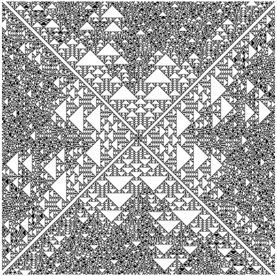
Cellular Automata, Lecture 2



Math Circles 2018
University of Waterloo

Part I

Multistate and Totalistic Automata

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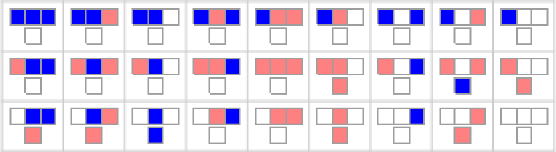

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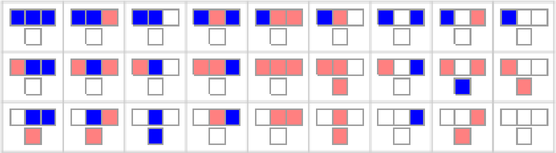

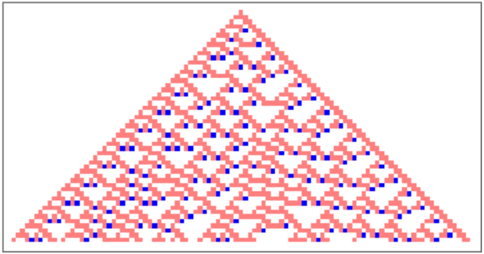

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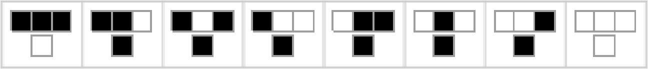

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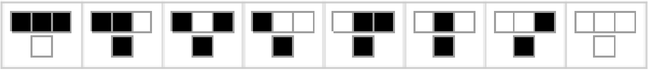

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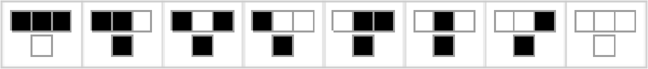

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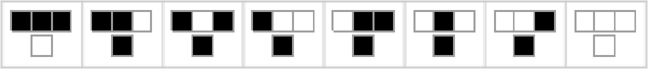

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How many *k*-color totalistic rules are there?

Totalistic automata: A Code Challenge

Exercise: Using Wolfram Language, write code that generates a 4-color totalistic rule with initial state 102321.

```
CellularAutomaton[{30,{4,1}},{{1,0,2,3,2,1},0},100]
```

Change 30 to another rule. See if you can find one that looks cool! Then use RulePlot to print out its transition rules.

Part II

The Mathematical Formalism of CA's

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We should be able to provide a *rigorous explanation* for each of the following:

- What is a **state**?
- What is a **neighborhood**?
- What do 1D and 2D automata have in common?

Suppose X is a set, $f : X \to X$ is a function, and $x_0 \in X$ is some point.

Then you can form a sequence

$$x_0,$$

Iterated function systems

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In general, the nth term is $f^n(x_0)$ which is obtained by **iterating** f on the initial value x_0.

This sequence is called the **orbit** of x_0.

Example: The Collatz Sequence

Consider the function $f : \mathbf{N} \to \mathbf{N}$ given as follows.

$$f(n) := \begin{cases} 3n+1 & \text{if } n \text{ is odd,} \\ n/2 & \text{if } n \text{ is even.} \end{cases}$$

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Exercise: Find the Collatz sequence starting at 9.

Example: The Collatz Sequence

Millenium Problem: Prove that every Collatz sequence reaches the $4, 2, 1$ loop, no matter what the initial value.

You'll win $1,000,000!

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Find the orbit of $(1, 1)$ under this function f.

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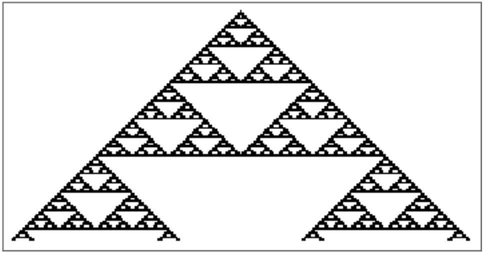
A cellular automaton starts with an initial value, and then we draw the *orbit* after iterating the rule.



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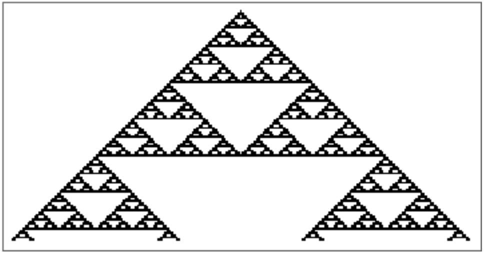

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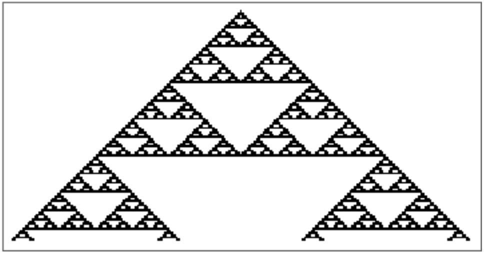

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(For k colors, a state is a function $X : \mathbf{Z} \to \{1, 2, \ldots, k\}$.)

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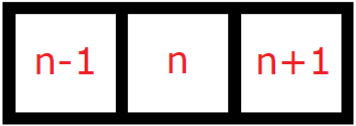
The neighborhood of a cell consists of itself, and the two cells to its left and right.



