# CIMC Sample Contest

## Part A

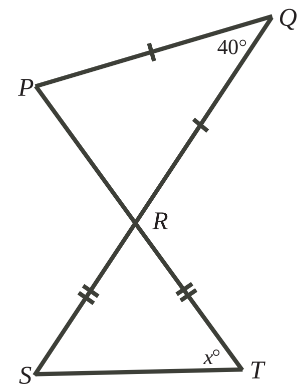
1. Determine the value of $\frac{\sqrt{25-16}}{\sqrt{25}-\sqrt{16}}$.

   {2008 Cayley #2}

2. In the diagram, $PT$ and $QS$ are straight lines intersecting at $R$ such that $QP = QR$ and $RS = RT$.

   Determine the value of $x$.

   {2008 Cayley #8}




3. If $x + y + z = 25$, $x + y = 19$ and $y + z = 18$, determine the value of $y$.

   {1998 Cayley #11}

4. The odd numbers from 5 to 21 are used to build a 3 by 3 magic square. (In a magic square, the numbers in each row, the numbers in each column, and the numbers on each diagonal have the same sum.) If 5, 9 and 17 are placed as shown, what is the value of $x$?

   {2010 Cayley #16}

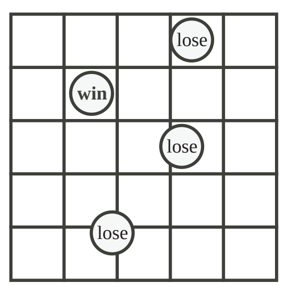
| | | |
|---|---|---|
| | 5 | |
| 9 | | 17 |
| $x$ | | |

5. What is the largest positive integer $n$ that satisfies $n^{200} < 3^{500}$?

   {2010 Cayley #20}

6. A coin that is 8 cm in diameter is tossed onto a 5 by 5 grid of squares each having side length 10 cm. A coin is in a winning position if no part of it touches or crosses a grid line, otherwise it is in a losing position. Given that the coin lands in a random position so that no part of it is off the grid, what is the probability that it is in a winning position?

   {2010 Cayley #24}

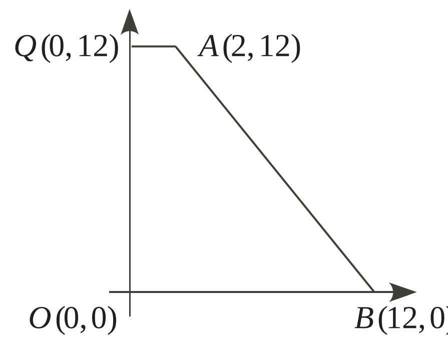


**Part B**

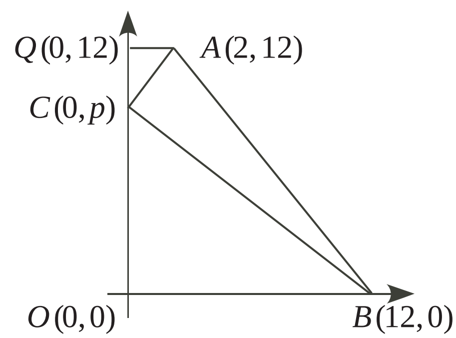
1. (a) Determine the average of the integers $71, 72, 73, 74, 75$.
   (b) Suppose that $n, n+1, n+2, n+3, n+4$ are five consecutive integers.
      (i) Determine a simplified expression for the sum of these five consecutive integers.
      (ii) If the average of these five consecutive integers is an odd integer, explain why $n$ must be an odd integer.
   (c) Six consecutive integers can be represented by $n, n+1, n+2, n+3, n+4, n+5$, where $n$ is an integer. Explain why the average of six consecutive integers is never an integer.

   {2010 Fryer #2}

2. (a) Quadrilateral $QABO$ is constructed as shown. Determine the area of $QABO$.




   (b) Point $C(0, p)$ lies on the y-axis between $Q(0, 12)$ and $O(0, 0)$ as shown. Determine an expression for the area of $\triangle COB$ in terms of $p$.




   (c) Determine an expression for the area of $\triangle QCA$ in terms of $p$.
   (d) If the area of $\triangle ABC$ is 27, determine the value of $p$.

   {2010 Galois #2}

**Part B (continued)**

3. If $m$ is a positive integer, the symbol $m!$ is used to represent the product of the integers from 1 to $m$. That is, $m! = m(m-1)(m-2)\dots(3)(2)(1)$. For example, $5! = 5(4)(3)(2)(1)$ or $5! = 120$. Some positive integers can be written in the form

$$n = a(1!) + b(2!) + c(3!) + d(4!) + e(5!).$$

In addition, each of the following conditions is satisfied:

- $a, b, c, d,$ and $e$ are integers
- $0 \le a \le 1$
- $0 \le b \le 2$
- $0 \le c \le 3$
- $0 \le d \le 4$
- $0 \le e \le 5$.

(a) Determine the largest positive value of $N$ that can be written in this form.

(b) Write $n = 653$ in this form.

(c) Prove that all integers $n$, where $0 \le n \le N$, can be written in this form.

(d) Determine the sum of all integers $n$ that can be written in this form with $c = 0$.

{2009 Galois #4}