



Grade 6 Math Circles
Winter 2015 - February 24/25
Angles

Introduction

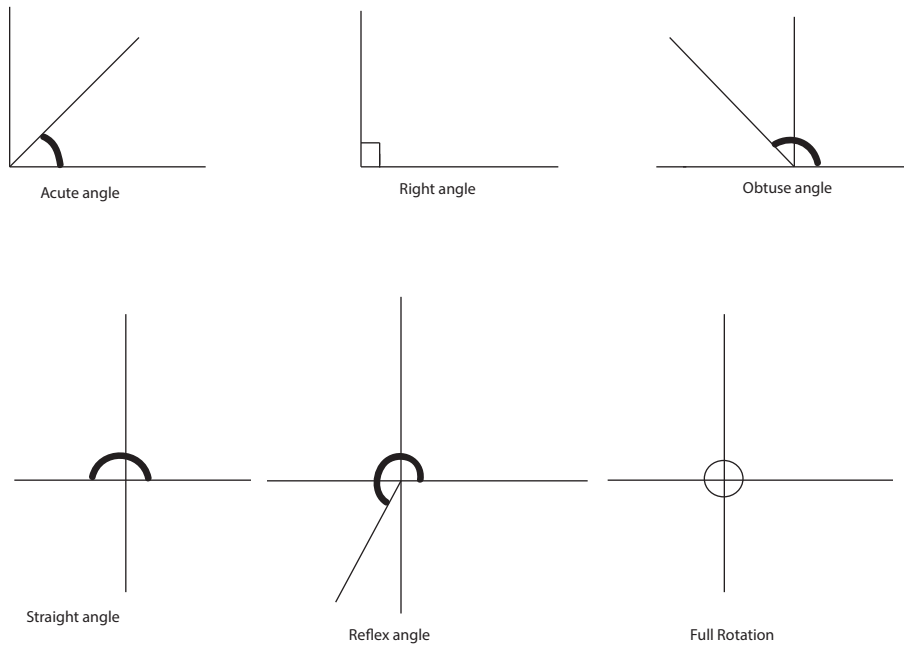
At this point in your mathematical career, you've been working with angles for a number of years. You should be comfortable measuring angles with a protractor. You also know some types and properties of angles. Today, we will look at some new properties that will help you find angles more easily. We will also take a look at triangles and other polygons.

Review: Types of Angles

An **angle** is the degree of space between two intersecting lines. We can classify angles based on how large they are:

Angle	Classification
$< 90^\circ$	Acute angle
$= 90^\circ$	Right angle
$> 90^\circ$ and $< 180^\circ$	Obtuse angle
$= 180^\circ$	Straight angle
$> 180^\circ$ and $< 360^\circ$	Reflex angle
$= 360^\circ$	Full rotation

Here are some examples of what these angles look like:



Negative Angles

A **negative angle** is an angle that begins on the same line that you would normally use to find an angle, but goes in the opposite direction. Usually (as in the examples above), we measure an angle counter-clockwise from the line that we measure as the 0° line. A negative angle will go in the clockwise direction.

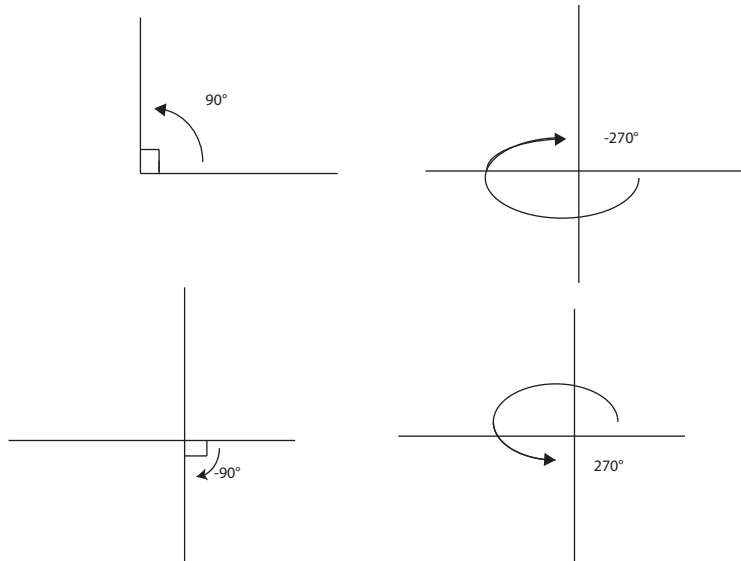
How many degrees are there in a rotation? From the table above, we see that this number is 360; this makes sense because a circle is 360° around.

When we calculate negative angles, we have to measure the angle the other way as well.

This means that any angle that is measured negatively is 360° minus the positive angle.

For example, consider the right angle, 90° . If we travelled in the opposite direction, what would the negative angle be?

$360^\circ - 90^\circ = 270^\circ$. Does this make sense?



