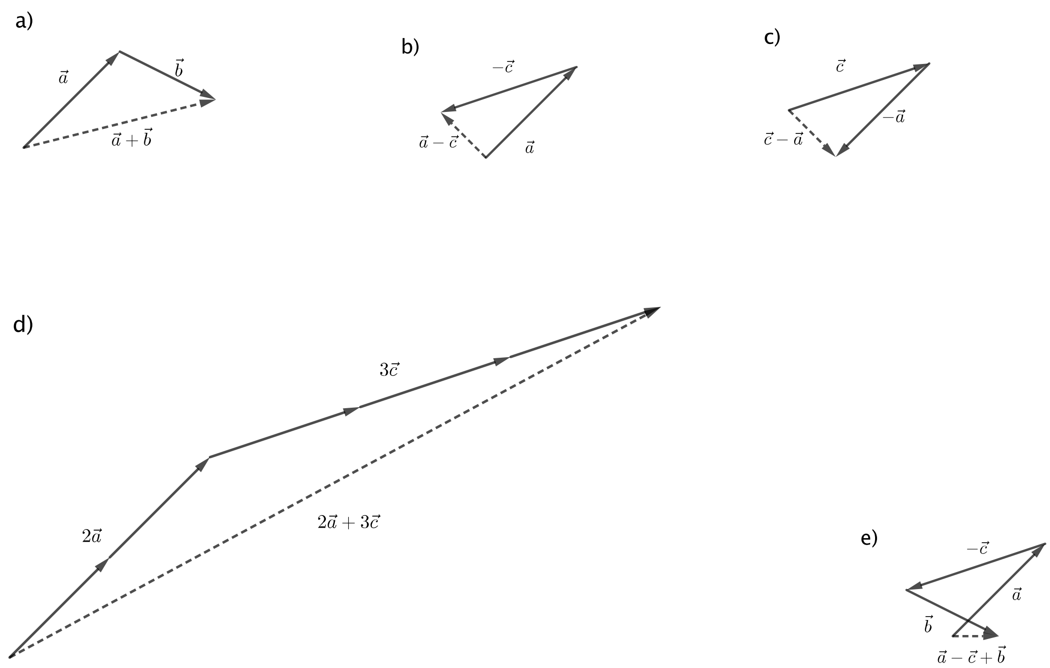
Intermediate Math Circles
Wednesday March 20, 2019
Problem Set 1 — Solutions

1) a) equal b) opposite c) neither

2) a) $\overrightarrow{CD}$ b) none c) $\overrightarrow{BC}$ d) $\overrightarrow{ED}$

3) a) $\overrightarrow{CD}$ or $\overrightarrow{BA}$ b) $\overrightarrow{DA}$ or $\overrightarrow{CB}$ c) $\overrightarrow{AE}$ or $\overrightarrow{EC}$ d) $\overrightarrow{EB}$ or $\overrightarrow{DE}$

4)



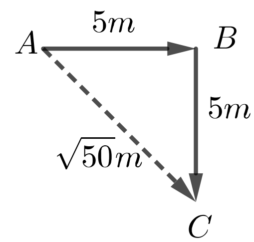
5)a) $\overrightarrow{DE}$ or $\overrightarrow{EB}$ b) $\overrightarrow{DA}$ or $\overrightarrow{CB}$ c) $\overrightarrow{DA}$ or $\overrightarrow{CB}$ d) $\overrightarrow{DB}$
e) $\overrightarrow{DE}$ or $\overrightarrow{EB}$ ($\frac{1}{2}\overrightarrow{DA}$) f) $\overrightarrow{DC}$ or $\overrightarrow{AB}$

6) All we know is that $\vec{u}$ and $\vec{v}$ have the same magnitude. We do not know if they are going in the same direction. Therefore, we do not necessarily know that they are the same vector

7)

Since Georgina walks north then east, $\angle ABC$ is a right angle. So we can us the Pythagorean Theorem to find $|\overrightarrow{AC}|$.

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= 5^2 + 5^2 \\ &= 25 + 25 \\ &= 50 \\ AC &= \sqrt{50} \end{aligned}$$



We know $\angle BAC + \angle ABC + \angle BCA = 180°$. (SATT). We also know that $\angle BAC = \angle BCA$. (ITT).Combining these two equations we get $\angle BAC + \angle ABC + \angle BAC = 180°$.
Replacing $\angle ABC = 90°$ we get

$$\begin{aligned} 2\angle BAC + 90° &= 180° \\ \angle BAC &= 90° \\ \angle BAC &= 45° \end{aligned}$$

Since we are on the compass, 45° is exactly southeast. Therefore $\overrightarrow{AC}$ (resultant displacement) is 7.07m southeast.

8) Since it is a parallelogram, $\overrightarrow{RS} = \overrightarrow{QP}$ and $\overrightarrow{RQ}$=$\overrightarrow{SP}$. We can now substitute to get:

$$\begin{aligned} \overrightarrow{RS} + \overrightarrow{RQ} &= \overrightarrow{QP} + \overrightarrow{SP} \\ &= \overrightarrow{SP} + \overrightarrow{QP} \end{aligned}$$

9) $\vec{u} = \vec{a} + \vec{b}$ and $\vec{v} = 2\vec{a} + 2\vec{b}$. Therefore $\vec{v} = 2(\vec{a} + \vec{b}) = 2\vec{u}$.
Since $2\vec{u} = \vec{v}$ then $\vec{u} = \frac{1}{2}\vec{v}$.
This solution will work for any triangle because in fact in an isosceles triangle $\vec{a} = \vec{b}$

10) We can show that $\overrightarrow{AP} = \frac{3}{5}\overrightarrow{AB}$ (1)
Here is what else we know
$\overrightarrow{OP} = \overrightarrow{OA} + \overrightarrow{AP}$ (2)
$\overrightarrow{OB} = \overrightarrow{OA} + \overrightarrow{AB}$
or $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ (3)

Using (1) and (2) we get, $\overrightarrow{OP} = \overrightarrow{OA} + \frac{3}{5}\overrightarrow{AB}$
Now substituting (3) we get

$$\begin{aligned} \overrightarrow{OP} &= \overrightarrow{OA} + \frac{3}{5}(\overrightarrow{OB} - \overrightarrow{OA}) \\ &= \overrightarrow{OA} + \frac{3}{5}\overrightarrow{OB} - \frac{3}{5}\overrightarrow{OA} \\ &= \frac{2}{5}\overrightarrow{OA} + \frac{3}{5}\overrightarrow{OB} \end{aligned}$$