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Grade 6 Math Circles

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Cool Geometry

Solutions

Problem Set

1. Using the Pythagorean theorem:

$$Hypotenuse = \sqrt{(Base)^2 + (Height)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{25} = 5\ units$$

Interestingly, our final answer is also a natural number. The numbers 3, 4 and 5 are called a Pythagorean triplet as they are natural number solutions to the Pythogorean theorem.

2. Using the Pythagorean theorem *backwards*:

$$Height = \sqrt{(Hypotenuse)^2 - (Base)^2} = \sqrt{(7)^2 - (2)^2} = \sqrt{45} = 6.708\ units$$

3. To see if such a right triangle is possible, we can take the longest side (call it the hypotenuse) and see if we can get it from the other two by using the Pythagorean theorem:

$$Hypotenuse = 12 = \sqrt{(Base)^2 + (Height)^2} = \sqrt{7^2 + 6^2} = \sqrt{85} = 9.219\ cm$$

Clearly $12 \neq 9.219$. This means that such a right triangle is impossible.

4. We're asked to write the hypotenuse in terms of the base. To do this, we don't really need any numbers. We start with the Pythagorean theorem and set the base equal to the height:

$$Hypotenuse = \sqrt{(Base)^2 + (Base)^2} = \sqrt{2 \times (Base)^2} = \sqrt{2} \times (Base)$$

So the hypotenuse of such a triangle is $\sqrt{2}$ times the base (or height).

5. Using the angle sum formula for a polygon and plugging in $n = 7$ we get:

$$\textit{Sum of interior angles} = (n-2) \times 180° = (7-2) \times 180° = 5 \times 180° = 900°$$

6. To find the number of sides given the sum of interior angles, we can use the angle sum formula for a polygon backwards:

$$\textit{Number of sides} = \frac{\textit{Sum of interior angles}}{180°} + 2 = \frac{1440°}{180°} + 2 = 8 + 2 = 10$$

   A 10-sided polygon is called a **Decagon**.

7. The Pythagorean theorem in Minkowski geometry is weird because of the negative sign (instead of the usual positive sign). If you had an isosceles right triangle (i.e. base = height), you'd find the length of the hypotenuse to be:

$$Hypotenuse = \sqrt{(Base)^2 - (Height)^2} = \sqrt{(Base)^2 - (Base)^2} = \sqrt{0} = 0$$

   This is a weird result because your theorem now tells you that the length of your *longest* side is now 0 i.e. there is no triangle!

   Don't worry, math hasn't broken down. What's happening here is that the above problem shows an interesting difference between the geometry you've learned so far (called **Euclidean geometry**) and Minkowski geometry which is **Non-Euclidean**.

   The above version of the Pythagorean theorem is correct in Minkowski geometry. In fact, Einstein based his theory of relativity on it. The zero length hypotenuse has some weird consequences in how time passes for us as we move faster and faster. To put it shortly - *as we go faster, time slows down for us.* The negative sign in the Pythagorean theorem represents this fact.

   Interestingly, if you were (hypothetically) able to go at the speed of light (which is about 300,000,000 m/s), **time would stand still for you**.

8. The construction is as shown in Figure 1 below:

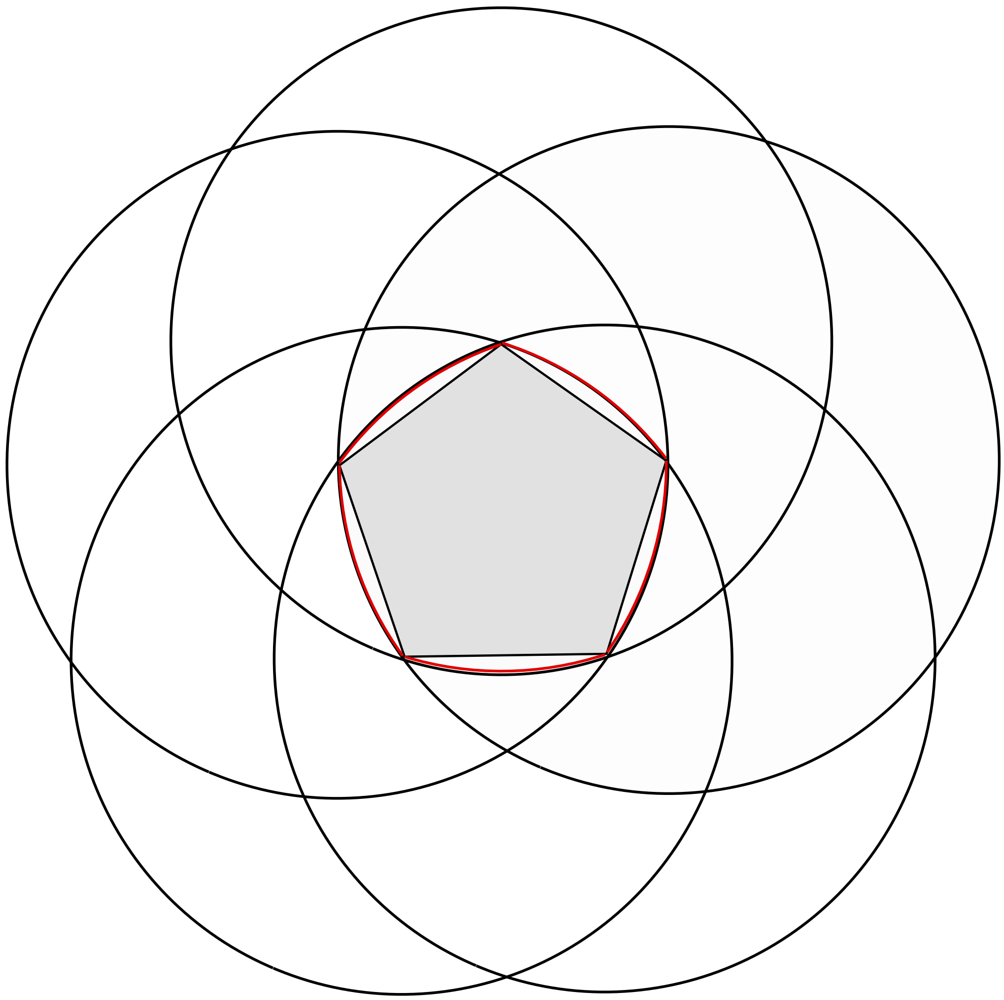

Figure 1: Contructing a Reuleaux Pentagon

9. A Reuleaux pentagon doesn't have a fixed center of rotation. This can easily be observed by cutting out the Reuleaux pentagon you made earlier and rolling it along a table while watching the center.

   Interestingly, the center of rotation of the Reuleaux pentagon moves in a smaller circle than a Reuleaux triangle. This has got to with the fact that a pentagon has more sides than a triangle.

   In fact, if you imagined an *infinite* sided polygon, it'd look very similar to a circle.

   So in a manner of speaking, the higher order Reuleaux polygons (ones with more sides) have a *more fixed* center of rotation than the lower order ones (ones with fewer sides)