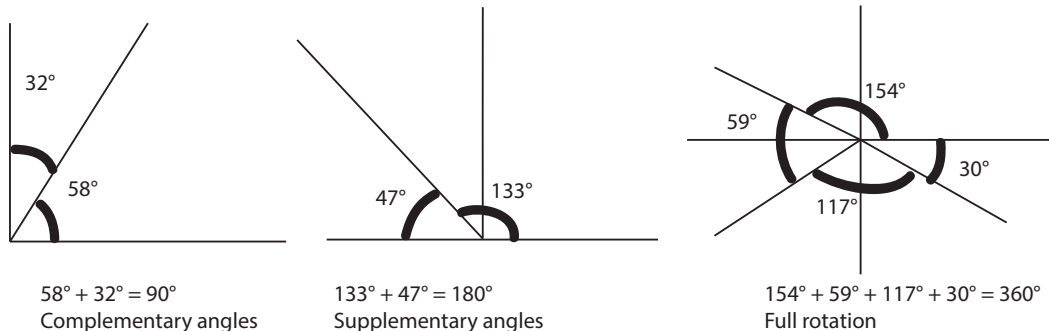
Notice that if we found the angle 270° the regular way, we would be measuring the same angle in a different direction (as shown above). So, $90^\circ = -270^\circ$, and $270^\circ = -90^\circ$.

Complementary and Supplementary Angles

Complementary angles are two angles that add up to 90° . These angles can be together or apart. When complementary angles are together, they make a right angle.

Supplementary angles are two angles that add up to 180° . They can also be together or apart. When supplementary angles are together, they make a straight angle.

In addition, the angles around any point always add up to 360° . That is, the **full rotation** around a point makes a circle, so naturally it is 360° .



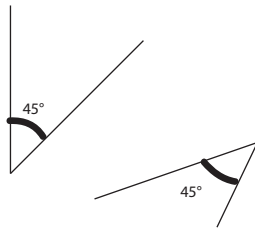
Notation

Before we begin to learn some cool angle properties, we need to learn some terminology that will come in handy.

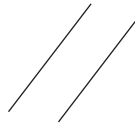
Congruent angles mean that angles are of equal measure, even if they are not in the same place.

Two lines are **parallel** if they never meet and are always the same distance apart.

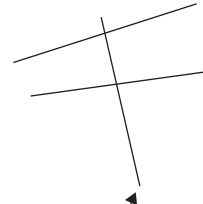
A **transversal** is a line that crosses at least two other lines.



Congruent angles

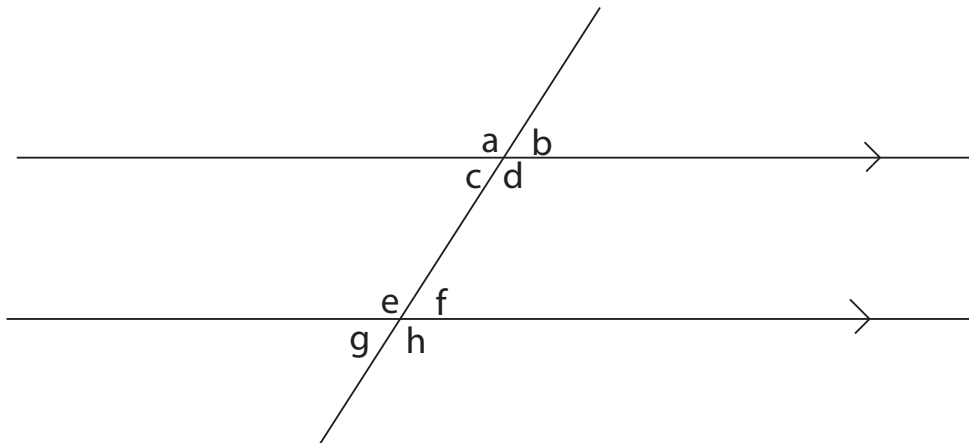


Parallel lines



Transversal

Angle Properties



Alternate angles are angles that are on opposite sides of the transversal, but in between the lines. When the lines are parallel, alternate angles are congruent.

In the diagram above, d and e are alternate angles, and c and f are alternate angles. So $d = e$ and $c = f$.

The easiest way to think of alternate angles is the “z pattern.”. If you draw the letter “z” from either direction of the top parallel line to the bottom, the angles which represent the insides of the “z” are congruent.

Corresponding angles are angles that are on the same side of the transversal, and on matching sides of their own line. When the lines are parallel, corresponding angles are congruent.

In the diagram above, a and e are corresponding angles, b and f are corresponding angles, c and g are corresponding angles, and d and h are corresponding angles. So, $a = e$, $b = f$, $c = g$ and $d = h$.

The easiest way to think of corresponding angles is the “F pattern.” If you draw the letter “F”, backwards or forwards, on the transversal, then the angles on top or below the horizontal lines will be congruent.

