) } 216 \]

3. The value of $\frac{6 + 6 \times 3 - 3}{3}$ is
   \[ \text{(A) } 11 \quad \text{(B) } 7 \quad \text{(C) } 3 \quad \text{(D) } 9 \quad \text{(E) } 17 \]

4. In the diagram, the value of $x$ is
   \[ \text{(A) } 40 \quad \text{(B) } 60 \quad \text{(C) } 100 \]
   \[ \text{(D) } 120 \quad \text{(E) } 80 \]

5. The value of $\frac{5^8}{8^5}$ is
   \[ \text{(A) } \frac{1}{16} \quad \text{(B) } 8 \quad \text{(C) } 4 \quad \text{(D) } \frac{1}{4} \quad \text{(E) } 2 \]

6. Which of the following is not equal to $\frac{18}{5}$?
   \[ \text{(A) } \frac{6^2}{10} \quad \text{(B) } \frac{1}{3} [6(3)] \quad \text{(C) } \frac{18 + 1}{5 + 1} \quad \text{(D) } 3.6 \quad \text{(E) } \frac{\sqrt{324}}{\sqrt{25}} \]

7. In the diagram, the numbers 1, 2, 4, 5, 6, and 8 are substituted, in some order, for the letters $A$, $B$, $C$, $D$, $E$, and $F$, so that the number between and below two numbers is the positive difference between those two numbers. For example, the 7 in the third row is the positive difference between $D$ and 9. Thus $D = 2$ because $9 - 2 = 7$. The value of $A + C$ is
   \[ \text{(A) } 7 \quad \text{(B) } 12 \quad \text{(C) } 13 \]
   \[ \text{(D) } 10 \quad \text{(E) } 14 \]
8. What is the area of rectangle $ABCD$?
   (A) 15 (B) 16 (C) 18 (D) 30 (E) 9

9. The largest prime number less than 30 that can be written as the sum of two primes is
   (A) 29 (B) 23 (C) 19 (D) 17 (E) 13

10. Which of the following numbers is the largest?
    (A) 3.2571 (B) $\overline{3.2571}$ (C) $\overline{3.2571}$ (D) $\overline{3.2571}$ (E) $\overline{3.2571}$

Part B: Each correct answer is worth 6.

11. If $x = 2$ and $y = -3$ satisfy the equation $2x^2 + kxy = 4$, then the value of $k$ is
    (A) $\frac{2}{3}$ (B) 0 (C) $\frac{4}{3}$ (D) $-\frac{2}{3}$ (E) $-2$

12. At a math conference, the following exchange rates are used:
    1 calculator = 100 rulers
    10 rulers = 30 compasses
    25 compasses = 50 protractors
    How many protractors are equivalent to 1 calculator?
    (A) 400 (B) 600 (C) 300 (D) 500 (E) 200

13. In the diagram, each of the 15 small squares is going to be coloured. Any two squares that have a vertex in common or share a side must be a different colour. What is the least number of different colours needed?
    (A) 3 (B) 4 (C) 5 (D) 8 (E) 9

14. If $x$ and $y$ are positive integers and $x + y = 5$, then a possible value for $2x - y$ is
    (A) 3 (B) $-3$ (C) 2 (D) $-2$ (E) 0

15. In the diagram, square $ABCD$ is made up of 36 squares, each with side length 1. The area of the square $KLMN$, in square units, is
    (A) 12 (B) 16 (C) 18 (D) 20 (E) 25
16. If \( n \) is any integer, \( n + 3, n - 9, n - 4, n + 6, \) and \( n - 1 \) are also integers. If \( n + 3, n - 9, n - 4, n + 6, \) and \( n - 1 \) are arranged from smallest to largest, the integer in the middle is

(A) \( n + 3 \) (B) \( n - 9 \) (C) \( n - 4 \) (D) \( n + 6 \) (E) \( n - 1 \)

17. In the diagram, \( AB \) is a straight line. The value of \( x \) is

(A) 67 (B) 59 (C) 62 (D) 40 (E) 86

18. The average (mean) of a list of \( n \) numbers is 7. When the number \(-11\) is added to the list, the new average is 6. What is the value of \( n \)?

(A) 13 (B) 14 (C) 15 (D) 16 (E) 17

19. In the diagram, what is the area of quadrilateral \( ABCD \)?

(A) 14 (B) 16 (C) 18 (D) 20 (E) 28

20. The people of Evenland never use odd digits. Instead of counting 1, 2, 3, 4, 5, 6, an Evenlander counts 2, 4, 6, 8, 20, 22. What is an Evenlander’s version of the integer 111?

(A) 822 (B) 828 (C) 840 (D) 842 (E) 824

Part C: Each correct answer is worth 8.

21. A straight one-way city street has 8 consecutive traffic lights. Every light remains green for 1.5 minutes, yellow for 3 seconds, and red for 1.5 minutes. The lights are synchronized so that each light turns red 10 seconds after the preceding one turns red. What is the longest interval of time, in seconds, during which all 8 lights are green?

(A) 10 (B) 15 (C) 20 (D) 25 (E) 30

22. In the diagram, two circles with centres \( A \) and \( B \) intersect at points \( P \) and \( Q \) so that \( \angle PAQ = 60^\circ \) and \( \angle PBQ = 90^\circ \). What is the ratio of the area of the circle with centre \( A \) to the area of the circle with centre \( B \)?

(A) 3:1 (B) 3:2 (C) 4:3 (D) 2:1 (E) 9:4

23. An escalator moves at a constant rate from one floor up to the next floor. Jack walks up 29 steps while travelling on the escalator between the floors. Jill takes twice as long to travel between the floors and walks up only 11 steps. When it is stopped, how many steps does the escalator have between the two floors?

(A) 47 (B) 51 (C) 40 (D) 36 (E) 69

continued...
24. An artist wants to completely cover a rectangle with identically sized squares which do not overlap and do not extend beyond the edges of the rectangle. If the rectangle is $60\frac{1}{2}$ cm long and $47\frac{2}{3}$ cm wide, what is the minimum number of squares required?

(A) 429  (B) 858  (C) 1573  (D) 1716  (E) 5148

25. In the cube shown, $L$ and $K$ are midpoints of adjacent edges $AD$ and $AB$. The perpendicular distance from $F$ to the line segment $LK$ is 10. What is the volume of the cube, to the nearest integer?

