Intermediate Math Circles
Wednesday April 3, 2019
Problem Set 3

1. If $\vec{u} = [-3, 2]$, $\vec{v} = [5, 1]$, and $\vec{w} = [1, -4]$, find the following:

   (a) i. $\vec{u} \cdot \vec{v}$

   ii. $\vec{v} \cdot \vec{u}$

   iii. $\hat{u}$

   iv. $(\vec{u} + \vec{v}) \cdot \vec{w}$

   v. $(3\vec{u}) \cdot \vec{v}$

   vi. $3(\vec{u} \cdot \vec{v})$

   vii. $\vec{w} \cdot \vec{w}$

   viii. $\vec{u} \times \vec{v}$

   ix. $\vec{v} \times \vec{w}$

   x. $\vec{w} \times \vec{u}$

   (b) Find $d(\vec{u}, \vec{v})$, $d(\vec{v}, \vec{w})$, and $d(\vec{w}, \vec{u})$.

   (c) Find the area of the triangle formed by $\vec{u}$ and $\vec{v}$ using the formula $|\vec{u} \times \vec{v}| = |\vec{u}|\,|\vec{v}|\sqrt{1 - (\hat{u} \cdot \hat{v})^2}$

2. Let $\vec{x} = [-2, -5]$ and $\vec{y} = [k, 4$, what value of k makes these vectors perpendicular?

3. Let $\vec{x} = [3k + 2, -3]$ and $\vec{y} = [6, 5]$, what value of k makes these vectors perpendicular?

4. Lines are parallel if their direction vectors are scalar multiples. They are perpendicular if their direction vectors are perpendicular.
   Show whether the following pairs of lines are parallel, perpendicular or neither

   (a) $\vec{r} = [3, 5] + t[2, -1]$

   $x = 5 - 4t$
   $y = 11 + 2t$

   (b) $\vec{r} = [3, 5] + t[-2, 3]$

   $2x - 3y - 19 = 0$

   (c) $x = 9 + 6t$
   $y = -4t$

   $5x + 3y - 17 = 0$

5. Recall that $|\vec{z}|^2 = \vec{z} \cdot \vec{z}$. Prove that if $\vec{u} \cdot \vec{v} = 0$, then $|\vec{u} + \vec{v}|^2 = |\vec{u}|^2 + |\vec{v}|^2$.

6. Prove $\vec{u} \cdot \vec{u} \geq 0$ and $\vec{u} \cdot \vec{u} = 0$ only if $\vec{u} = \vec{0}$.

7. Given that $|\vec{u} \cdot \vec{v}| \leq |\vec{u}|\ |\vec{v}|$, use the properties of dot products to prove $|\vec{u} + \vec{v}|^2 \leq (|\vec{u}| + |\vec{v}|)^2$

   This property is called the Minkowski Inequality and can be simplified by taking the square root of both sides.