Intermediate Math Circles
Wednesday March 27, 2019
Problem Set 2

1) Let $\vec{u} = [3, 7], \vec{v} = [0, 4]$, and $\vec{w} = [2, -5]$. Find the following:

(a) $\vec{u} + \vec{v} + \vec{w}$

(b) $3\vec{u} - 2\vec{v}$

(c) $-2\vec{u} + \frac{1}{8}\vec{v} + 3\vec{w}$

2) Simplify $(a + b + c)[1, 1] + (-a + b)[1, 2] + (-a + b + c)[-1, -1]$

3.) Find a and b.

(a) $[a, 6] = [4, a + b]$

(b) $[2, b] = a[1, 3]$

4) Given $\vec{v} = [4, 3]$.
A)Find the following

(a) $|\vec{v}|$

(b) $\vec{u} = \frac{1}{|\vec{v}|}\vec{v}$

(c) $|\vec{u}|$

B) Graph $\vec{v}$ and $\vec{u}$. What do you notice?

5) In 4, we should have seen that $|\vec{u}| = 1$. Since the length is 1, we call $\vec{u}$ a *unit vector*. For any vector, $\vec{v}$, the associated unit vector $\vec{u}$ is defined as:

$$\vec{u} = \frac{1}{|\vec{v}|}\vec{v}$$

Find the unit vector of the following vectors, then verify its length is 1.

(a) $[5, 12]$

(b) $[4, 7]$

(c) $[a, b]$

6) Sketch a graph of each of the following:

(a) $\vec{r} = [3, -1] + t[3, 2]$

(b) $x = 4 + t$
$y = 3 - 2t$

7) A) On the same set of axes, sketch a graph of each of the following:

(a) $\vec{r} = [3, 5] + t[2, -1]$

(b) $x = 5 - 4t$
$y = 4 + 2t$

B) What do you notice?

8) In 7, we should have seen that they were the same line. We can verify this by showing they have the same direction and that one point on the first line is on the second line.

(a) Show the two vectors are heading in the same direction

(b) Show that the point (3,5) from a) is on the line in b)

9) Determine if the following are the same line or not

(a) $\vec{r} = [6, 5] + t[3, -2]$

(b) $x = 9 + 6t$
$y = -4t$

10) Determine if the following are the same line or not

(a) $\vec{r} = [3, 5] + t[-2, 3]$

(b) $3x + 2y - 19 = 0$

11)

(a) Show the vector [8,1] can be found by adding scalar multiples of [2,5] and [4,3].

(b) Show that any vector of the form [a,b] can be found by adding scalar multiples of [2,5] and [4,3].