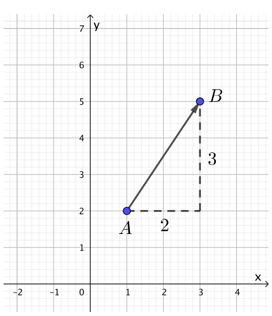
Intermediate Math Circles
Wednesday March 27, 2019
Introduction to Vectors II

Review of last week.
We looked at

1. naming vectors
2. equal and opposite vectors
3. adding vectors
4. scalar multiplication
5. subtracting vectors
6. real world applications
7. vector proofs

Vector Notation



When we look at $\overrightarrow{AB}$, we notice to get from A to B we move 2 units right and 3 units up.
We can represent $\overrightarrow{AB}$ as

$\overrightarrow{AB} =$

or

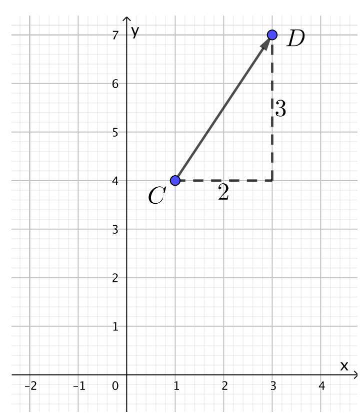
$\overrightarrow{AB} =$

or

$\overrightarrow{AB} =$

We are going to use $\overrightarrow{AB} =$

For each vector, there is a related vector called the ______________



The position vector [a,b] starts at __________ and ends at the point __________.

Find the value for each variable.

a) [3,b]=[a,5]   b) [c+d,-2]=[5,d]

Adding, Subtracting and Scalar Multiplication

Adding Vectors

Find the resultant of the following vectors.

a) [1,2] + [5,7]   b) [3,-2] + [-4,5]   c) [2,-3] + [-2,3]

Notice that the answer for c) is ___________. This is known as the ___________ vector written as ___________.
Note: that the ___________vector has ___________ magnitude and ___________ direction.

Adding and Scalar Multiplication

1) Simplify the following:

a) 3[4,2]   b) a[5,2]   c) 2[3,2]+4[1,-2]

2) Given $\vec{u} = [1, 5]$ and $\vec{v} = [3, -2]$, find the following

a) $3\vec{u}$.   b) $4\vec{v}$.   c) $2\vec{u} + 3\vec{v}$.   d) $a\vec{u} + b\vec{v}$

Adding, Subtracting and Scalar Multiplication

1) Simplify the following:

a) -3[4,-11]   b) [12,1] - [4,-3]   c) 2[5,3] - 4[2,-2]

2) Given $\vec{u} = [3, 2]$ and $\vec{v} = [-4, 3]$, find the following

a) $2\vec{u} - 3\vec{v}$ b) $a\vec{u} - b\vec{v}$

Magnitude of Vectors

1) Find the following:

a) $|\vec{u}|$ when $\vec{u} = [8, 15]$ b) $|\vec{v}|$ when $\vec{v} = [-12, 5]$

c) $|\vec{a}|$ when $\vec{a} = [3, 6]$ d) $|\vec{b}|$ when $\vec{u} = [0, 0]$

2) Given $\vec{u} = [4, -3]$ and $\vec{v} = [5, 12]$, find the following:

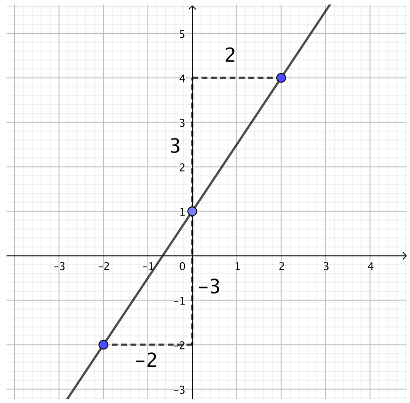
a) $|\vec{u}| + |\vec{v}|$ b)$|\vec{u} + \vec{v}|$ c)$|3\vec{u} - 2\vec{v}|$

Vectors and Lines

Review Given the line $y = \frac{3}{2}x + 1$. What do we know about this line?