(A) 323  (B) 324  (C) 325  (D) 326  (E) 327

************
Students and parents who enjoy solving problems for fun and recreation may find the following publications of interest. They are an excellent resource for enrichment, problem solving and contest preparation.

Copies of Previous Canadian Mathematics Competitions
Copies of previous contests and solutions are available at no cost in both English and French at http://www.cemc.uwaterloo.ca

Problems Problems Problems Books
Each volume is a collection of problems (multiple choice and full solution), grouped into 9 or more topics. Questions are selected from previous Canadian Mathematics Competition contests, and full solutions are provided for all questions. The price is $15. (Available in English only.)

Volume 1
• over 300 problems and full solutions
• 10 topics
• for students in Grades 9, 10, & 11
• French version of Volume 1 is available

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Orders should be addressed to: Canadian Mathematics Competition
Faculty of Mathematics, Room 5181
University of Waterloo
Waterloo, ON N2L 3G1
Include your name, address (with postal code), and telephone number.

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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circle on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.

Canadian Mathematics Competition
An activity of The Centre for Education in Mathematics and Computing, University of Waterloo, Waterloo, Ontario

Pascal Contest (Grade 9)
Wednesday, February 20, 2002

C.M.C. Sponsors: University of Waterloo
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C.M.C. Contributors: Manulife Financial
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Sponsors: Anywhere Solutions
Contributors: Canadian Institute of Actuaries
Supporters: Chartered Accountants

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
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8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.

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Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 10 unanswered questions.

Part A: Each correct answer is worth 5.

1. \( \frac{15 + 9 - 6}{3\times 2} \) equals
   (A) 11  (B) 4  (C) 3  (D) 23  (E) 12

2. 50% of 2002 is equal to
   (A) 1001  (B) 3003  (C) 2001  (D) 1952  (E) 1001

3. If \( x + 2 = 10 \) and \( y - 1 = 6 \), then the numerical value of \( x + y \) is
   (A) 15  (B) 15  (C) 16  (D) 17  (E) 19

4. The value of \( (3^2 - 3)^2 \) is
   (A) 36  (B) 72  (C) 9  (D) 3  (E) 0

5. Sofia entered an elevator. The elevator went up seven floors, then down six floors, and finally up five floors. If Sofia got out on the twentieth floor, then she entered the elevator on floor number
   (A) 14  (B) 2  (C) 16  (D) 38  (E) 26

6. In the diagram, the value of \( x \) is
   (A) 20  (B) 60  (C) 70  (D) 40  (E) 50

7. If \( n \) is \( \frac{5}{6} \) of 240, then \( \frac{2}{5} \) of \( n \) is
   (A) 288  (B) 80  (C) 96  (D) 200  (E) 500

8. The value of \( 1 - \left(5^{-2}\right) \) is
   (A) \( \frac{24}{25} \)  (B) \(-24\)  (C) \( \frac{26}{25} \)  (D) 26  (E) \( \frac{9}{10} \)

9. A rectangle is divided into four smaller rectangles. The areas of three of these rectangles are 6, 15 and 25, as shown. The area of the shaded rectangle is
   (A) 7  (B) 15  (C) 12  (D) 16  (E) 10

\[
\begin{array}{c|c|c}
6 & 15 & 25
\end{array}
\]
10. Toothpicks are used to form squares in the pattern shown:

Four toothpicks are used to form one square, seven to form two squares, and so on. If this pattern continues, how many toothpicks will be used to form 10 squares in a row?

(A) 39  (B) 40  (C) 31  (D) 35  (E) 28

Part B: Each correct answer is worth 6.

11. $ABCD$ is a square with $AB = x + 16$ and $BC = 3x$, as shown. The perimeter of $ABCD$ is

(A) 16  (B) 32  (C) 96

(D) 48  (E) 24

12. In a sequence of numbers, each number, except the first, equals twice the previous number. If the sum of the second and third numbers in the list is 24, then the sixth number is

(A) 112  (B) 192  (C) 64  (D) 40  (E) 128

13. Triangle $ABC$ has vertices $A(1,2)$, $B(4,0)$ and $C(1,-4)$. The area of $\triangle ABC$ is

(A) 18  (B) 12  (C) 8  (D) 10  (E) 9

14. A class of 30 students wrote a history test. Of these students, 25 achieved an average of 75%. The other 5 students achieved an average of 40%. The class average on the history test was closest to

(A) 46  (B) 69  (C) 63  (D) 58  (E) 71

15. In the diagram, $ABC$ represents a triangular jogging path. Jack jogs along the path from $A$ to $B$ to $F$. Jill jogs from $A$ to $C$ to $F$. Each jogs the same distance. The distance from $F$ to $B$, in metres, is

(A) 40  (B) 120  (C) 100  (D) 80  (E) 200

16. When the product $5^3 \cdot 7^{52}$ is expanded, the units digit (that is, the last digit) is

(A) 5  (B) 3  (C) 9  (D) 7  (E) 0
17. The number 1000 can be written as the product of two positive integers, neither of which contains zeros. The sum of these two integers is

(A) 65  (B) 110  (C) 133  (D) 205  (E) 1001

18. Together Akira and Jamie weigh 101 kg. Together Akira and Rabia weigh 91 kg. Together Rabia and Jamie weigh 88 kg. How many kilograms does Akira weigh?

(A) 48  (B) 46  (C) 50  (D) 52  (E) 38

19. The natural numbers from 1 to 2100 are entered sequentially in 7 columns, with the first 3 rows as shown. The number 2002 occurs in column \( m \) and row \( n \). The value of \( m + n \) is

<table>
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<tr>
<th>Column 1</th>
<th>Column 2</th>
<th>Column 3</th>
<th>Column 4</th>
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</table>

(A) 290  (B) 291  (C) 292  (D) 293  (E) 294

20. For how many integer values of \( x \) is \( \sqrt{25 - x^2} \) equal to an integer?

(A) 7  (B) 6  (C) 5  (D) 3  (E) 2

Part C: Each correct answer is worth 8.

21. A rectangular block, with dimensions 4 cm, 5 cm and 6 cm, is made up of cubes each with side length 1 cm. If 1 cm\(^3\) cubes are removed from this larger rectangular block, what is the minimum number of these cubes that must be removed so that the resulting solid is itself a cube?

