

Math Circles - Problem Set 1

Introduction to Sequences and Series

Zack Cramer - zcramer@uwaterloo.ca

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- If the sequence $7, a, b, 43, \dots$ is arithmetic, what are the values of a and b ?
 - The 6^{th} term of an arithmetic sequence is 59, and the 21^{st} term is 14. What is the common difference?
- The sum of the first n terms of a sequence is $n(n+1)(n+2)$.
 - Write down the first 5 terms in this sequence.
 - What is the 180^{th} term?
 - Find an expression for the n^{th} term in the sequence.
- The sum of 100 consecutive integers is 9350. What is the largest of these integers?
 - The sum of the first 6 terms in an arithmetic sequence is -81 , and the sum of the first 40 terms is 4220. What is the 14^{th} term in the sequence?
- Find the sum of the first 1000 positive integers.
 - Find the sum of the numbers between 1 and 1000 (including 1 and 1000) that are not multiples of 3.
 - Determine the value of $1 - 2 + 3 - 4 + \dots + 99 - 100$.
- The numbers 2, 5, 8, 11, 14, \dots are written in order in a book, beginning on page 1. There are 100 numbers on each page. On what page can the number 11 111 be found?
- The 3^{rd} term in a geometric sequence is 8 and the 6^{th} term is 17 576. What is the common ratio?

- (b) The 10th term of a geometric sequence is -6655 and the 13th term is 5 . What is the common ratio?
7. (a) Consider the recursive sequence defined by $a_1 = 9$ and $a_n = a_{n-1} - 4$ for all $n \geq 2$. Find a formula for a_n that depends only on n .
- (b) Consider the recursive sequence defined by $a_1 = 1$, $a_2 = -1$, and $a_n = \left(\frac{n-3}{n-1}\right)a_{n-2}$ for $n \geq 3$. Determine the values of a_{2019} and a_{2020} .

Challenge Problems

8. Can a sequence with infinitely many terms be both arithmetic and geometric? If so, describe all sequences with this property.
9. Determine an expression for the sum of the first n terms in a geometric sequence. (We'll derive this next lesson to solve the pizza problem!)