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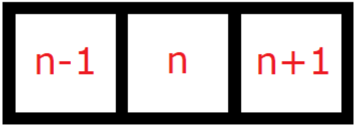

So the **neighborhood** of cell n is defined to be

$$\{n-1, n, n+1\}.$$

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This is a function whose inputs and outputs are ALSO functions!

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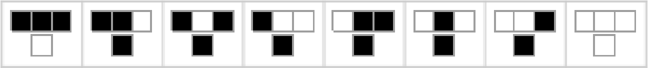
Here are the transition rules for Rule 126.



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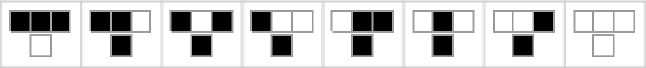

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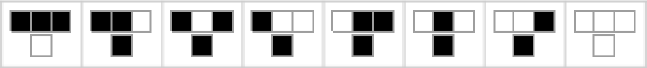

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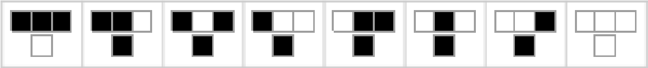

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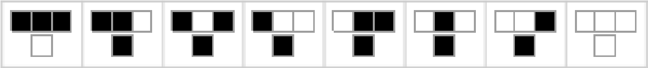
- Since Rule 126 is totalistic, we can define its **memory rule** by

$$\mu(a,b,c) := \begin{cases} 1 & \text{if } a+b+c \in \{0,3\} \\ 0 & \text{if } a+b+c \in \{1,2\} \end{cases}$$

Example: Rule 126

Here are the transition rules for Rule 126.



Let's write this as a function $\tau : \{0,1\}^{\mathbf{Z}} \to \{0,1\}^{\mathbf{Z}}$.

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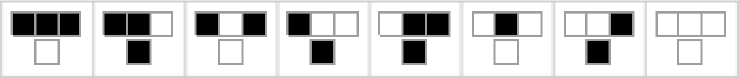
- For a state $x : \mathbf{Z} \to \{0,1\}$, define τ_x to be the state

$$\tau(x)_n := \mu(x_{n-1}, x_n, x_{n+1}).$$

Exercise For You: Rule 90

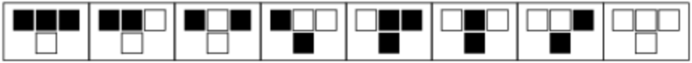
Here are the transition rules for Rule 90 (the XOR automaton).



Find the memory function μ for Rule 90, and write the cellular automaton τ in terms of μ.

Exercise For You, 2: Rule 30

Here are the transition rules for Rule 30.



Find the memory function μ for Rule 30, and write the cellular automaton τ in terms of μ.

(Hint: the memory function involves XOR.)

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Now we've seen that a cellular automaton τ should have a memory function.

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which is determined by a **memory rule** $\mu : \{0,1\}^3 \to \{0,1\}$:

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(If you want k colors, replace $\{0,1\}$ by $\{1,\dots,k\}$.)

Part III

(A little bit on) 2-Dimensional Automata

Return to Planet Friendship

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Zoink is back!

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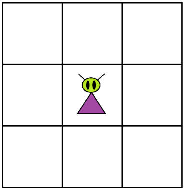
Zoink has a new power: he can grow friends **in all eight directions around them!**



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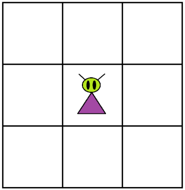

This is called **2-dimensional friendification**.

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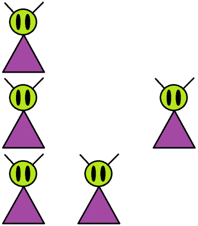
4. **Birth:** If a cell has *exactly three* live neighbors, an alien will be born there!

Exercise

Suppose the aliens stand in the following arrangement. Figure out who will live, die, or survive over the next two days.



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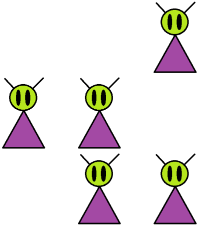
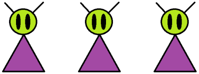


More exercises

(a) For each of these two states, see what happens over the course of seven days.



(b) Find an example of a state that never changes.

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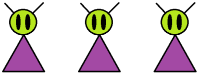

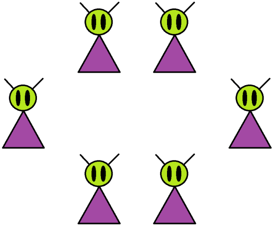

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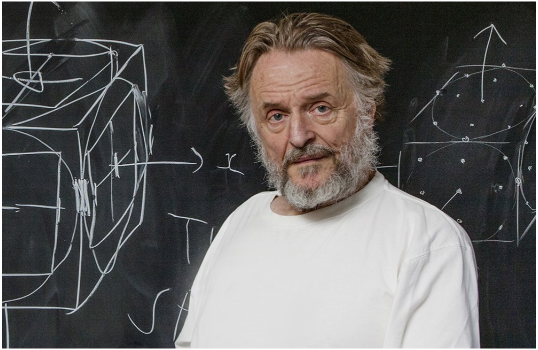

John Conway

[VIDEO]

A Useful App

http://conwaylife.appspot.com/new

2D Cellular Automata

Each cell has **eight** neighbors, which can be either alive or dead (black or white).

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- How many 2D cellular automata are there?
- If there are k colors, how many 2D cellular automata are there?
- How many 2D totalistic (two-state) automata are there?

Have a nice holiday!