(A) 40  (B) 93  (C) 46  (D) 64  (E) 56

22. In a school, 500 students voted on each of two issues. Of these students, 375 voted in favour of the first issue, 275 voted in favour of the second, and 40 students voted against both issues. How many students voted in favour of both issues?

(A) 110  (B) 150  (C) 190  (D) 95  (E) 230

23. The number of ordered pairs \( (a,b) \) of integers which satisfy the equation \( a^b = 64 \) is

(A) 3  (B) 5  (C) 8  (D) 6  (E) 7

continued ...
24. In the diagram, $ABC$ is a semi-circle with diameter $AC$, centre $O$ and radius 1. Also, $OB$ is perpendicular to $AC$. Using $AB$ as a diameter, a second semi-circle $AEB$ is drawn. The region inside this second semi-circle that lies outside the original semi-circle is shaded, as shown. The area of this shaded region is

\[(A) \frac{\pi}{4} \quad (B) \frac{1}{2} \quad (C) \frac{3\pi}{4} + \frac{1}{2} \]

\[(D) \frac{3}{4} \quad (E) \frac{\pi}{2} - \frac{1}{2}\]

25. A student has two open-topped cylindrical containers. (The walls of the two containers are thin enough so that their width can be ignored.) The larger container has a height of 20 cm, a radius of 6 cm and contains water to a depth of 17 cm. The smaller container has a height of 18 cm, a radius of 5 cm and is empty. The student slowly lowers the smaller container into the larger container, as shown in the cross-section of the cylinders in Figure 1. As the smaller container is lowered, the water first overflows out of the larger container (Figure 2) and then eventually pours into the smaller container. When the smaller container is resting on the bottom of the larger container, the depth of the water in the smaller container will be closest to

\[(A) 2.82 \text{ cm} \quad (B) 2.84 \text{ cm} \quad (C) 2.86 \text{ cm}\]

\[(D) 2.88 \text{ cm} \quad (E) 2.90 \text{ cm}\]
Pascal Contest (Grade 9)

Wednesday, February 21, 2001

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C.
   There is no penalty for an incorrect answer.
   Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.
Scoring: There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.

Part A: Each correct answer is worth 5.

1. The value of $\frac{5(6) - 3(4)}{6 + 3}$ is
   (A) 1   (B) 2   (C) 6   (D) 12   (E) 31

2. When 12345678 is divided by 10, the remainder is
   (A) 0   (B) 2   (C) 4   (D) 6   (E) 8

3. Evaluate $\frac{2^5 - 2^3}{2^2}$.
   (A) 6   (B) 1   (C) $\frac{1}{4}$   (D) 0   (E) 30

4. If $x = \frac{1}{4}$, which of the following has the largest value?
   (A) $x$   (B) $x^2$   (C) $\frac{1}{2}x$   (D) $\frac{1}{x}$   (E) $\sqrt{x}$

5. In the diagram, the value of $x$ is
   (A) 100   (B) 65   (C) 80
   (D) 70   (E) 50

6. Anna’s term mark was 80%. Her exam mark was 90%. In calculating her final mark, the term mark was given a weight of 70% and the exam mark a weight of 30%. What was her final mark?
   (A) 81%   (B) 83%   (C) 84%   (D) 85%   (E) 87%

7. The least value of $x$ which makes $\frac{24}{x - 4}$ an integer is
   (A) −44   (B) −28   (C) −20   (D) −8   (E) 0

8. The 50th term in the sequence 5, $6x$, $7x^2$, $8x^3$, $9x^4$, … is
   (A) $54x^{49}$   (B) $54x^{50}$   (C) $45x^{50}$   (D) $55x^{49}$   (E) $46x^{51}$
9. The perimeter of \( \triangle ABC \) is
   \[
   (A) \ 23 \quad (B) \ 40 \quad (C) \ 42 \\
   (D) \ 46 \quad (E) \ 60
   \]

10. Dean scored a total of 252 points in 28 basketball games. Ruth played 10 fewer games than Dean. Her scoring average was 0.5 points per game higher than Dean’s scoring average. How many points, in total, did Ruth score?
   \[
   (A) \ 153 \quad (B) \ 171 \quad (C) \ 180 \quad (D) \ 266 \quad (E) \ 144
   \]

Part B: Each correct answer is worth 6.

11. Sahar walks at a constant rate for 10 minutes and then rests for 10 minutes. Which of these distance, \( d \), versus time, \( t \), graphs best represents his movement during these 20 minutes?
   \[
   (A) \quad (B) \quad (C) \\
   (D) \quad (E)
   \]

12. A bag contains 20 candies: 4 chocolate, 6 mint and 10 butterscotch. Candies are removed randomly from the bag and eaten. What is the minimum number of candies that must be removed to be certain that at least two candies of each flavour have been eaten?
   \[
   (A) \ 6 \quad (B) \ 10 \quad (C) \ 12 \quad (D) \ 16 \quad (E) \ 18
   \]

13. Pierre celebrated his birthday on February 2, 2001. On that day, his age equalled the sum of the digits in the year in which he was born. In what year was Pierre born?
   \[
   (A) \ 1987 \quad (B) \ 1980 \quad (C) \ 1979 \quad (D) \ 1977 \quad (E) \ 1971
   \]

14. Twenty tickets are numbered from one to twenty. One ticket is drawn at random with each ticket having an equal chance of selection. What is the probability that the ticket shows a number that is a multiple of 3 or 5?
   \[
   (A) \ \frac{3}{10} \quad (B) \ \frac{11}{20} \quad (C) \ \frac{2}{5} \quad (D) \ \frac{9}{20} \quad (E) \ \frac{1}{2}
   \]
15. The line $L$ crosses the $x$-axis at $(-8, 0)$. The area of the shaded region is 16. What is the slope of the line $L$?