Opposite angles are angles that are opposite each other when two lines cross. Opposite angles are equal. In the diagram above, a and d, b and c, e and h, and f and g are opposite angles. So, $a = d$, $b = c$, $e = h$, and $f = g$.

The easiest way to think of opposite angles is the “X pattern.” Picture an X between the transversal and the line. The angles on top and on the bottom of the “X” are congruent, as are the angles on either side.

Interior angles are angles that are on the same side of the transversal, in between the lines. When the lines are parallel, interior angles are supplementary (remember, this means that they add up to 180°).

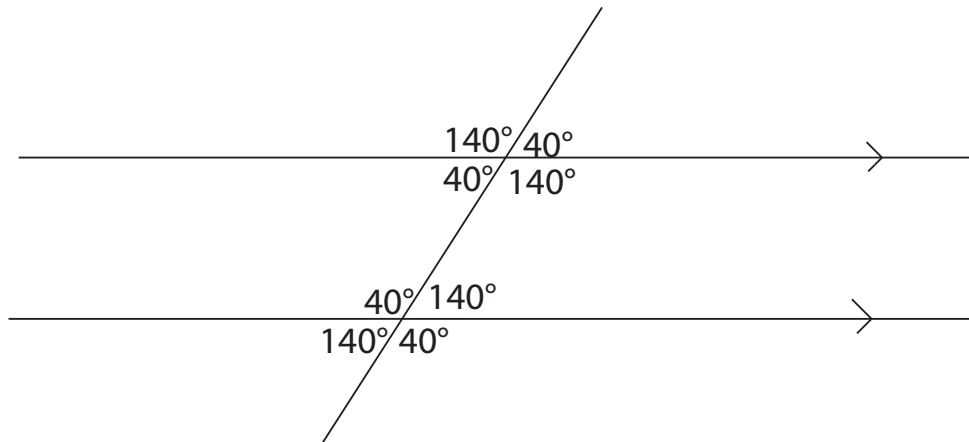
In the diagram above, d and f are interior angles, and c and e are interior angles. So, $c + e = 180^\circ$, and $d + f = 180^\circ$.

The easiest way to think of interior angles is the “C pattern.” If the space between the two parallel lines and the transversal is a very robotic letter “C”, then the two angles in the “C” are supplementary.

Notice that since $a = d$ and $d = h$ (because they are opposite and corresponding angles), $a = h$. Similarly, $b = c$ and $c = g$, so $b = g$.

Practice

Find angles b, c, d, e, f, g and h.



Triangles

The sum of angles in a triangle is always 180° .

This leads to different types of triangles, categorized by angles:

Equilateral triangles have 3 equal sides, and therefore 3 equal angles.

Since there are 180° in a triangle and all angles are equal in an equilateral triangle, what is this angle?

60°

Isosceles triangles have 2 equal sides, and therefore 2 equal angles.

Scalene triangles have no equal sides, and therefore no equal angles.

Right triangles have one right angle.

Isosceles right triangles have 2 equal sides, and a right angle. Can you find their other angle?

45°

Acute triangles have all angles less than 90° .

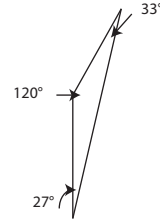
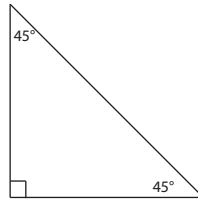
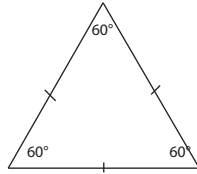
Obtuse triangles have at least 1 angle greater than 90° .

Why is it impossible to have a triangle with a reflex angle?

The sum of angles in a triangle is 180° so we can't have an angle greater than 180° .

Practice

Classify the triangle by number of similar angles (equilateral, isosceles, scalene) and size of angle (right, acute, obtuse). Then determine the missing angle(s):



Equilateral acute, Isosceles right, Scalene obtuse.

Polygons

We have discovered properties of angles in a triangle. Now let's move to any n-sided shape (polygons).

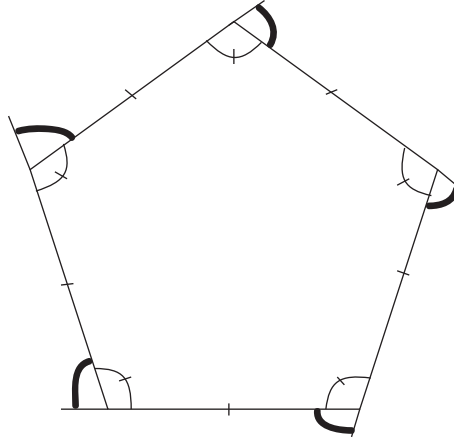
The sum of angles in any polygon is $180^\circ \times (n-2)$, where n is the number of sides. Recall that a **regular polygon** is a polygon where all of the side lengths are the same, and all of the angles are the same.