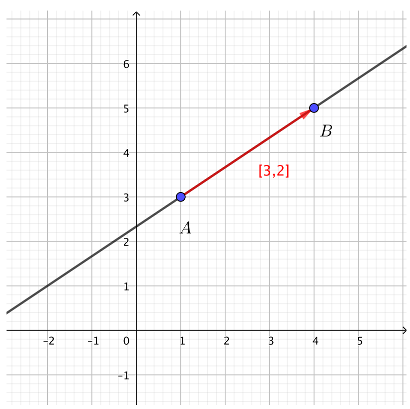
a) slope = _____ b) y-int = _____

We can graph it as well. Label y-int. Move up _____and over _____ to next point. Or down _____ and back _____. Then add line.



Vector Equation

The direction vector is [3,2]. A(1,3) is a point on the line.



This will give the following equation __________.

Let's find another 'point' on the line by putting letting t = 1 .

$$\begin{aligned}\vec{r} &= \_\_\_\_\_ + \_\_\_\_\_ \\ &= \_\_\_\_\_ + \_\_\_\_\_ \\ &= \_\_\_\_\_\end{aligned}$$

This is the position vector for the point _____ from the graph.

To find other position vectors, and hence points, we just need to give different values for t.

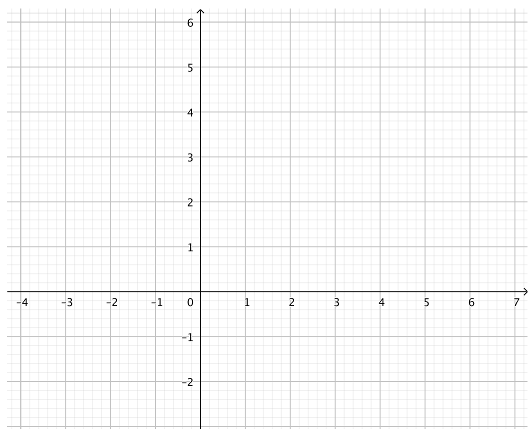
Let's find 3 points on the line $\vec{r} = [-1, -2] + t[1, 1]$
by letting t =-1, t=0 and t=1. Then graph the line.

$\vec{r} =$ __________ = __________ = _____
So one point on the line is _____

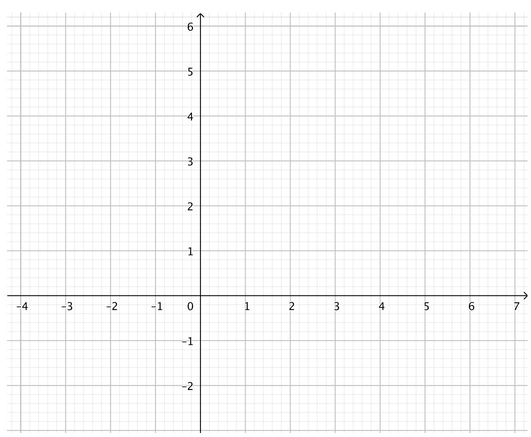
$\vec{r} =$ __________ = __________ = _____
So another point on the line is _____

$\vec{r} =$ _____ = _____ = __________
A third point on the line is _____

So the three points are



Let's find 3 points on the line $\vec{r} = [1, 2] + t[-3, 1]$ by letting t =-1, t=0 and t=1. Then graph the line.



In general the vector equation passing through the point P(x,y) and with direction vector $\vec{m}$ is _______________, where _____ is the position vector of P.

Why do we use $\vec{m}$ for direction?

Recall in the first line we saw had a slope of _____ and the direction vector was _____.

So if the direction vector is _____ the slope will be _____.