(A) $\frac{1}{2}$  (B) 4  (C) $-\frac{1}{2}$
(D) 2  (E) $-2$

16. In the diagram, all triangles are equilateral. The total number of equilateral triangles of any size is

(A) 18  (B) 20  (C) 24
(D) 26  (E) 28

17. In the rectangle shown, the value of $a - b$ is

(A) $-3$  (B) $-1$  (C) 0
(D) 3  (E) 1

18. The largest four-digit number whose digits add to 17 is 9800. The 5th largest four-digit number whose digits have a sum of 17 is

(A) 9521  (B) 9620  (C) 9611  (D) 9602  (E) 9530

19. Two circles with equal radii are enclosed by a rectangle, as shown. The distance between their centres is $\frac{2x}{3}$. The value of $x$ is

(A) $\frac{15}{4}$  (B) 5  (C) 6
(D) $\frac{60}{7}$  (E) $\frac{15}{2}$

20. Square $ABCD$ has an area of 4. $E$ is the midpoint of $AB$. Similarly, $F, G, H,$ and $I$ are the midpoints of $DE$, $CF$, $DG$, and $CH$, respectively. The area of $\triangle IDC$ is

(A) $\frac{1}{4}$  (B) $\frac{1}{8}$  (C) $\frac{1}{16}$
(D) $\frac{1}{32}$  (E) $\frac{1}{64}$
Part C: Each correct answer is worth 8.

21. Cindy leaves school at the same time every day. If she cycles at 20 km/h, she arrives home at 4:30 in the afternoon. If she cycles at 10 km/h, she arrives home at 5:15 in the afternoon. At what speed, in km/h, must she cycle to arrive home at 5:00 in the afternoon?

(A) $16\frac{2}{3}$  (B) 15  (C) $13\frac{1}{3}$  (D) 12  (E) $18\frac{3}{4}$

22. In the diagram, $AB$ and $BD$ are radii of a circle with centre $B$. The area of sector $ABD$ is $2\pi$, which is one-eighth of the area of the circle. The area of the shaded region is

(A) $2\pi - 4$  (B) $\pi$  (C) $2\pi - 2$

(D) $2\pi - 4.5$  (E) $2\pi - 8$

23. Five points are located on a line. When the ten distances between pairs of points are listed from smallest to largest, the list reads: 2, 4, 5, 7, 8, $k$, 13, 15, 17, 19. What is the value of $k$?

(A) 11  (B) 9  (C) 13  (D) 10  (E) 12

24. A sealed bottle, which contains water, has been constructed by attaching a cylinder of radius 1 cm to a cylinder of radius 3 cm, as shown in Figure A. When the bottle is right side up, the height of the water inside is 20 cm, as shown in the cross-section of the bottle in Figure B. When the bottle is upside down, the height of the liquid is 28 cm, as shown in Figure C. What is the total height, in cm, of the bottle?

(A) 29  (B) 30  (C) 31  (D) 32  (E) 48

25. A palindrome is a positive integer whose digits are the same when read forwards or backwards. For example, 2882 is a four-digit palindrome and 49194 is a five-digit palindrome. There are pairs of four-digit palindromes whose sum is a five-digit palindrome. One such pair is 2882 and 9339. How many such pairs are there?

(A) 28  (B) 32  (C) 36  (D) 40  (E) 44
Calculators are permitted, providing they are non-programmable and without graphic displays.

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

Instructions
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2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 in Part A, 6 in Part B, and 8 in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2, to a maximum of 20.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.
Scoring: There is *no penalty* for an incorrect answer.
Each unanswered question is worth 2 credits, to a maximum of 20 credits.

**Part A: Each correct answer is worth 5.**

1. The value of $5^2 + 2(5 - 2)$ is
   (A) 16  (B) 19  (C) 31  (D) 36  (E) 81

2. The sum of $29 + 12 + 23$ is
   (A) $32^2$  (B) $2^6$  (C) $3^4$  (D) $1^{64}$  (E) $64^0$

3. If $x = 4$ and $y = -3$, then the value of $\frac{x - 2y}{x + y}$ is
   (A) $-\frac{1}{2}$  (B) $-2$  (C) $\frac{10}{7}$  (D) $-\frac{2}{7}$  (E) 10

4. If the following sequence of five arrows repeats itself continuously, what arrow would be in the 48th position?

   
   
   (A)  (B)  (C)  (D)  (E)

5. If $y = 6 + \frac{1}{6}$, then $\frac{1}{y}$ is
   (A) $\frac{6}{37}$  (B) $\frac{37}{6}$  (C) $\frac{6}{7}$  (D) $\frac{7}{6}$  (E) 1

6. If $\frac{2}{3}$, $\frac{23}{30}$, $\frac{9}{10}$, $\frac{11}{15}$, and $\frac{4}{5}$ are written from smallest to largest then the middle fraction will be
   (A) $\frac{23}{30}$  (B) $\frac{4}{5}$  (C) $\frac{2}{3}$  (D) $\frac{9}{10}$  (E) $\frac{11}{15}$

7. Three squares with the same centre and corresponding parallel sides are drawn. The distance between the sides of successive squares is 3 and the side length of the largest square is 22, as shown. What is the perimeter of the smallest square?
   (A) 40  (B) 100  (C) 10  (D) 64  (E) 20
8. In the diagram, the value of $y$ is
   \[
   \begin{array}{ccc}
   \text{(A) } 30 & \text{(B) } 20 & \text{(C) } 80 \\
   \text{(D) } 60 & \text{(E) } 40
   \end{array}
   \]

9. The ages of three contestants in the Pascal Contest are 14 years, 9 months; 15 years, 1 month; and 14 years, 8 months. Their average (mean) age is
   \[
   \begin{array}{ccc}
   \text{(A) } 14 \text{ years, 8 months} & \text{(B) } 14 \text{ years, 9 months} & \text{(C) } 14 \text{ years, 10 months} \\
   \text{(D) } 14 \text{ years, 11 months} & \text{(E) } 15 \text{ years}
   \end{array}
   \]

10. The number of integers between $-\sqrt{8}$ and $\sqrt{32}$ is
   \[
   \begin{array}{ccc}
   \text{(A) } 5 & \text{(B) } 6 & \text{(C) } 7 \\
   \text{(D) } 8 & \text{(E) } 19
   \end{array}
   \]

Part B: Each correct answer is worth 6.