In a regular polygon, each angle is $\frac{180^\circ(n-2)}{n}$, since we distribute the angles equally.

The sum of the **exterior angles** in a polygon (that is, the angles outside of each corner if a straight line is joined to each side) is always 360° . If the polygon is regular, then each exterior angle is $\frac{360^\circ}{n}$.

Since an exterior angle and an interior angle make a straight line, each pair of interior and exterior angles are supplementary.

If $n = 5$, then we have a pentagon (5 sides). Let's consider a regular pentagon.



The sum of the angles on the inside of the pentagon is $180^\circ \times (n-2)$. In this case, we have $180^\circ \times (5-2) = 540^\circ$. Each angle will then be $540^\circ \div 5 = 108^\circ$.

The sum of the angles on the outside of the pentagon is 360° . Each angle will then be $360^\circ \div 5 = 72^\circ$. Notice $108^\circ + 72^\circ = 180^\circ$.

Practice

What is the sum of interior angles in an octagon?

1080°.

If the octagon is regular, what are the measures of each interior and exterior angle?

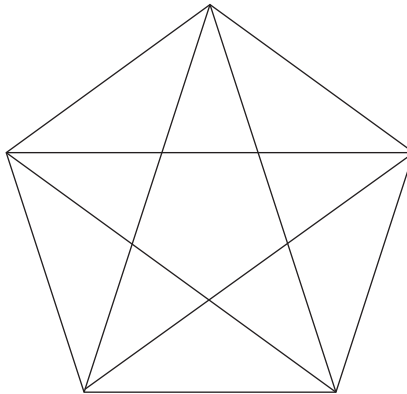
Interior angles are 135°, exterior angles are 45°.

Polygon Diagonals

We can also find the number of diagonals of a polygon based on the number of sides it has. We define a **diagonal** as a line segment that goes from one corner to another, but is not one of the sides.

The formula for the number of diagonals in an n -sided polygon is $\frac{n \times (n - 3)}{2}$.

For example, in an octagon, there are $\frac{8 \times 5}{2} = 20$ diagonals. Draw the diagonals on a pentagon. How many are there?



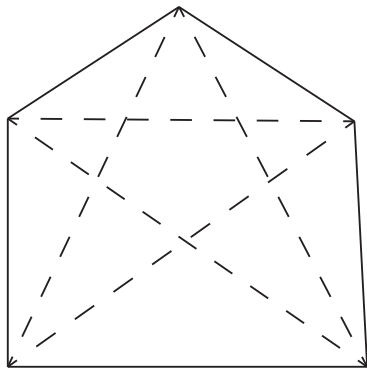
There are 5.

Concave and Convex

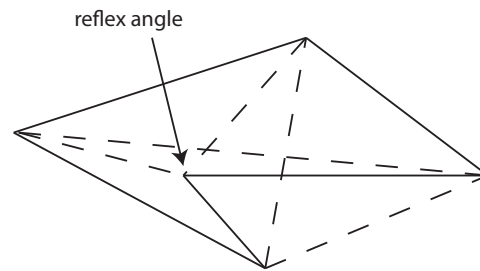
We mentioned that a triangle can not have a reflex angle. However, it is possible for other polygon to have one. If a polygon has a reflex angle, it is called **concave**. Otherwise, we call it **convex**.

Regular polygons are always convex. When we draw diagonals in a concave polygon, at least one will be outside the polygon.

Identify which pentagon is convex, which is concave, and where the reflex angle is on the concave pentagon.



convex

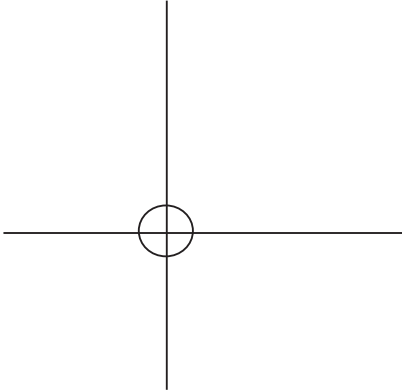


concave

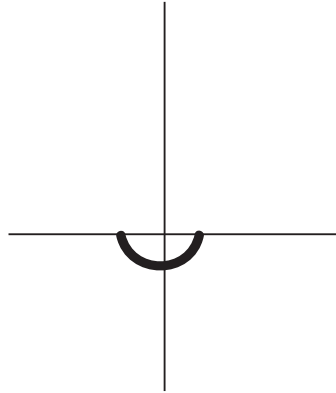
Problems

1. Classify the angle.

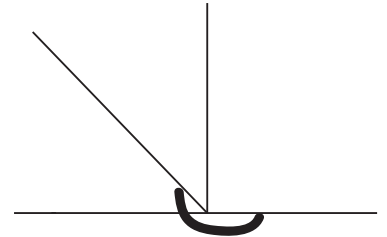
a)



b)



