Intermediate Math Circles
Wednesday March 20, 2019
Introduction to Vectors I

A vector is used to describe such things as velocity and force.

A scalar only has ________________.
A vector has both ________________ and ________________.

Example of scalars ________________, ________________, and ________________.

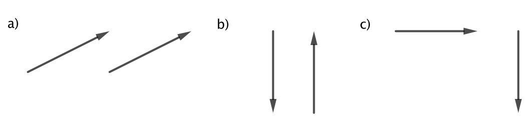
Example of vectors ________________, ________________, and ________________.

We will name the vector in two ways.

1. ________________- label starting from the *tail* to the *tip*.

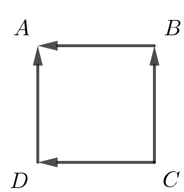
2. ________________ - label with a single lower case level.

We define ________________ vectors as vectors that have the same magnitude and direction. Conversely if two vectors have the same magnitude and direction then they are ________________. So two directed line segments with the same length and the same direction represent the same vector.
Which of the following pairs of vectors appear to be equal?



a)
b)
c)

Note the two vectors in part b) are said to be ________________ vectors because they have the same length but they are in the opposite directions.
Given square $ABCD$ labelled as shown. State a) two pairs of equal vectors and b) two pairs of opposite vectors.

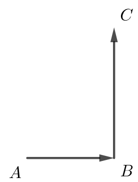


a)

b)

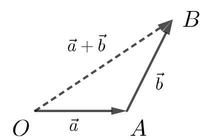
SUM OF VECTORS

John walks from A to B. He then walks from B to C.

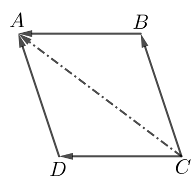


The *definition of the sum of vectors* is:

Suppose $\vec{a}$ and $\vec{b}$ are any two vectors. Choose points O and A so that $\vec{a} = \overrightarrow{OA}$. Choose a point B so that $\vec{b} = \overrightarrow{AB}$. The sum $\vec{a} + \vec{b}$, of $\vec{a}$ and $\vec{b}$ is represented by $\overrightarrow{OB}$.



In the following diagram, $ABCD$ is a parallelogram. Express $\overrightarrow{CA}$ as the sum of two vectors in as many ways as possible.



SCALAR MULTIPLICATION

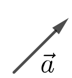
$k > 0$

In general, if we multiply $\vec{a}$ by a scalar $k, k > 0$, then $k\vec{a}$ is a vector in the same direction of __________ but __________ times as long. This is written as __________ = __________.

$k < 0$

In general, if we multiply $\vec{a}$ by a scalar $k, k < 0$, then $k\vec{a}$ is a vector in the opposite direction of $\vec{a}$ but __________ (absolute value of k) times as long. This is written as __________ =__________.

Given $\vec{a}$, draw



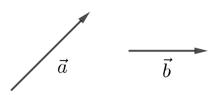
a) $3\vec{a}$. b) -2$\vec{a}$

VECTOR SUBTRACTION

To subtract a vector, add its ______________.

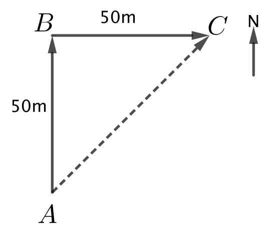
VECTOR COMBINATION

Given $\vec{a}$ and $\vec{b}$ draw the following:
a) $\vec{a}+\vec{b}$ b) $\vec{a}-\vec{b}$. c) $2\vec{a}-3\vec{b}$

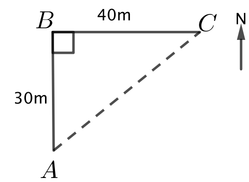


Example 1:

Federico walks 50m north(from A to B). He then walks 50m east (from B to C). What is the resultant displacement? (i.e. $\overrightarrow{AC}$) This means we need to find the magnitude of $\overrightarrow{AC}$ (We write this as $|\overrightarrow{AC}|$). Therefore $|\overrightarrow{AB}| = 50$ and $|\overrightarrow{BC}| = 50$. We will also need to find the direction of $\overrightarrow{AC}$.

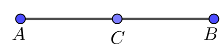


Example 2:
Federico walks 30m north(from A to B). He then walks 40m east (from B to C). What is the resultant displacement? (i.e. $\overrightarrow{AC}$)

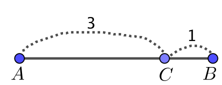


VECTOR PROOFS

1.) Given C is the midpoint of AB explain why $\overrightarrow{AC} = \overrightarrow{CB}$



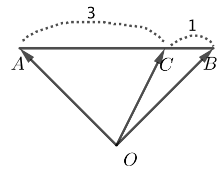
2.) Given C divides AB in the ratio 3:1 explain why $\overrightarrow{AC} = \frac{3}{4}\overrightarrow{AB}$



Explanation:
Since C is on the line segment AB then $\overrightarrow{AC}$ is in the same direction as $\overrightarrow{AB}$. Therefore $|\overrightarrow{AC}| = 3|\overrightarrow{CB}|$ or $\frac{1}{3}|\overrightarrow{AC}| = |\overrightarrow{CB}|$.

$$\begin{aligned}
\text{Now: } |\overrightarrow{AB}| &= |\overrightarrow{AC}| + |\overrightarrow{CB}|, \\
|\overrightarrow{AB}| &= |\overrightarrow{AC}| + \frac{1}{3}|\overrightarrow{AC}| \\
|\overrightarrow{AB}| &= \frac{4}{3}|\overrightarrow{AC}| \\
\text{or } \frac{3}{4}|\overrightarrow{AB}| &= |\overrightarrow{AC}| \\
\text{therefore } \overrightarrow{AC} &= \frac{3}{4}\overrightarrow{AB}
\end{aligned}$$

3.) Given C divides AB in the ratio 3:1 and O is not on AB then express $\overrightarrow{OC}$ in terms of $\overrightarrow{OA}$ and $\overrightarrow{OB}$



Solution:
From the previous question we know $\overrightarrow{AC} = \frac{3}{4}\overrightarrow{AB}$ (1)
Here is what else we know
$\overrightarrow{OC} = \overrightarrow{OA} + \overrightarrow{AC}$ (2)
$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
or $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (3)

Using (1) and (2) we get, $\overrightarrow{OC} = \overrightarrow{OA} + \frac{3}{4}\overrightarrow{AB}$
Now substituting (3) we get

$$\begin{aligned}\overrightarrow{OC} &= \overrightarrow{OA} + \frac{3}{4}(\overrightarrow{OB} - \overrightarrow{OA}) \\ &= \overrightarrow{OA} + \frac{3}{4}\overrightarrow{OB} - \frac{3}{4}\overrightarrow{OA} \\ &= \frac{1}{4}\overrightarrow{OA} + \frac{3}{4}\overrightarrow{OB}\end{aligned}$$