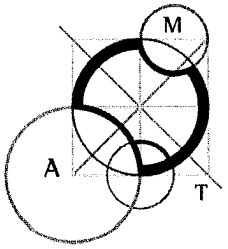


AUSTRALIAN MATHEMATICS COMPETITION
WARM-UP PAPER
INTERMEDIATE 8

Questions 1 - 4, 3 marks each

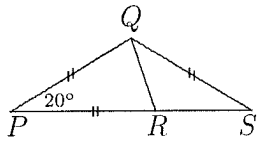
1. $(0.01)^2$ equals

(A) 0.1 (B) 0.01 (C) 0.001 (D) 0.0001 (E) 0.0002

2. The value of $\dfrac{0.75}{15}$ is

(A) 5 (B) 0.5 (C) 0.05 (D) 0.005 (E) 0.0005

3. In the diagram $PQ = PR = QS$ and $\angle QPR = 20°$. The size of $\angle RQS$, in degrees, is



(A) 20 (B) 40 (C) 60 (D) 80 (E) 100

4. My children are aged six, eight and ten years. Between them they receive \$12 pocket money each week, proportional to their ages. How much does the eldest receive per week?

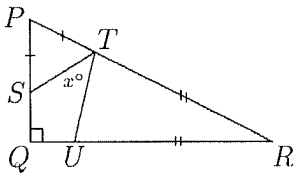
(A) \$3 (B) \$4 (C) \$5 (D) \$6 (E) \$10

Questions 5 - 8, 4 marks each

5. Two fractions are equally spaced between $\frac{1}{4}$ and $\frac{2}{3}$. The smaller of the two fractions is

(A) $\dfrac{13}{24}$ (B) $\dfrac{7}{18}$ (C) $\dfrac{29}{36}$ (D) $\dfrac{5}{12}$ (E) $\dfrac{1}{3}$

6. Triangle PQR is right angled at Q and triangles PST and RTU are isosceles as shown. If $\angle STU$ measures $x°$ then the value of x is



(A) 30 (B) 45 (C) 50 (D) 55 (E) 60

7. The diagram shows a 5 by 5 table.

P	Q	R	S	T
	P	Q	R	

The top row contains the symbols P, Q, R, S and T. The fourth row contains the symbols P, Q and R at the centre. The remaining squares can be filled with Ps, Qs, Rs, Ss and Ts such that no row, column or diagonal contains the same symbol more than once. The symbol that must go into the shaded square is

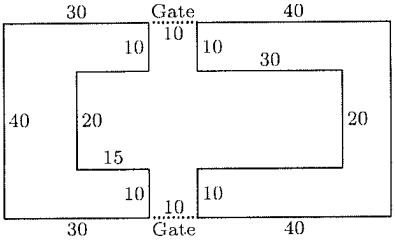
(A) P (B) Q (C) R (D) S (E) T

8. If $a^2 = a + 2$, then a^3 equals

(A) $a + 4$ (B) $2a + 8$ (C) $3a + 2$ (D) $4a + 8$ (E) $27a + 8$

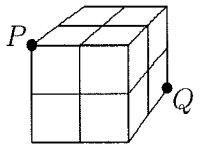
Questions 9 - 10, 5 marks each

9. Here is the plan of a building which has a courtyard with two entrance gates. Passers-by can look through the gates but may not enter. Dimensions of the building are given in metres, and all corners are right angles. What is the area, in square metres, of that part of the courtyard which cannot be seen by passers-by?