11. A store had a sale on T-shirts. For every two T-shirts purchased at the regular price, a third T-shirt was bought for $1.00. Twelve T-shirts were bought for $120.00. What was the regular price for one T-shirt?
   \[
   \begin{array}{ccc}
   \text{(A) } $10.00 & \text{(B) } $13.50 & \text{(C) } $14.00 \\
   \text{(D) } $14.50 & \text{(E) } $15.00
   \end{array}
   \]

12. In the diagram, every number beginning at 30 equals twice the sum of the two numbers to its immediate left. The value of $c$ is
   \[
   \begin{array}{ccc}
   \text{(A) } 50 & \text{(B) } 70 & \text{(C) } 80 \\
   \text{(D) } 100 & \text{(E) } 200
   \end{array}
   \]

13. In the expression $\frac{a}{b} + \frac{c}{d} + \frac{e}{f}$ each letter is replaced by a different digit from 1, 2, 3, 4, 5, and 6. What is the largest possible value of this expression?
   \[
   \begin{array}{ccc}
   \text{(A) } 8\frac{2}{3} & \text{(B) } 9\frac{5}{6} & \text{(C) } 9\frac{4}{5} \\
   \text{(D) } 9\frac{4}{3} & \text{(E) } 10\frac{1}{3}
   \end{array}
   \]

14. The numbers 6, 14, $x$, 17, 9, $y$, 10 have a mean of 13. What is the value of $x + y$?
   \[
   \begin{array}{ccc}
   \text{(A) } 20 & \text{(B) } 21 & \text{(C) } 23 \\
   \text{(D) } 25 & \text{(E) } 35
   \end{array}
   \]

15. The digits 1, 1, 2, 2, 3, and 3 are arranged to form an odd six digit integer. The 1’s are separated by one digit, the 2’s by two digits, and the 3’s by three digits. What are the last three digits of this integer?
   \[
   \begin{array}{ccc}
   \text{(A) } 312 & \text{(B) } 123 & \text{(C) } 131 \\
   \text{(D) } 121 & \text{(E) } 213
   \end{array}
   \]

16. The area of square $ABCD$ is 64. The midpoints of its sides are joined to form the square $EFGH$. The midpoints of its sides are $J, K, L, \text{ and } M$. The area of the shaded region is
   \[
   \begin{array}{ccc}
   \text{(A) } 32 & \text{(B) } 24 & \text{(C) } 20 \\
   \text{(D) } 28 & \text{(E) } 16
   \end{array}
   \]
17. In the diagram, the value of the height $h$ is

(A) 6  (B) 9  (C) 10
(D) 12  (E) 15

18. In the diagram the five smaller rectangles are identical in size and shape. The ratio of $AB:BC$ is

(A) 3:2  (B) 2:1  (C) 5:2
(D) 5:3  (E) 4:3

19. The year 2000 is a leap year. The year 2100 is not a leap year. The following are the complete rules for determining a leap year:

(i) Year $Y$ is not a leap year if $Y$ is not divisible by 4.
(ii) Year $Y$ is a leap year if $Y$ is divisible by 4 but not by 100.
(iii) Year $Y$ is not a leap year if $Y$ is divisible by 100 but not by 400.
(iv) Year $Y$ is a leap year if $Y$ is divisible by 400.

How many leap years will there be from the years 2000 to 3000 inclusive?

(A) 240  (B) 242  (C) 243  (D) 244  (E) 251

20. A straight line is drawn across an 8 by 8 checkerboard. What is the greatest number of 1 by 1 squares through which this line could pass?

(A) 12  (B) 14  (C) 16  (D) 11  (E) 15

Part C: Each correct answer is worth 8.

21. $ABCD$ is a rectangle with $AD=10$. If the shaded area is 100, then the shortest distance between the semicircles is

(A) $2.5\pi$  (B) $5\pi$  (C) $\pi$
(D) $2.5\pi + 5$  (E) $2.5\pi - 2.5$

22. A wooden rectangular prism has dimensions 4 by 5 by 6. This solid is painted green and then cut into 1 by 1 by 1 cubes. The ratio of the number of cubes with exactly two green faces to the number of cubes with three green faces is

(A) 9:2  (B) 9:4  (C) 6:1  (D) 3:1  (E) 5:2

23. The left most digit of an integer of length 2000 digits is 3. In this integer, any two consecutive digits must be divisible by 17 or 23. The 2000th digit may be either ‘a’ or ‘b’. What is the value of $a+b$?

(A) 3  (B) 7  (C) 4  (D) 10  (E) 17

24. There are seven points on a piece of paper. Exactly four of these points are on a straight line. No other line contains more than two of these points. Three of these seven points are selected to form the vertices of a triangle. How many triangles are possible?

(A) 18  (B) 28  (C) 30  (D) 31  (E) 33

continued ...
25. \( \triangle ABC \) is an isosceles triangle in which \( AB = AC = 10 \) and \( BC = 12 \). The points \( S \) and \( R \) are on \( BC \) such that \( BS:SR:RC = 1:2:1 \). The midpoints of \( AB \) and \( AC \) are \( P \) and \( Q \) respectively. Perpendiculars are drawn from \( P \) and \( R \) to \( SQ \) meeting at \( M \) and \( N \) respectively. The length of \( MN \) is

(A) \( \frac{9}{\sqrt{13}} \)  \hspace{1cm} (B) \( \frac{10}{\sqrt{13}} \)  \hspace{1cm} (C) \( \frac{11}{\sqrt{13}} \)

(D) \( \frac{12}{\sqrt{13}} \)  \hspace{1cm} (E) \( \frac{5}{2} \)
Pascal Contest (Grade 9)

Wednesday, February 24, 1999

Instructions

1. Do not open the contest booklet until you are told to do so.
2. You may use rulers, compasses and paper for rough work.
3. Be sure that you understand the coding system for your response form. If you are not sure, ask your teacher to clarify it. All coding must be done with a pencil, preferably HB. Fill in circles completely.
4. On your response form, print your school name, city/town, and province in the box in the upper right corner.
5. Be certain that you code your name, age, sex, grade, and the contest you are writing on the response form. Only those who do so can be counted as official contestants.
6. This is a multiple-choice test. Each question is followed by five possible answers marked A, B, C, D, and E. Only one of these is correct. When you have decided on your choice, fill in the appropriate circles on the response form.
7. Scoring: Each correct answer is worth 5 credits in Part A, 6 credits in Part B, and 8 credits in Part C. There is no penalty for an incorrect answer. Each unanswered question is worth 2 credits, to a maximum of 20 credits.
8. Diagrams are not drawn to scale. They are intended as aids only.
9. When your supervisor instructs you to begin, you will have sixty minutes of working time.

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Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.

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C.M.C. Contributors: Sybase Inc. (Waterloo)
Part A: Each question is worth 5 credits.

