This is a problem set of assorted problems related to counting/probability made for Math Circles at UW on February 20, 2019.

Extra Problems

1. The two digit number 36 has the property that when the digits are switched, the resulting number (in this case, 63) is larger than the original number. How many two-digit numbers have this property? (Ans: 36)

2. (Gauss G8 2006, 19) Bethany, Chun, Dominic, and Emily go to the movies. They choose a row with four consecutive empty seats. If Chun and Emily must sit next to each other, in how many different ways can the four friends sit? (Ans: 12)

3. (Pascal 2015, 20) Andre has an unlimited supply of \$1 coins, \$2 coins, and \$5 bills. Using only these coins and bills, and not necessarily using some of each kind, in how many different ways can he form exactly \$10? (Ans: 10)

4. How many different ways may the letters b,b,c,c,c,c be ordered from left to right? (Ans: 15)

5. How many different ways may the letters a,a,b,b,c,c be ordered from left to right? (Ans: 90)

6. (Cayley 2010, 19) How many 3-digit positive integers have exactly one even digit? (Ans: 350)

7. (Cayley 2009, 19) How many integers n are there with the property that the product of the digits of n is 0, where $5000 \leq n \leq 6000$? (Ans: 272)

8. (Euclid 2018, 7b) Eight people, including triplets Barry, Carrie and Mary, are going for a trip in four canoes. Each canoe seats two people. The eight people are to be randomly assigned to the four canoes in pairs. What is the probably that no two of Barry, Carrie and Mary will be in the same canoe? (Ans: $\frac{4}{7}$)

Extra Problems (Challenging)

1. (CIMC 2017, A6) In the diagram below, six squares form a 2×3 grid. The middle square in the top row is marked with an R. Each of the five remaining squares is to be marked with an R, S or T. In how many ways can the grid be completed so that it includes at least one pair of squares side-by-side in the same row or same column that contain the same letter? (Ans: 225)
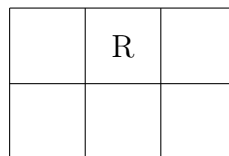
2. (Cayley 2003, 24) In how many ways can a, b, c, and d be chosen from the set $\{0, 1, 2, ..., 8, 9\}$ so that $a < b < c < d$ and $a + b + c + d$ is a multiple of 3? (Ans: 72)

3. (Pascal 2016, 25) A 0 or 1 is to be placed in each of the nine 1×1 squares in a 3×3 grid so that each row contains at least one 0 and at least one 1, and each column contains at least one 0 and at least one 1. How many ways can this be done? (Ans: 102)

4. (CSMC 2012, A6) Lynne is tiling her long and narrow rectangular front hall. The hall is exactly 2 tiles wide and 13 tiles long. She is going to use exactly 11 black tiles and exactly 15 white tiles. Determine the number of distinct ways of tiling the hall so that no tow black tiles are adjacent (that is, share an edge). (Ans: 486)

5. (Euclid 2016, 9a) $BBABBAABBA$ is a 10 letter string(word) of A's and B's that contains $ABBA$ as a substring (subword), while $AAABBBBBAA$ is a 10 letter string of A's and B's that does not contain the substring $ABBA$. How many 10 letter strings of A's and B's are there which do not contain "$ABBA$" as a substring? (Ans: 631)