Answers to Practice Set Number 3

Pascal

1) E 2) D 3) D 4) A 5) C 6) C 7) E 8) B 9) B 10) E

Cayley

1) C 2) E 3) C 4) E 5) B 6) C 7) C 8) A 9) D 10) B

Fermat

1) D 2) E 3) B 4) B 5) C 6) C 7) A 8) E 9) A 10) A

Hints, suggestions, and some solutions:

Pascal

1. $\left(\frac{6}{5}\right)\left(\frac{7}{6}\right)\left(\frac{8}{7}\right) = \left(\frac{8}{5}\right)$.
2. Since the three angles add to 180, $2b + b + 3b = 180$ and $b = 30$. So $3b = 90$ and we have a right triangle.
3. Trial and error shows the numbers are 45, 47, 49.
4. Since most of these cancel in pairs we just add $24 + 25 + 26 \ldots + 31$. These 8 integers average $(24 + 31)/2$ and so the total is $4 \times 55 = 220$.
5. Think of the suits as 100 groups of 2 expensive and 1 cheap suit. The average of each group is $(24 + 31)/2$ and so the total is $4 \times 55 = 220$.
6. The numerator must be increased by 1700.
7. The area is an a by b rectangle plus a triangle of height $c - a$ and base b. The area is $ab + a(c - b)/2 = ab/2 + ac/2 = a(b + c)/2$.
8. The areas of triangles with the same altitudes are in the same ratio as their bases. So if the required area is A then $\frac{A}{54} = \frac{12}{36}$ and $A = 18$.
9. If the distance is D then $\frac{D}{60} - \frac{D}{100} = 2$ and $D = 300$. So the required speed is $\frac{300}{4} = 75$ km/h.
10. Join E to the midpoint M and AC. Then the triangle CEM is equilateral of side 1 and triangle EMA has a 120 angle at M and $MA = ME = 1$. Bisect angle M in the triangle to form two 30-60-90 triangles. Using the ratio of the sides of a 30-60-90 triangle $AE = 2\left(\frac{\sqrt{3}}{2}\right)$.

Cayley

1. The answer is approximately $14 \times 365 \times 24 \times 60 \times 60/1000000 = 400$.
2. The perimeter is just the same as a 9 by 27 rectangle!
3. Using the standard formula $2(8x15 + 4x15 + 8x4)$.

4. $x + 47 = 2(25)$ and $11 + y = 28$ so $x + y = 20$.
5. Solve $32x + 72(25 - x) = 64(25)$.
6. Since the correct answer involves multiplying and then adding 5 the error must involve reversing these operations. So $x = 5$ and the correct answer is $95 + 5 = 100$.
7. Divide the area into two semicircles around a rectangle!
8. Only decimals in lowest terms whose denominators involve only powers of 2 and 5 terminate. But $144 = 9 \times 16$ so $\frac{9}{144} = \frac{1}{16}$ is the first terminating decimal.
9. First $CZ = 6$, $BY = k$ and $CY = 8 - k$. Using Pythagoras

   $$PA^2 + PB^2 + PC^2 = PX^2 + 9 + PY^2 + k^2 + PZ^2 + 36 = PX^2 + 25 + PY^2 + (8 - k)^2 + PZ^2 + 4$$

   Thus $16k = 48$ and $k = 3$.
10. $2004 = 2 \times 2 \times 3 \times 167$. Let $n = k^2$ and $n + 2004 = m^2$ so $m^2 - k^2 = 2004 \Rightarrow (m+k)(m-k) = 2004$. But $m + k$ and $m - k$ must have same parity and so must be even. So $\{m + k = 2(501)$ and $m - k = 2(1)\}$ or $\{m + k = 2(167)$ and $m - k = 2(3)\}$. Thus $k = 500$ or 164, leading to just 2 values for n.

Fermat

1. Since the triangle is scalene the best we can do is 75, 74, 31.
2. A little arithmetic shows that for a, b, c and d to be integers a must be a multiple of 9.
3. $(a - b)^2 = a^2 + b^2 - 2ab = 9$.
4. $432 = 16 \times 27$. Since 16 is a perfect square we need only make 27 one also.
5. Since the ratio $16 : 24 = 24 : 36 = 2 : 3$ the triangles are similar and their areas are in ratio $4 : 9$.
6. The number '*ddd*' has a factor $111 = 3 \times 37$. So 37 must be one number. The other is $9 \times 3 = 27$ and indeed $27 \times 37 = 999$.
7. Using sum and product $ab = b$ and $-a = a + b$. Since there are two roots, a and b are not zero so solving these equations $a = 1$ and $b = -2$.
8. The areas are: a) 12.25 b) 12 c) 12 d) $4\sqrt{5}$ e) $\frac{9}{2}\pi$
9. Since $30^{30} = 2^{30} \cdot 3^{30} \cdot 5^{30}$ and perfect squares have even exponents the answer is $16 \times 16 \times 16$.
10. Note triangle ABC is a $30, 60, 90$ triangle! Let the other point of circle intersection be D. The required area is then sector ACD plus sector BCD minus twice triangle ABC.
    Since the sector angles are 60 and 120 we get $\frac{1}{3}(3\pi) + \frac{1}{6}(9\pi) = 2\left(\frac{1}{2}3\sqrt{3}\right) = \frac{5}{2}\pi - 3\sqrt{3}$.