Intermediate Math Circles
Wednesday March 27, 2019
Problem Set 2 — Solutions

1. **Solution**

   (a) $\vec{u} + \vec{v} + \vec{w} = [3, 7] + [0, 4] + [2, -5] = [5, 6]$

   (b) $3\vec{u} - 2\vec{v} = 3[3, 7] - 2[0, 4] = [9, 21] - [0, 8] = [9, 13]$

   (c) $-2\vec{u} + \dfrac{1}{8}\vec{v} + 3\vec{w} = [-6, -14] + [0, \frac{1}{2}] + [6, -15] = [0, -\frac{59}{2}]$

2. **Solution**

$$\begin{aligned} &a[1,1] + b[1,1] + c[1,1] - a[1,2] + b[1,2] - a[-1,-1] + b[-1,-1] + c[-1,-1] \\ &= a\left([1,1] - [1,2] + [1,1]\right) + b\left([1,1] + [1,2] + [-1,-1]\right) + c\left([1,1] + [-1,-1]\right) \\ &= a[1,0] + b[1,2] \end{aligned}$$

3. **Solution**

   (a) $a = 4, b = 4$
   We have $a = 4$. Thus, $6 = a + b \implies 6 = 4 + b$. Therefore, $b = 2$.

   (b) $a = 2, b = 6$
   We have $2 = a$. Thus, $b = 3a \implies b = 3(2) = 6$.

4. **Solution** A)

   (a) $|\vec{v}| = \sqrt{9 + 16} = 5$

   (b) $\vec{u} = \frac{1}{5}[4, 3] = [\frac{4}{5}, \frac{3}{5}]$

   (c) $|\vec{u}| = \sqrt{\left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2} = \sqrt{\frac{16}{25} + \frac{9}{25}} = \sqrt{\frac{25}{25}} = 1$

   B) $\vec{u}$ has the same direction of $\vec{v}$ but $\frac{1}{5}$ the magnitude of $\vec{v}$

5. **Solution**

   (a) $|\vec{v}| = \sqrt{25 + 144} = 13$; $\vec{u} = \frac{1}{13}[5, 12] = [\frac{5}{13}, \frac{12}{13}]$
   $|\vec{u}| = \sqrt{\left(\frac{5}{13}\right)^2 + \left(\frac{12}{13}\right)^2} = \sqrt{\frac{25}{169} + \frac{144}{169}} = \sqrt{\frac{169}{169}} = 1$

(b) $|\vec{v}| = \sqrt{16+49} = \sqrt{65}$; $\vec{u} = \frac{1}{\sqrt{65}}[4,7] = [\frac{4}{\sqrt{65}}, \frac{7}{\sqrt{65}}]$

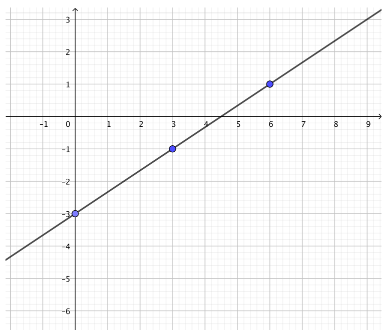
$|\vec{u}| = \sqrt{\left(\frac{4}{\sqrt{65}}\right)^2 + \left(\frac{7}{\frac{4}{\sqrt{65}}}\right)^2} = \sqrt{\frac{16}{65} + \frac{65}{65}} = \sqrt{\frac{65}{65s}} = 1$

(c) $|\vec{v}| = \sqrt{a^2+b^2}$; $\vec{u} = \frac{1}{\sqrt{a^2+b^2}}[a,b] = [\frac{a}{\sqrt{a^2+b^2}}, \frac{b}{\sqrt{a^2+b^2}}]$

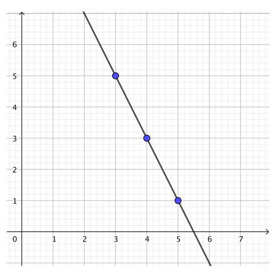
$|\vec{u}| = \sqrt{\left(\frac{a}{\sqrt{a^2+b^2}}\right)^2 + \left(\frac{b}{\sqrt{a^2+b^2}}\right)^2} = \sqrt{\frac{a^2}{\sqrt{a^2+b^2}} + \frac{b}{\sqrt{a^2+b^2}}} = \sqrt{\frac{a^2+b^2}{a^2+b^2}} = 1$

6. **Solution**

(a) t=-1, pt(0,-3); $t = 0$, pt(3,-1); $t = 1$, pt(6,1)



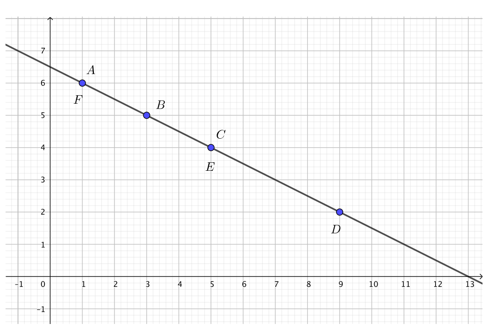
(b) t=-1, pt(3,5); $t = 0$, pt(4,3); $t = 1$, pt(5,1)



7. **Solution** A)

(a) t=-1, pt A(1,6); $t = 0$, pt B(3,5); $t = 1$, pt C(5,4)

(b) t=-1, pt D(9,2); $t = 0$, pt E(5,4); $t = 1$, pt F(1,6)



B) They are the same line.

8. **Solution**

   (a) The two direction vectors are [2,-1] and [-4,2]. Since [-4,2] = -2[2,-1], they are parallel to each other and have the 'same' direction

   (b) Sub 3 into the x of second equation $3 = 5 - 4t \implies -2 = -4t \implies t = \frac{1}{2}$
   Check with y; $y = 4 + 2\frac{1}{2} = 4 + 1 = 5 \implies$ the point is on the line.
   Therefore the lines are the same.

9. **Solution**

   The two direction vectors are [3,-2] and [6,-4]. Since [6,-4]=2[3,-2] they have the 'same' direction.
   Show [6,5] is in the other line.
   Sub 6 into the $x \implies 6 = 9 + 6t \implies -3 = 6t \implies t = \frac{-1}{2}$
   Check with y; $y = -4\left(\frac{-1}{2}\right) = 2 \implies$ the point is not on the line.
   Therefore the lines are not the same.

10. **Solution**

    Rewrite the second equation in slope y-int form. $3x + 2y - 19 = 0 \implies 2y = -3x + 19 \implies y = -\frac{3}{2}x + \frac{19}{2}$. Therefore the slope is $\frac{-3}{2}$ which gives a direction vector of [-2,3]. Therefore the two lines have the same direction
    Now sub in the point from a) into b): 3(3)+2(5) - 19 = 19 - 19 = 0. This is a true statement and therefore the point is on the second line. And they are the same line.