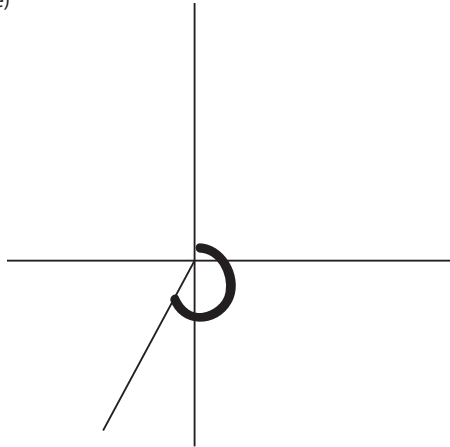
c)



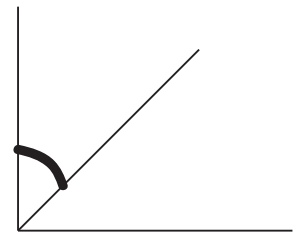
d)



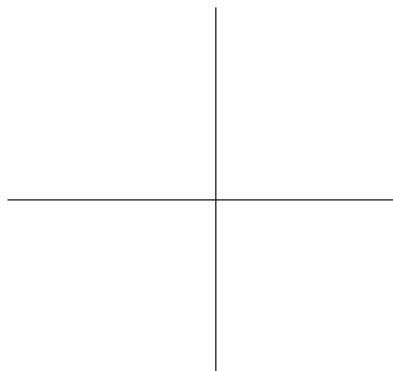
e)



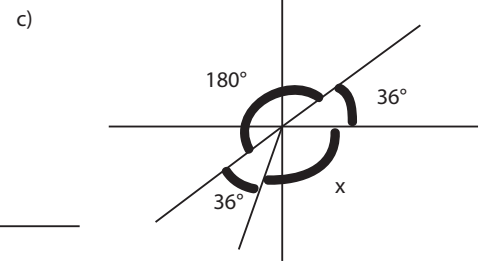
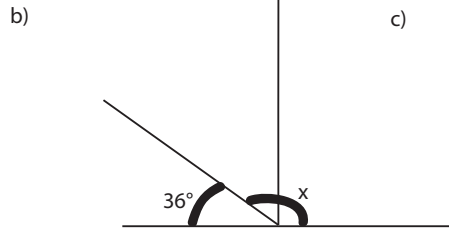
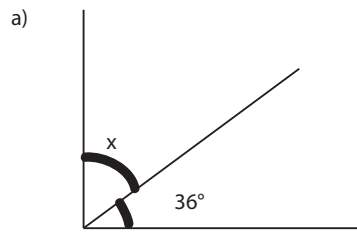
f)



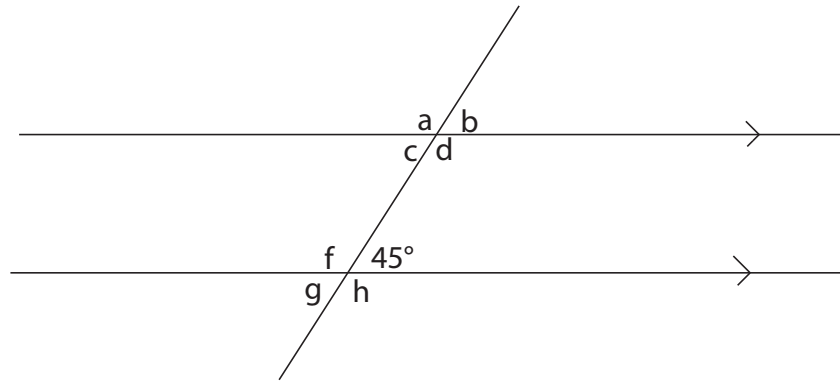
2. Find and draw -170° .



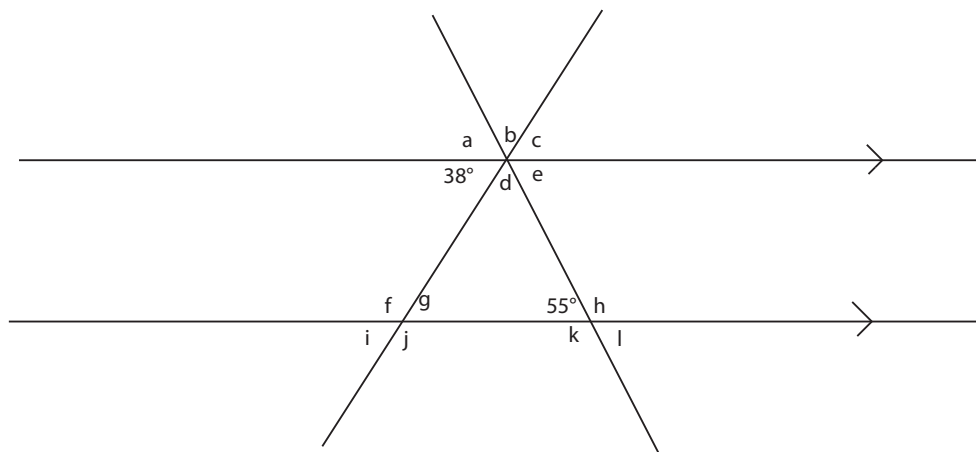
3. Find the missing angle x in each diagram.