1. The value of $\frac{4 \times 4 + 4}{2 \times 2 - 2}$ is
   (A) 2               (B) 6               (C) 10               (D) 12               (E) 18

2. If $k = 2$, then $(k^3 - 8)(k + 1)$ equals
   (A) 0               (B) 3               (C) 6               (D) 8               (E) $-6$

3. If $4(\heartsuit)^2 = 144$, then a value of $\heartsuit$ is
   (A) 3               (B) 6               (C) 9               (D) 12               (E) 18

4. Which of the following numbers divide exactly into $\left(15 + \sqrt{49}\right)$?
   (A) 3               (B) 4               (C) 5               (D) 7               (E) 11

5. If 10% of 400 is decreased by 25, the result is
   (A) 15               (B) 37.5               (C) 65               (D) 260               (E) 3975

6. In the diagram, $a + b$ equals
   (A) 10               (B) 85               (C) 110
   (D) 170               (E) 190

7. If $2x - 1 = 5$ and $3y + 2 = 17$, then the value of $2x + 3y$ is
   (A) 8               (B) 19               (C) 21               (D) 23               (E) 25

8. The average of four test marks was 60. The first three marks were 30, 55 and 65. What was the fourth mark?
   (A) 40               (B) 55               (C) 60               (D) 70               (E) 90

9. In the diagram, each small square is 1 cm by 1 cm. The area of the shaded region, in square centimetres, is
   (A) 2.75               (B) 3               (C) 3.25
   (D) 4.5               (E) 6
10. \(10 + 10^3 \) equals

(A) \(2.0 \times 10^3\)  
(B) \(8.0 \times 10^3\)  
(C) \(4.0 \times 10^1\)  
(D) \(1.0 \times 10^4\)  
(E) \(1.01 \times 10^3\)

Part B: Each question is worth 6 credits.

11. Today is Wednesday. What day of the week will it be 100 days from now?
   (A) Monday  
   (B) Tuesday  
   (C) Thursday  
   (D) Friday  
   (E) Saturday

12. The time on a digital clock is 5:55. How many minutes will pass before the clock next shows a time with all digits identical?
   (A) 71  
   (B) 72  
   (C) 255  
   (D) 316  
   (E) 436

13. In Circle Land, the numbers 207 and 4520 are shown in the following way:

\[
\begin{array}{c}
\text{207} \\
\text{7} \\
\end{array} \quad \begin{array}{c}
\text{4520} \\
\text{2} \\
\end{array}
\]

In Circle Land, what number does the following diagram represent?

\[
\begin{array}{c}
\text{5} \\
\end{array}
\]

(A) 30105  
(B) 30150  
(C) 3105  
(D) 3015  
(E) 315

14. An 8 cm cube has a 4 cm square hole cut through its centre, as shown. What is the remaining volume, in \(\text{cm}^3\)?
   (A) 64  
   (B) 128  
   (C) 256  
   (D) 384  
   (E) 448

15. For how many different values of \(k\) is the 4-digit number \(7k52\) divisible by 12?
   (A) 0  
   (B) 1  
   (C) 2  
   (D) 3  
   (E) 4

16. In an election, Harold received 60\% of the votes and Jacque received all the rest. If Harold won by 24 votes, how many people voted?
   (A) 40  
   (B) 60  
   (C) 72  
   (D) 100  
   (E) 120
17. In the parallelogram, the value of $x$ is

(A) 30 
(B) 50 
(C) 70 
(D) 80 
(E) 150

18. In the diagram, $AD < BC$. What is the perimeter of $ABCD$?

(A) 23 
(B) 26 
(C) 27 
(D) 28 
(E) 30

19. The numbers 49, 29, 9, 40, 22, 15, 53, 33, 13, 47 are grouped in pairs so that the sum of each pair is the same. Which number is paired with 15?

(A) 33 
(B) 40 
(C) 47 
(D) 49 
(E) 53

20. The units (ones) digit in the product $(5 + 1)(5^3 + 1)(5^6 + 1)(5^{12} + 1)$ is

(A) 6 
(B) 5 
(C) 2 
(D) 1 
(E) 0

Part C: Each question is worth 8 credits.

21. A number is Beprisque if it is the only natural number between a prime number and a perfect square (e.g. 10 is Beprisque but 12 is not). The number of two-digit Beprisque numbers (including 10) is

(A) 1 
(B) 2 
(C) 3 
(D) 4 
(E) 5

22. If $w = 2^{129} \times 3^{81} \times 5^{128}$, $x = 2^{127} \times 3^{81} \times 5^{128}$, $y = 2^{126} \times 3^{82} \times 5^{128}$, and $z = 2^{125} \times 3^{82} \times 5^{129}$, then the order from smallest to largest is

(A) $w, x, y, z$ 
(B) $x, w, y, z$ 
(C) $x, y, z, w$ 
(D) $z, y, x, w$ 
(E) $x, w, z, y$

23. Al and Bert must arrive at a town 22.5 km away. They have one bicycle between them and must arrive at the same time. Bert sets out riding at 8 km/h, leaves the bicycle and then walks at 5 km/h. Al walks at 4 km/h, reaches the bicycle and rides at 10 km/h. For how many minutes was the bicycle not in motion?

(A) 60 
(B) 75 
(C) 84 
(D) 94 
(E) 109

24. A number is formed using the digits 1, 2, ..., 9. Any digit can be used more than once, but adjacent digits cannot be the same. Once a pair of adjacent digits has occurred, that pair, in that order, cannot be used again. How many digits are in the largest such number?

(A) 72 
(B) 73 
(C) 144 
(D) 145 
(E) 91

continued ...
25. Two circles \( C_1 \) and \( C_2 \) touch each other externally and the line \( l \) is a common tangent. The line \( m \) is parallel to \( l \) and touches the two circles \( C_1 \) and \( C_3 \). The three circles are mutually tangent. If the radius of \( C_2 \) is 9 and the radius of \( C_3 \) is 4, what is the radius of \( C_1 \)?

(A) 10.4  (B) 11  (C) \( 8\sqrt{2} \)

(D) 12  (E) \( 7\sqrt{3} \)
Instructions

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9. When your supervisor instructs you to begin, you will have sixty minutes of working time.