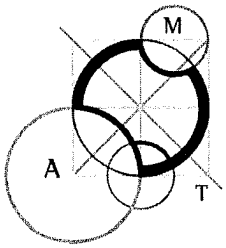
(A) 250 (B) 200 (C) 300 (D) 400 (E) 325

10. Each face of a solid cube is divided into four as indicated in the diagram.



Starting from vertex P, paths can be travelled to vertex Q along connected line segments. If each movement along the path takes one closer to Q, the number of possible paths from P to Q is

(A) 46 (B) 90 (C) 36 (D) 54 (E) 60



AUSTRALIAN MATHEMATICS COMPETITION
WARM-UP PAPER
INTERMEDIATE 9

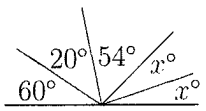
Questions 1 - 4, 3 marks each

1. $(7a + 5b) - (5a - 7b)$ equals

(A) $12a - 12b$ (B) $2a - 2b$ (C) 0 (D) $2a + 12b$ (E) $12a - 2b$

2. In the diagram x equals

(A) 34 (B) 33 (C) 46 (D) 67 (E) 23



3. The value of

$$\frac{\sqrt{20 + x^2}}{\sqrt{20 - x^2}},$$

when $x = 4$, is

(A) $\sqrt{\frac{3}{2}}$ (B) $\frac{9}{4}$ (C) 3 (D) $\frac{9}{2}$ (E) 9

4. What is the highest power of 2 which divides exactly into 1 000 000?

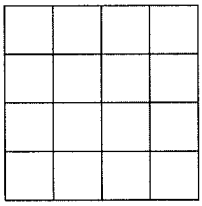
(A) 2^3 (B) 2^4 (C) 2^5 (D) 2^6 (E) 2^8

Questions 5 - 8, 4 marks each

5. A litre of orange fruit juice drink contains 10% orange juice. How many millilitres of orange juice must be added to produce a mixture containing 50% orange juice?

(A) 450 (B) 800 (C) 600 (D) 400 (E) 500

6. Piran has a 4×4 grid of squares on which he is trying to place as many counters as possible.



No more than one counter may be placed on any square and no more than three on any row, column or diagonal. What is the maximum number of counters he can place in this way?

(A) 9 (B) 10 (C) 11 (D) 12 (E) 13

7. How many positive integers less than 1000 have the sum of their digits equal to 6?

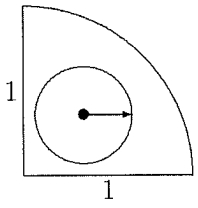
(A) 28 (B) 19 (C) 111 (D) 18 (E) 27

8. Three jolly swagmen, John, Kevin and Robert, are gambling by a billabong. They start with sums of money in the ratio $7 : 6 : 5$ and finish with sums of money in the ratio $6 : 5 : 4$ (in the same order of wealth). One of the swagmen won \$12. How many dollars did he start with?

(A) 420 (B) 1080 (C) 432 (D) 120 (E) 90

Questions 9 - 10, 5 marks each

9. What is the radius of the largest circle that can be drawn inside a quarter circle of radius 1 unit?



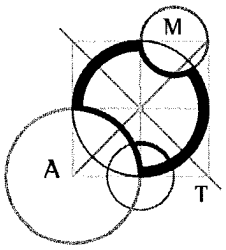
(A) $\sqrt{2}-1$ (B) $\frac{1}{2}$ (C) $\frac{\sqrt{2}}{3}$ (D) $\frac{1}{\pi}$ (E) $\frac{1}{2\sqrt{2}}$

10. If p and q are positive integers such that

$$\frac{7}{10} < \frac{p}{q} < \frac{11}{15}$$

then the smallest possible value of q is

(A) 25 (B) 60 (C) 30 (D) 7 (E) 6



AMC WARM-UP PAPER
INTERMEDIATE PAPER 8
SOLUTIONS

1. $(0.01)^2 = 0.0001$,

hence (D).

2. The value of $\dfrac{75}{15}$ is 5. Hence the value of $\dfrac{0.75}{15}$ is 0.05,

hence (C).

3. It can be seen that $\angle PQR = \angle PRQ = 80°$ and that, since $\triangle QPS$ is isosceles, $\angle PSQ = 20°$. Thus $\angle PQS = 140°$ and $\angle RQS = 140° - 80° = 60°$,

hence (C).

4. Let the eldest child receive $\$x$ per week. Then the others receive $\$0.6x$ and $\$0.8x$. Adding these, we have $(0.6 + 0.8 + 1)x = 2.4x = 12$, i.e. $x = 12/2.4 = 5$,

hence (C).

