



Grade 6 Math Circles

Fall 2018 - December 4/5

Jeopardy

Vectors

\$100 What is a scalar? What is a vector? What is the difference between them?

Scalars only have size. Vectors have size and direction. The difference is that vectors have direction.

\$200 If $\vec{A} = 3 \text{ km [West]}$, what is $-2\vec{A}$?

6 km [East]

\$300 Alice goes to the supermarket 5 km North of her house and back. Assuming she took a straight path, what is the distance she travelled? What is her displacement?

The total distance she travelled is 10 km. Her displacement is $\vec{0}$ because she ends up where she started from.

\$400 Sam wants to row across a river. She can row at 3 km/h. The river has a current of 5 km/h [N]. She wants to end up directly East across the river from where she started. What information are we given (in terms of speed, velocity, and direction) for this vector addition?

We are given Sam's rowing speed (3 km/h), the river's velocity (5 km/h [N]), and the direction of her final velocity (East).

\$500 Carl is out cycling. He first goes 3 km [NE] and then cycles 10 km [NW]. Finally, he goes 4 km [SW]. Determine his final displacement.

Carl's final displacement is 10.3 km [50° W of N].

Matrices

\$100 What is A^T ?

$$A = \begin{bmatrix} 13 & 2 & 1 \\ 5 & 3 & 4 \end{bmatrix}$$
$$A^T = \begin{bmatrix} 13 & 5 \\ 2 & 3 \\ 1 & 4 \end{bmatrix}$$

\$200 What is the matrix that results from this addition?

$$\begin{bmatrix} 4 & -2 \\ 24 & 0 \\ 7 & 30 \end{bmatrix} + \begin{bmatrix} 9 & 11 \\ 5 & 16 \\ 8 & -4 \end{bmatrix} = \begin{bmatrix} 13 & 9 \\ 29 & 16 \\ 15 & 26 \end{bmatrix}$$

\$300 Evaluate the following.

$$3 \begin{bmatrix} 2 & 4 & 1 \\ 12 & 0 & 5 \\ 20 & 3 & 4 \end{bmatrix} - \begin{bmatrix} 3 & 6 & 1 \\ 17 & 4 & 8 \\ 53 & 5 & 10 \end{bmatrix} = \begin{bmatrix} 3 & 6 & 2 \\ 19 & -4 & 7 \\ 7 & 4 & 2 \end{bmatrix}$$

\$400 Given the following vectors, find the area of the parallelogram formed by them.

$$\vec{p} = \begin{bmatrix} 12 \\ 7 \end{bmatrix} \quad \vec{q} = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$
$$\text{Area} = \left| \det \begin{bmatrix} 12 & 5 \\ 7 & 4 \end{bmatrix} \right| = |12(4) - 5(7)| = |48 - 35| = |13| = 13$$

The area of the parallelogram is 13 units²

\$500 How does Google PageRank determine the importance of a webpage?

Google PageRank ranks how important webpages are by following links on pages. A link from webpage j to webpage k is a vote of importance of page k by page j.

Area of Triangles

\$100 Name 3 methods that can be used to find the area of a triangle.

Any three of Heron's Formula, Complete the Rectangle, Shoelace Theorem, and Basic Formula.

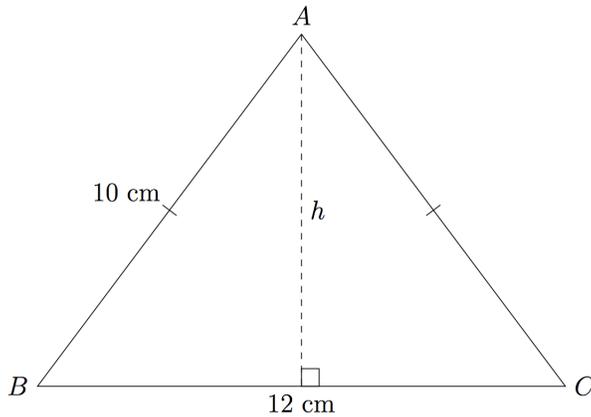
\$200 What is the area of a triangle with side lengths of 12 cm, 16 cm, and 20 cm?

Using Heron's Formula, we get:

$$s = \frac{1}{2}(12 + 16 + 20) = 24$$

$$A_{\triangle DEF} = \sqrt{24(24 - 12)(24 - 16)(24 - 20)} = \sqrt{24(12)(8)(4)} = \sqrt{9216} = 96 \text{ cm}^2$$

\$300 What is the area of this triangle?



Using Pythagorean Theorem,

$$6^2 + h^2 = 10^2$$

$$36 + h^2 = 100$$

$$h^2 = 64$$

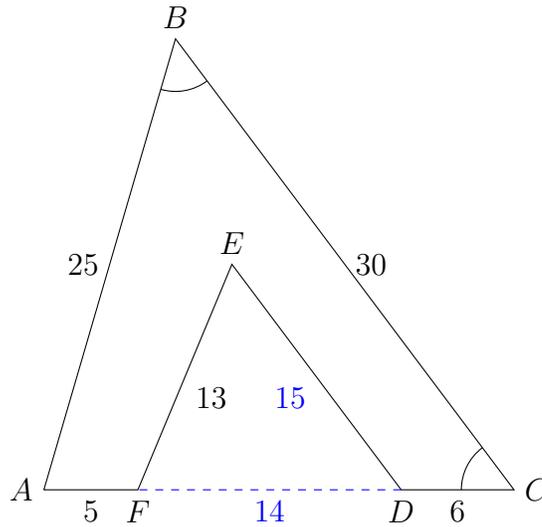
$$h = 8 \text{ cm}$$

$$\text{So, } A_{\triangle ABC} = \frac{12 \times 8}{2} = 48 \text{ cm}^2$$

\$400 What type of triangle only requires one triangle be subtracted from the rectangle in the complete the rectangle method?