Pascal Contest (Grade 9)
Wednesday, February 18, 1998

Time: 1 hour

Calculators are permitted, providing they are non-programmable and without graphic displays.
Part A: Each question is worth 5 credits.

1. The value of \(\frac{1+3+5}{10+6+2}\) is
   
   (A) \(\frac{1}{6}\)  (B) 2  (C) \(\frac{1}{2}\)  (D) \(1\frac{1}{2}\)  (E) \(3\frac{1}{10}\)

2. If \(3(x - 5) = 3(18 - 5)\), then \(x\) is
   
   (A) \(\frac{44}{3}\)  (B) \(\frac{32}{3}\)  (C) 9  (D) 18  (E) 81

3. The pie chart shows a percentage breakdown of 1000 votes in a student election. How many votes did Sue receive?
   
   (A) 550  (B) 350  (C) 330  (D) 450  (E) 935

4. The value of \((\sqrt{169} - \sqrt{25})^2\) is
   
   (A) 64  (B) 8  (C) 16  (D) 144  (E) 12

5. The value of \(\frac{5^6 \times 5^9 \times 5}{5^3}\) is
   
   (A) \(5^{18}\)  (B) \(25^{18}\)  (C) \(5^{13}\)  (D) \(25^{13}\)  (E) \(5^{51}\)

6. If \(x = 3\), which of the following expressions is an even number?
   
   (A) \(9x\)  (B) \(x^3\)  (C) \(2\left(x^2 + 9\right)\)  (D) \(2x^2 + 9\)  (E) \(3x^2\)

7. The value of \(490 - 491 + 492 - 493 + 494 - 495 + \ldots - 509 + 510\) is
   
   (A) 500  (B) \(-10\)  (C) \(-11\)  (D) 499  (E) 510

8. The average (mean) of a list of 10 numbers is 0. If 72 and \(-12\) are added to the list, the new average will be
   
   (A) 30  (B) 6  (C) 0  (D) 60  (E) 5

9. What is one-half of \(1.2 \times 10^{30}\) ?
   
   (A) \(6.0 \times 10^{30}\)  (B) \(6.0 \times 10^{29}\)  (C) \(0.6 \times 5^{30}\)  (D) \(1.2 \times 10^{15}\)  (E) \(1.2 \times 5^{30}\)

10. If \(x + y + z = 25\) and \(y + z = 14\), then \(x\) is
    
    (A) 8  (B) 11  (C) 6  (D) \(-6\)  (E) 31
Part B: Each question is worth 6 credits.

11. The number in an unshaded square is obtained by adding the numbers connected to it from the row above. (The ‘11’ is one such number.) The value of $x$ is
(A) 4 (B) 6 (C) 9
(D) 15 (E) 10

12. In the diagram, $DA = CB$. What is the measure of $\angle DAC$?
(A) 70° (B) 100° (C) 95°
(D) 125° (E) 110°

13. A three-wheeled vehicle travels 100 km. Two spare wheels are available. Each of the five wheels is used for the same distance during the trip. For how many kilometres is each wheel used?
(A) 20 (B) 25 (C) $33\frac{1}{3}$ (D) 50 (E) 60

14. The sum of the digits of a five-digit positive integer is 2. (A five-digit integer cannot start with zero.) The number of such integers is
(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

15. Four points are on a line segment, as shown.
If $AB : BC = 1 : 2$ and $BC : CD = 8 : 5$, then $AB : BD$ equals
(A) 4 : 13 (B) 1 : 13 (C) 1 : 7
(D) 3 : 13 (E) 4 : 17

16. On a rectangular table 5 units long and 2 units wide, a ball is rolled from point $P$ at an angle of 45° to $PQ$ and bounces off $SR$. The ball continues to bounce off the sides at 45° until it reaches $S$. How many bounces of the ball are required?
(A) 9 (B) 8 (C) 7
(D) 5 (E) 4

17. If $1998 = p^5q^r$, where $p$, $q$ and $r$ are prime numbers, what is the value of $p + q + r$?
(A) 222 (B) 48 (C) 42 (D) 66 (E) 122

18. In the diagram, $DEFG$ is a square and $ABCD$ is a rectangle. A straight line is drawn from $A$, passes through $C$ and meets $FG$ at $H$. The area of the shaded region is
(A) 8 (B) 8.5 (C) 10
(D) 9 (E) 10.5
19. Using only digits 1, 2, 3, 4, and 5, a sequence is created as follows: one 1, two 2’s, three 3’s, four 4’s, five 5’s, six 1’s, seven 2’s, and so on.
The sequence appears as: 1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 1, 1, 1, 1, 1, 1, 2, 2, … .
The 100th digit in the sequence is
(A) 1  (B) 2  (C) 3  (D) 4  (E) 5

20. Driving between two towns at 110 km/h instead of 100 km/h saves 9 minutes. The distance in kilometres between the two towns is
(A) 210  (B) 99  (C) 165  (D) 9900  (E) 150

Part C: Each question is worth 8 credits.

21. \(Q\) is the point of intersection of the diagonals of one face of a cube whose edges have length 2 units. The length of \(QR\) is
(A) 2  (B) \(\sqrt{8}\)  (C) \(\sqrt{5}\)
(D) \(\sqrt{12}\)  (E) \(\sqrt{6}\)

22. A deck of 100 cards is numbered from 1 to 100. Each card has the same number printed on both sides. One side of each card is red and the other side is yellow. Barsby places all the cards, red side up, on a table. He first turns over every card that has a number divisible by 2. He then examines all the cards, and turns over every card that has a number divisible by 3. How many cards have the red side up when Barsby is finished?
(A) 83  (B) 17  (C) 66  (D) 50  (E) 49

23. The numbers 123 456 789 and 999 999 999 are multiplied. How many of the digits in the final result are 9’s?
(A) 0  (B) 1  (C) 2  (D) 3  (E) 17

24. Three rugs have a combined area of 200 \(m^2\). By overlapping the rugs to cover a floor area of 140 \(m^2\), the area which is covered by exactly two layers of rug is 24 \(m^2\). What area of floor is covered by three layers of rug?
(A) 12 \(m^2\)  (B) 18 \(m^2\)  (C) 24 \(m^2\)  (D) 36 \(m^2\)  (E) 42 \(m^2\)

25. One way to pack a 100 by 100 square with 10 000 circles, each of diameter 1, is to put them in 100 rows with 100 circles in each row. If the circles are repacked so that the centres of any three tangent circles form an equilateral triangle, what is the maximum number of additional circles that can be packed?
(A) 647  (B) 1442  (C) 1343  (D) 1443  (E) 1344
Part A: Each question is worth 5 credits.