5. Let the spacing be h and the smaller fraction be x.
Then $\frac{2}{3} = \frac{1}{4} + 3h$, i.e. $3h = \frac{2}{3} - \frac{1}{4} = (8 - 3)/12 = 5/12$.
Thus $h = 5/36$ and so $x = \frac{1}{4} + \frac{5}{36} = (9 + 5)/36 = 14/36 = 7/18$,

hence (B).

6. Let $\angle SPT = y°$. Then $\angle TRU = (90 - y)°$. Since $\triangle PST$ is isosceles,

$$\angle PTS = \frac{1}{2}(180 - y)° = (90 - \frac{1}{2}y)°.$$

Similarly,

$$\angle UTR = (90 - \frac{1}{2}(90 - y))° = 45 + \frac{1}{2}y°.$$

Summing the angles at T gives

$$x + 90 - \frac{1}{2}y + 45 + \frac{1}{2}y = 180,$$

i.e. $x = 45$,

hence (B).

7. The square marked * must be filled with Q, because there are already R and S on the same column and P and T on the same main diagonal. Now the only square on the first column that can be filled with Q is the shaded one

P	Q	R	S	T
			*	
	P	Q	R	

P	Q	R	S	T
S	T	P	Q	R
Q	R	S	T	P
T	P	Q	R	S
R	S	T	P	Q

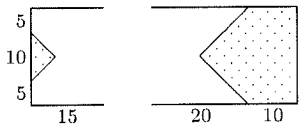
hence (B).

Note that the solution is unique and is a 5×5 Latin Square.

8. Since $a^2 = a + 2$, $a^3 = a(a + 2) = a^2 + 2a = a + 2 + 2a = 3a + 2$,

hence (C).

9. Given that the gateways are square (i.e. 10 metres by 10 metres) the maximum diagonal angles through which the passers-by can see are 45° and the visible region of the internal courtyard is that which is not shaded below.

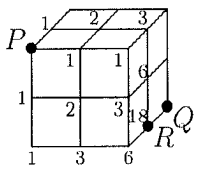


It can be seen that the area remaining is, in square metres,

$$25 + 300 = 325$$

hence (E).

10. The number of paths to each node shown on the diagram (drawn schematically) are found by successive addition. It is then easily seen that the number of paths to point R is 18. Hence by symmetry (there are 3 ways of approaching Q), the total number is $3 \times 18 = 54$,



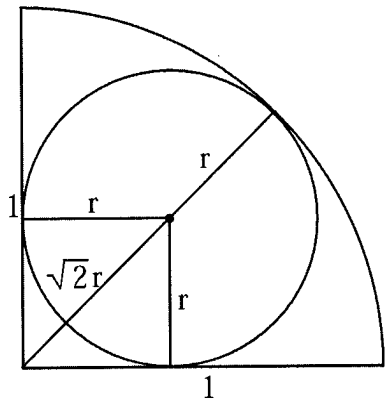
hence (D).

Pascal and Cayley Contest Preparation

Answers to Australian Mathematics Competition Intermediate 9

1. D
2. E
3. C
4. D
5. B
6. D
7. A
8. A
9. A

   Hint: In the diagram below, we can see that the largest circle will touch the quarter circle at 3 different points. We can draw 3 radius lines from these points to the centre of the circle and label them r as shown below. Then we can draw a line from the corner of the quarter-circle to the centre and using Pythagorean Theorem, we can label it $\sqrt{2}r$. We can see that this is an extension of one of the radius lines, and that this extended line is the radius of the quarter circle, 1. Use this to determine r.



10. D

    Hint: Since the question asks for the smallest possible value, start with the smallest answer given, E) 6, and see if that answer works. The first fraction greater than $\frac{7}{10}$ is $\frac{5}{6}$ but it is not less than $\frac{11}{15}$ so q cannot be 6. Repeat this process with the rest of the answers given, from smallest to largest, until you find one that works.