UNIVERSITY OF
WATERLOO

FACULTY OF MATHEMATICS
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CENTRE FOR EDUCATION IN
MATHEMATICS AND COMPUTING

Grade 6 Math Circles
February 12th/13th
Angles

Types of Angles

As can be seen in the warm-up, the space between two lines measured from a point of intersection is called an **angle**. We can classify angles based on their size:

Angle	Classification
less than 90°	Acute angle
= 90°	Right angle
greater than 90° and less than 180°	Obtuse angle
= 180°	Straight angle
greater than 180° and less than 360°	Reflex angle
= 360°	Full rotation

Here are some examples of what these angles look like:

Acute angle

Right angle

Obtuse angle
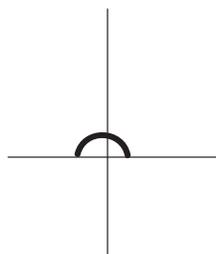
Straight angle
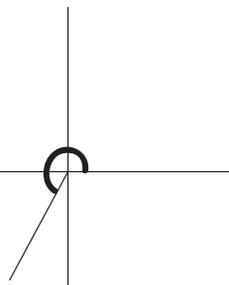
Reflex angle
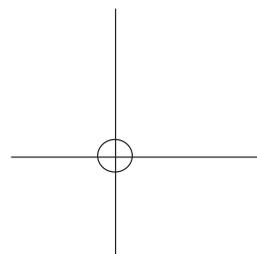
Full Rotation

Labelling Angles

Angles are labeled according to the three points involved, in order in-middle-out. For example, the angle below could be called either $\angle ABC$ or $\angle CBA$.
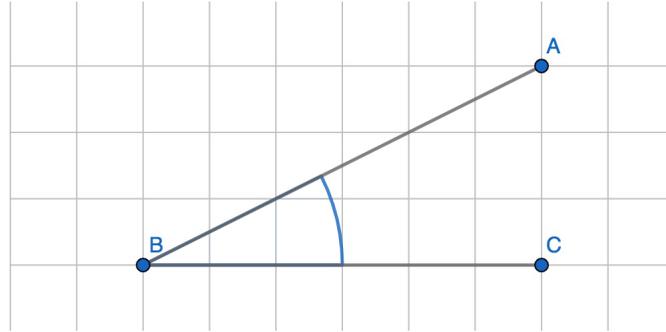

Classifying Angles

Additionally, there are some other ways that we classify angles:

- **Complementary angles** are two angles that add up to _____. These angles can be beside each other or apart. When complementary angles are together, they make a right angle.

- **Supplementary angles** are two angles that add up to _____. They can also be beside each other or apart. When supplementary angles are together, they make a straight angle.

- In addition, the angles around any intersection point always add up to _____. The **full rotation** around a point makes a circle, so naturally it is _____.
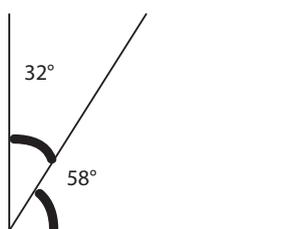

$58° + 32° = 90°$
Complementary angles
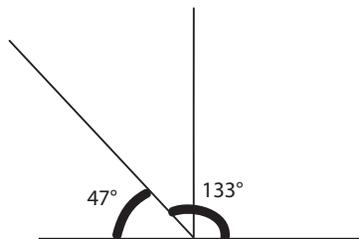
$133° + 47° = 180°$
Supplementary angles
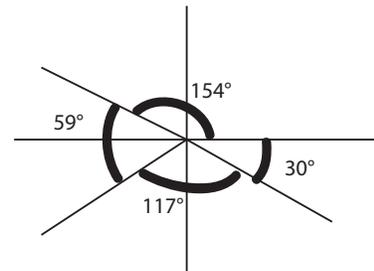
$154° + 59° + 117° + 30° = 360°$
Full rotation

2

- **Congruent** angles mean that angles are of _____ measure, even if they are not in the same place.

- Two lines are **parallel** if they never meet and are always the same _____ _____.

  When looking at a diagram, if there are arrows along two or more lines, those lines are parallel to one another.

- A **transversal** is a line that crosses at least _____ other lines.


45°

45°
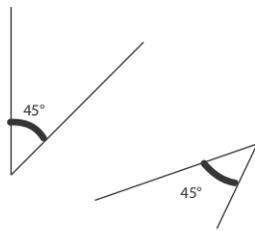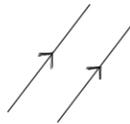
Congruent angles   Parallel lines   Transversal
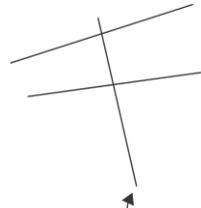
Angle Properties