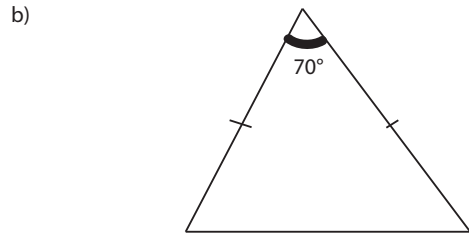
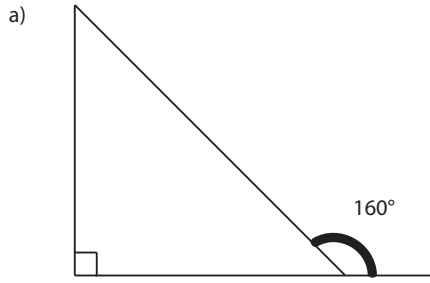
4. Find angles a , b , c , d , f , g and h .



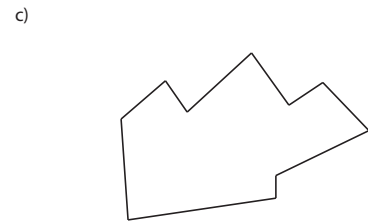
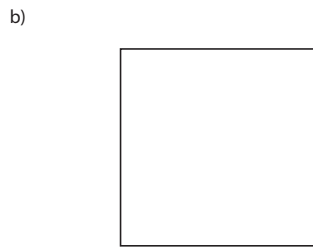
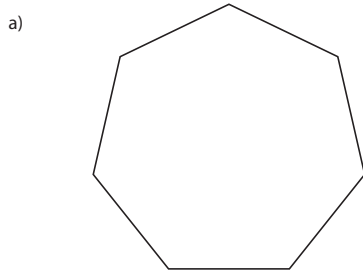
5. Find angles a , b , c , d , e , f , g , h , i , j , k and l .



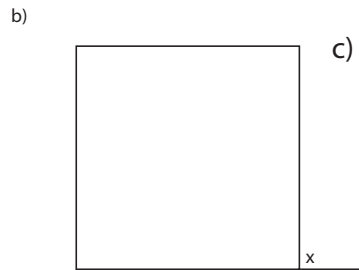
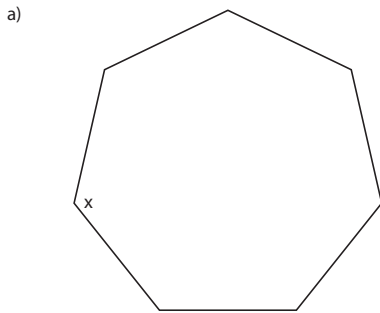
6. Classify the triangle and find the missing angle(s):



7. What is the sum of interior angles in each of the following shapes?

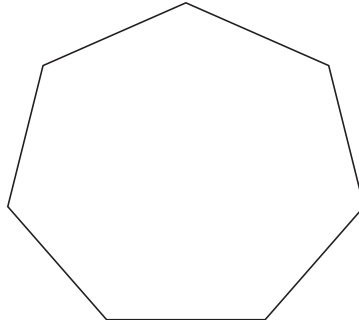


8. Find the measure of angle x in each of the following regular shapes, to 2 decimals.

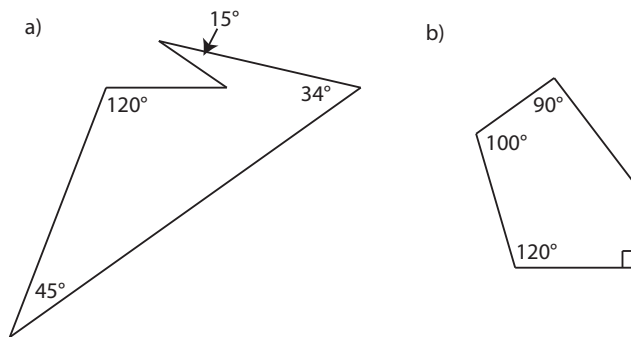


c) x is an interior angle in a 46-gon

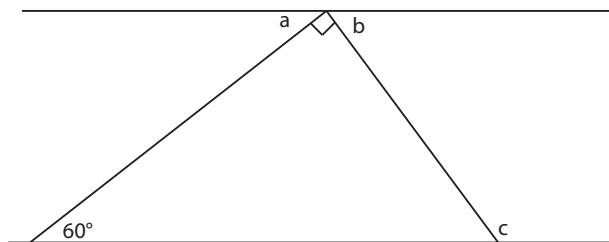
9. Using the formula, find the number of diagonals in a heptagon. Then draw the diagonals in the following heptagon. Is the number of diagonals from both methods the same?