1. \( \frac{4 + 35}{8 - 5} \) equals
   (A) \( \frac{11}{7} \)  (B) 8  (C) \( \frac{7}{2} \)  (D) -12  (E) 13

2. In the subtraction question \( \frac{4}{8} \frac{7}{9} \frac{2}{6} \frac{8}{8} \), the \( \heartsuit \) represents the digit
   (A) 2  (B) 8  (C) 7  (D) 5  (E) 4

3. The value of \( 2 \frac{1}{10} + 3 \frac{11}{100} + 4 \frac{111}{1000} \) is
   (A) 9.321  (B) 9.111  (C) 9.123  (D) 9.111111  (E) 9.11081081

4. \((1)^{10} + (-1)^8 + (-1)^7 + (1)^5\) equals
   (A) 0  (B) 1  (C) 2  (D) 16  (E) 4

5. If 60% of a number is 42, what is 50% of the same number?
   (A) 25  (B) 28  (C) 30  (D) 35  (E) 40

6. If \( x = -2 \), the value of \( (x)(x^2)(\frac{1}{x}) \) is
   (A) 4  (B) \(-8 \frac{1}{2}\)  (C) -4  (D) -8  (E) 16

7. In the diagram the triangle shown is isosceles with \( AB = AC \). The value of \( x \) is
   (A) 40  (B) 55  (C) 35  (D) 50  (E) 35

8. The greatest number of Mondays that can occur in the first 45 days of a year is
   (A) 5  (B) 6  (C) 7  (D) 8  (E) 9

9. When a certain number is divided by 9, the quotient is 6 and the remainder is 4. The number is
   (A) 58  (B) 42  (C) 33  (D) 67  (E) 49

10. The sum of nine consecutive positive integers is 99. The largest of these integers is
    (A) 9  (B) 11  (C) 19  (D) 7  (E) 15
Part B: Each question is worth 6 credits.

11. Twelve balloons are arranged in a circle as shown. Counting clockwise, every third balloon is popped. C is the first one popped. This continues around the circle until two unpopped balloons remain. The last two remaining balloons are

(A) $B, H$  
(B) $B, G$  
(C) $A, E$  
(D) $E, J$  
(E) $F, K$

12. The graph shows the number of students who selected each of five possible choices in responding to a question. The correct response was the one most frequently chosen. The percentage of students who selected the correct response was

(A) 14  
(B) 56  
(C) 50  
(D) 11  
(E) 44

13. Janet has 10 coins consisting of nickels, dimes, and quarters. Seven of the coins are either dimes or quarters, and eight of the coins are either dimes or nickels. How many dimes does Janet have?

(A) 2  
(B) 3  
(C) 4  
(D) 5  
(E) 6

14. In the game "TRISQUARE", three points are awarded for each triangle found, and four points for each square. The highest number of points that can be achieved for the given diagram is

(A) 38  
(B) 36  
(C) 34  
(D) 32  
(E) 28

15. Each of the numbers 1, 2, 3, and 4 is substituted, in some order for $p, q, r, s$. The greatest possible value of $p^q + r^s$ is

(A) 14  
(B) 19  
(C) 66  
(D) 83  
(E) 162
16. In the diagram, all triangles are equilateral. What fraction of \( \triangle ABC \) is coloured black?
(A) \( \frac{3}{4} \)  
(B) \( \frac{1}{2} \)  
(C) \( \frac{9}{16} \)
(D) \( \frac{4}{9} \)  
(E) \( \frac{27}{64} \)

17. The digits 1, 2, 3, 4 can be arranged to form twenty-four different four-digit numbers. If these twenty-four numbers are then listed from the smallest to largest, in what position is 3142?
(A) 13th  
(B) 14th  
(C) 15th  
(D) 16th  
(E) 17th

18. The product of 20^50 and 50^20 is written as an integer in expanded form. The number of zeros at the end of the resulting integer is
(A) 70  
(B) 71  
(C) 90  
(D) 140  
(E) 210

19. Three balls numbered 1, 2, and 3 are placed in a bag. A ball is drawn from the bag and the number is recorded. The ball is returned to the bag. After this has been done three times, what is the probability that the sum of the three recorded numbers is less than 8?
(A) \( \frac{23}{27} \)  
(B) \( \frac{5}{7} \)  
(C) \( \frac{5}{9} \)  
(D) \( \frac{8}{9} \)  
(E) \( \frac{22}{27} \)

20. A beam of light shines from point \( S \), reflects off a reflector at point \( P \), and reaches point \( T \) so that \( PT \) is perpendicular to \( RS \). Then \( x \) is
(A) 26°  
(B) 32°  
(C) 37°  
(D) 38°  
(E) 45°
Part C: Each question is worth 8 credits.

21. In the diagram adjacent edges are at right angles. The four longer edges are equal in length, and all of the shorter edges are also equal in length. The area of the shape is 528. What is the perimeter?
   (A) 132  (B) 264  (C) 144
   (D) 72    (E) 92

22. If $\frac{97}{19} = w + \frac{1}{x + \frac{1}{y}}$, where $w, x, y$ are all positive integers, then $w + x + y$ equals
   (A) 16  (B) 17  (C) 18  (D) 19  (E) 26

23. Determine all ordered pairs that satisfy $(x - y)^2 + x^2 = 25$, where $x$ and $y$ are integers and $x \geq 0$. The number of different values of $y$ that occur is
   (A) 3  (B) 4  (C) 5  (D) 6  (E) 7

24. Two ships, one 200 metres in length and the other 100 metres in length, travel at constant but different speeds. When travelling in opposite directions, it takes 10 seconds for them to completely pass each other. When travelling in the same direction, it takes 25 seconds for them to completely pass each other. The speed of the faster ship, in metres per second, is
   (A) 12  (B) 14  (C) 18  (D) 21  (E) 30

25. In the diagram, the right prism has quadrilateral base $EFGH$ with right angles at $E$ and $G$. The height $AE$ is 32. The distance from $A$ to $G$ is
   (A) 41  (B) 40  (C) 34
   (D) 36  (E) 44