Parametric Equation

For the vector equation, $\vec{r} = [3,2] + t[4,5]$ let's let $\vec{r} = [x,y]$. We now have the equation: ________________.

This can be rewritten as $\quad$ _____ = _____ + _____

= _____

These can be taken apart to get the equations:

____

____

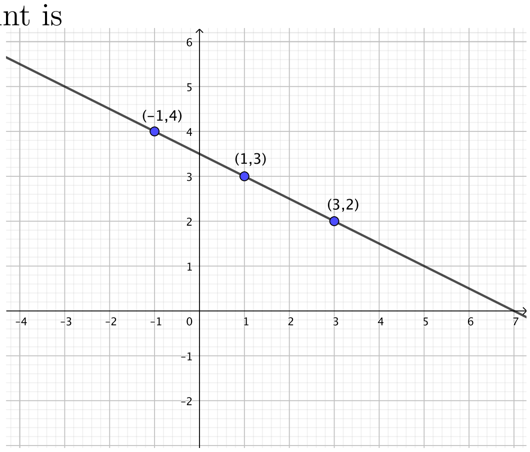
These are called the ________________ of the line.
Let's find 3 points on the line

$$x = 1 + 2t$$
$$y = 3 - t$$

When t = -1,
point is

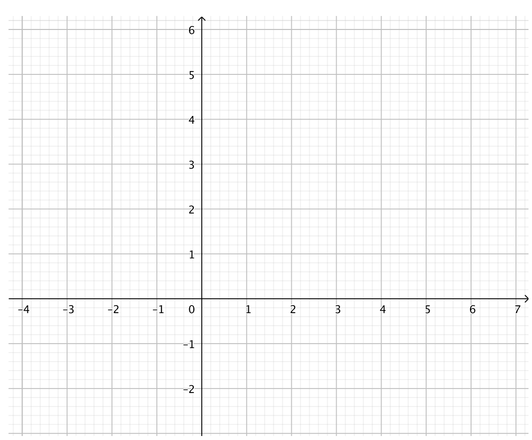
When t = 0,
point is

When t = 1,
point is



Try graphing

$$x = 3 - 2t$$
$$y = -2t$$



Is the point (5,1) on the line

$x = 4 + t$
$y = 3 - 2t$

For the point to be on the line then the t in both equations must be true. There are two ways to do this.

1) Let $x = 5$.
Solve for t
Check in y
$5 = 4 + t$
$t = 1$
$y = 3 - 2(1) = 5$
This is the value of y in the point. So (5,1) is on the line.

2) Let $x = 5$ and $y = 1$.
Solve for t in both
Compare the ts.
$x = 5 = 4 + t$
$t = 1$
$y = 1 = 3 - 2t$
$-2 = -2t$
$t = 1$
Since these ts are the same the point is on the line.

Is the point (5,5) on the line?

$x = 4 + t$
$y = 3 - 2t$

Is the point (7,0) on the line

$x = 3 - 2t$
$y = 4 + 2t$

Parametric to Vector Equation

Rewrite the following into a vector equation.

$x = 4 + 7t$
$y = 3 + 6t$

Just reverse the process we did to get to parametric.

Therefore the vector equation is ________________.

Find the direction vectors for the following lines?

a) $\vec{r} = [3, 2] + t[4, 7]$

b) $x = 3 - 2t$
$y = 4 + 2t$

c) $y = \frac{3}{5}x + 2$

Scalar Equation

Starting with the equation

$$x = 3 + 4t$$
$$y = 2 + 5t$$

Now set the two ts equal to each other.

$$\underline{\hspace{3cm}} = \underline{\hspace{3cm}}$$
$$=$$
$$=$$
$$=$$
$$=$$
$$=$$

This is called the scalar form of a line.
But we know it as the standard form of a line.
We have now come full circle because we can rewrite this as ________________.

Try rewriting the following into scalar form.

a) $x = 5 - 3t$
$y = 4 + t$

b) $\vec{r} = [5, 3] + t[-1, 2]$