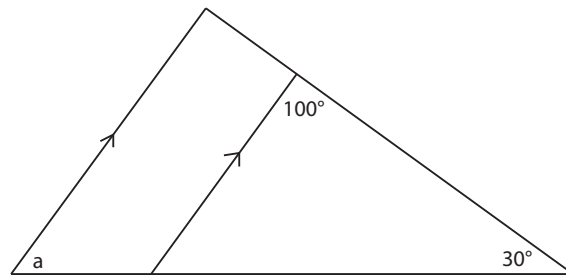
10. Identify whether the shape is convex or concave. If it is concave, find the measure of the reflex angle.



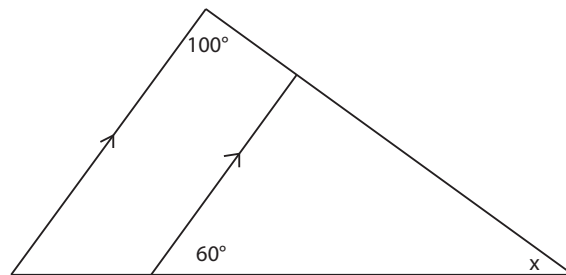
11. Find angles a, b and c.



12. Find the measure of angle a.



13. Find the measure of angle x.



14. If the sum of angles in a polygon is 1260° , how many sides does the polygon have?

15. a) The final round of FIFA World Cup Qualifying in North America is played in a “round robin” style, where 6 teams must play each other twice. At the end of the games, the top 4 teams go to the World Cup. In total, how many games are played? Show your answer algebraically, using a polygon.

b) Now consider 12 teams who must all play each other once. In total, how many games are played?

Solutions

1. a) Full rotation

b) Straight angle

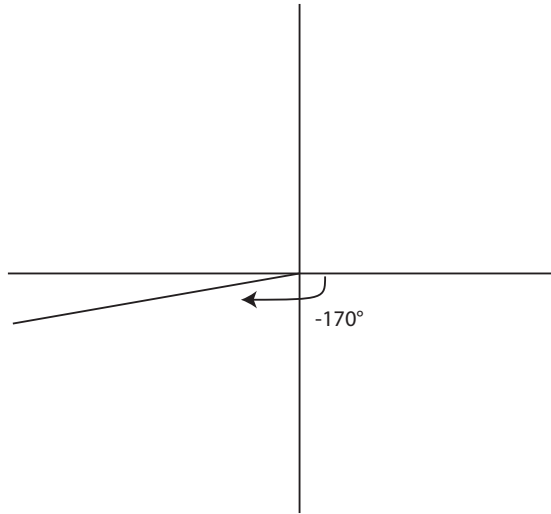
c) Reflex angle

d) Right angle

e) Obtuse angle

f) Acute angle

2. $360^\circ - 170^\circ = 190^\circ$

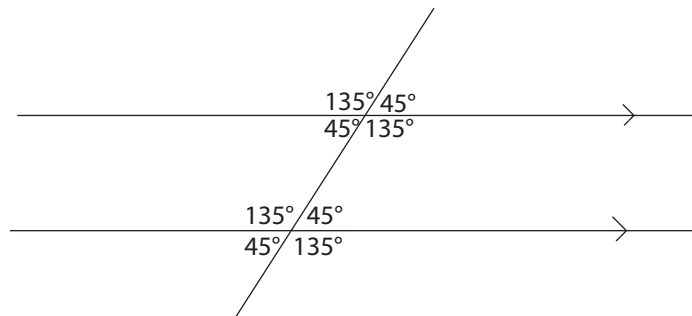


3. a) $x = 90^\circ - 36^\circ = 54^\circ$

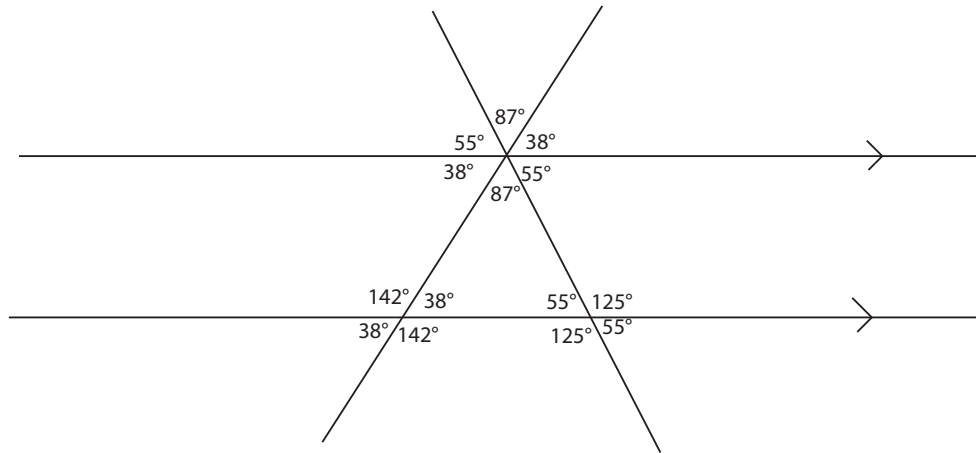
b) $x = 180^\circ - 36^\circ = 144^\circ$