A right triangle.

\$500 Given that the perimeter of $\triangle DEF$ is 42, $AB=AC$, and the area of $\triangle ABC$ is 300 units², what is the area of hexagon $ABCDEF$?



Since $AC = AB = 25$, $DF = 25 - 5 - 6 = 14$. We also know that the perimeter of $\triangle DEF$ is 42, so $DE = 42 - 13 - 14 = 15$. Use Heron's formula to find the area of $\triangle DEF$:

$$s_{\triangle DEF} = \frac{1}{2}(13 + 14 + 15) = 21$$

$$A_{\triangle DEF} = \sqrt{21(21 - 13)(21 - 14)(21 - 15)} = \sqrt{21(8)(7)(6)} = \sqrt{7056} = 84$$

The area of hexagon $ABCDEF$ is the area of $\triangle ABC$ minus the area of $\triangle DEF$ so:

$$A_{ABCDEF} = A_{\triangle ABC} - A_{\triangle DEF} = 300 - 84 = 216 \text{ units}^2$$

Structure of Math

\$100 What is an axiom? What is a definition? How are they similar?

Axioms are statements that are so basic that there is no way to prove them, you can only believe that they are true. A definition tells you the exact meaning of something. They are similar in that they are both not proven.

\$200 How are inductive and deductive logic different? Which one does math normally use?

Deductive logic is always certain about the outcomes it gives you. Inductive logic is "everyday" logic. Math uses deductive logic.

\$300 What does it mean for a definition to be "well-defined"?

"Well-defined" means that it is clear what is and what is not part of the definition. There is no grey area.

\$400 “For all puppies, for any puppy you choose there exists a collar such that if the puppy is wearing the collar, then the puppy won’t get lost.” What kind of statement is this?
[Implication inside existential inside universal statement.](#)

\$500 Consider this group (set) of numbers $\{1,3,6,19\}$. Prove that there exists a number in the set such that this number is not triple another number in the set.
[We only need to show that one thing in the set exists that makes this statement true. So, \$6 = 3 \times 2\$ means that 6 is triple 2. 2 is not another number in this set. This proves the theorem.](#)

Misc.

\$100 What is the earliest year of past Grade 6 Math Circles material on the Math Circles website?
[2010](#)

\$200 What does University of Waterloo’s DC building stand for?
[Davis Centre](#)

\$300 What is the name of a Grade 11 Math Contest that the CEMC creates?
[Fermat or Hypatia](#)

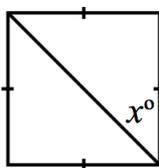
\$400 How old are the Math Circles instructors?
[Sydney is 19 and Hussam is 21.](#)

\$500 What is the capital of Denmark?
[Copenhagen](#)

Gauss Contest

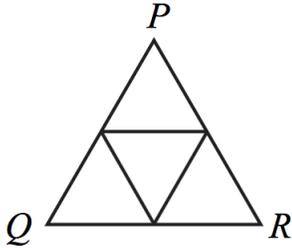
[Solutions can be found in past Gauss contests which are available on the CEMC website at \[www.cemc.uwaterloo.ca/contests/past_contests.html\]\(http://www.cemc.uwaterloo.ca/contests/past_contests.html\)](#)

\$100 In the square shown, what is the value of x ? (*Gauss 2017 Question 6*)



- A. 0 B. 45 C. 60 D. 180 E. 360
[B. 45](#)

\$200 In the diagram, $\triangle PQR$ is equilateral and is made up of 4 smaller equilateral triangles. If each of the smaller triangles has a perimeter of 9 cm, what is the perimeter of $\triangle PQR$? (*Gauss 2017 Question 10*)



- A. 15 cm B. 9 cm C. 36 cm D. 27 cm E. 18 cm
E. 18 cm

\$300 Five students ran a race. Ryan was faster than Henry and Faiz. Henry was slower than Faiz. Toma was faster than Ryan but slower than Omar. Which student finished fourth? (*Gauss 2017 Question 15*)

- A. Faiz B. Henry C. Omar D. Ryan E. Toma
A. Faiz

\$400 If $\frac{1}{2}$ of the number represented by x is 32, what is $2x$? (*Gauss Grade 7 2000 Question 8*)

- A. 128 B. 64 C. 32 D. 256 E. 16
A. 128

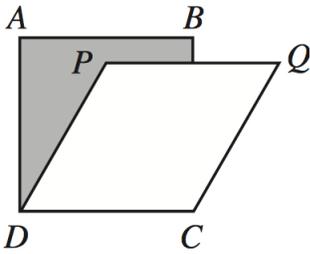
\$500 In the diagram all rows, columns, and diagonals sum to 12. What is the sum of the four corner numbers? (*Gauss Grade 7 2000 Question 15*)

		4
	4	
	3	

- A. 14 B. 15 C. 16 D. 17 E. 12
C. 16

Final Jeopardy

In the diagram, $ABCD$ is a square with area 25 cm^2 . If $PQCD$ is a rhombus with area 20 cm^2 , what is the area of the shaded region in cm^2 ? (*Gauss Grade 8 2003 Question 24*)



11 cm^2