c) $x = 360^\circ - 180^\circ - 36^\circ - 36^\circ = 108^\circ$

4.

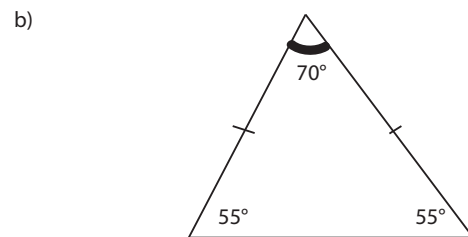
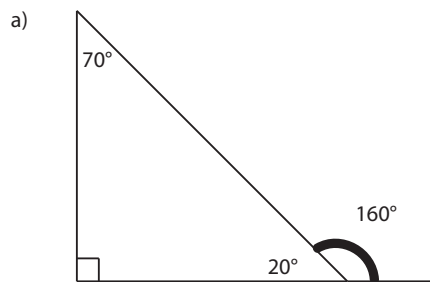


5.



6. a) Scalene right

b) Isosceles acute



7. a) 900°

b) 360°

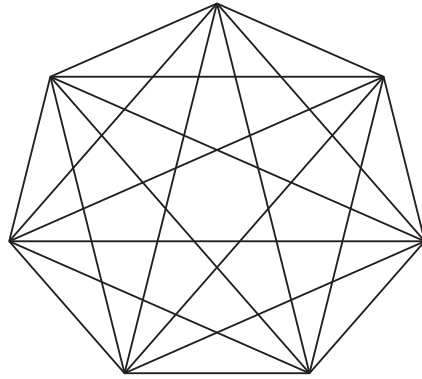
c) 1440°

8. a) $x = 128.57^\circ$

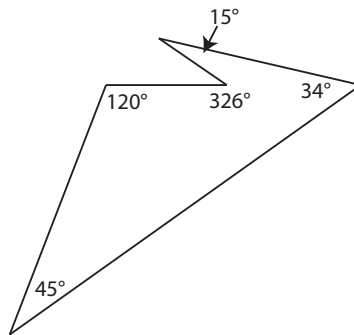
b) $x = 90^\circ$

c) $x = 172.11^\circ$

9. There are 14 diagonals in a heptagon.



10. a) Concave



b) Convex

11. $a = 60^\circ$, $b = 30^\circ$, $c = 150^\circ$

12. $a = 50^\circ$

13. $x = 20^\circ$

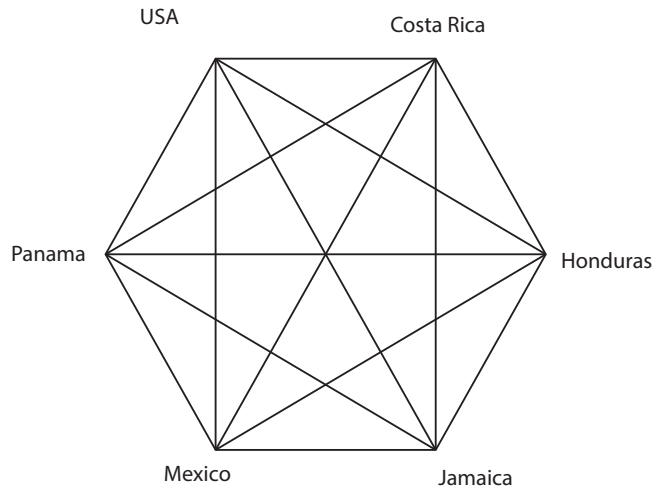
14. $1260^\circ = 180(n-2)$

$$7 = n - 2$$

$$n = 9$$

15. a) Consider a hexagon where each team in the round is a point of the hexagon. Each line in the hexagon will represent a game. We know that there are 6 sides, and there are $\frac{6 \times 3}{2} = 9$ diagonals in a hexagon. So, there are $6 + 9 = 15$ lines, or 15 games, in one round. However, each team plays each other twice, so there are $15 \times 2 = 30$ games.

An example is shown below:



b) We have a 12-gon, with $\frac{12 \times 9}{2} = 54$ diagonals, so there are $54 + 12 = 66$ games.