a b
c d

e f
g h
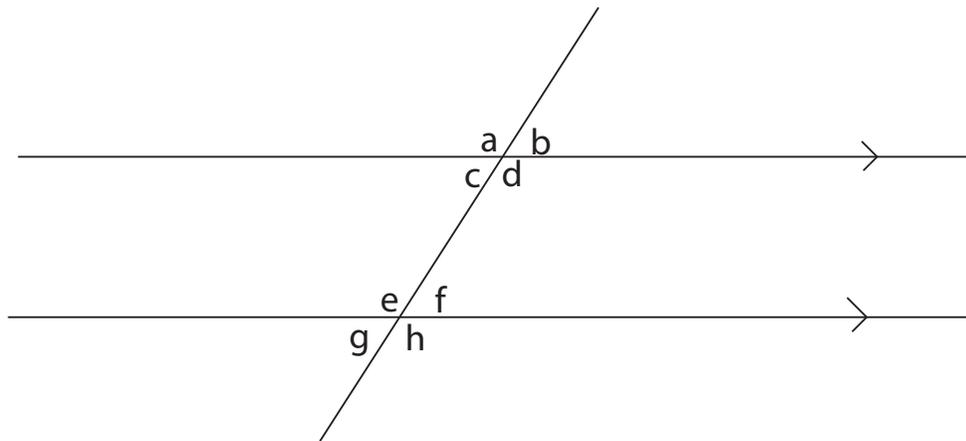
Alternate angles are angles that are on opposite sides of the transversal and not next to each other, but in between the other two lines. When the lines are parallel, alternate angles are congruent.

Which angles from the diagram above satisfy this property?

The easiest way to think of alternate angles is the **"Z pattern"**. If you draw the letter "z" from either direction of the top parallel line to the bottom, the angles which represent the insides of the "z" are congruent.

Corresponding angles are angles that are on the same side of the transversal, and on matching sides of their own line. When the lines are parallel, corresponding angles are congruent.
Which angles from the diagram above satisfy this property?

The easiest way to think of corresponding angles is the **"F pattern"**. If you draw the letter "F", backwards or forwards, on the transversal, then the angles on top or below the horizontal lines will be congruent.

Opposite angles are angles that are opposite each other when two lines cross. Opposite angles are equal.
Which angles from the diagram above satisfy this property?

The easiest way to think of opposite angles is the **"X pattern"**. Picture an X between the transversal and the line. The angles on top and on the bottom of the "X" are congruent, as are the angles on either side.

Interior angles are angles that are on the same side of the transversal, in between the lines. When the lines are parallel, interior angles are supplementary (remember, this means that they add up to 180°).
Which angles from the diagram above satisfy this property?

The easiest way to think of interior angles is the **"C pattern"**. If the space between the two parallel lines and the transversal is a very robotic letter "C", then the two angles in the "C" are supplementary.

Notice that since $a = d$ and $d = h$ (because they are opposite and corresponding angles), $a = h$. Similarly, $b = c$ and $c = g$, so $b = g$.

Exercise. Find angles a, c, d, e, f, g and h.
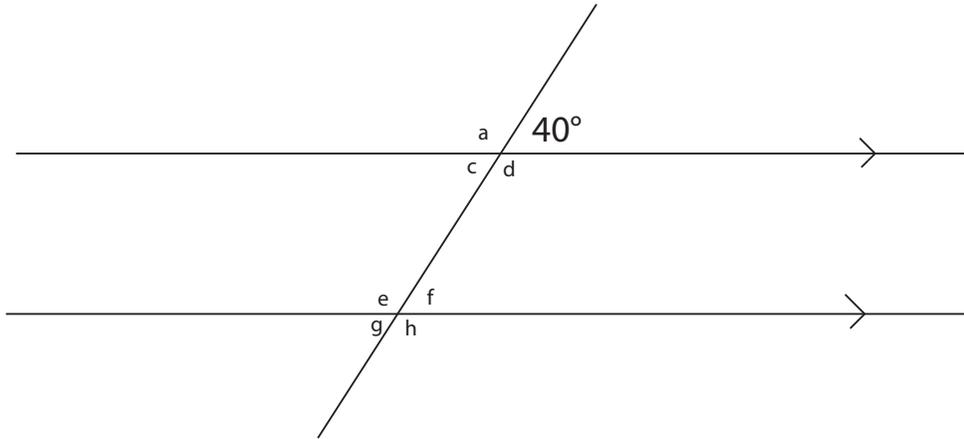
Triangles

The sum of angles in a triangle is always 180°.

This leads to different types of triangles, categorized by angles:

Equilateral triangles: Triangles with _____ equal sides and therefore _____ equal angles

When looking at a diagram, if there is a dash mark through two or more lines, those lines are of equal length.

Exercise.

Since there are 180° in a triangle and all angles are equal in an equilateral triangle, what is this angle?

Isosceles triangles: Triangles with _____ equal sides and therefore _____ equal angles

Scalene triangles: Triangles with _____ equal sides and therefore _____ equal angles

Right triangles: Triangles with _____ right angle

Exercise.
Isosceles right triangles have 2 equal sides, and a right angle. Can you find their other angle?

Acute triangles: Triangles in which _____ angles are acute, less than 90°

Obtuse triangles: Triangles in which _____ angle is obtuse, greater than 90°

Exercise.
Why is it impossible to have a triangle with a reflex angle?

Classify the triangle by number of similar angles (equilateral, isosceles, scalene) and size of angle (right, acute, obtuse). Then determine the missing angle(s):
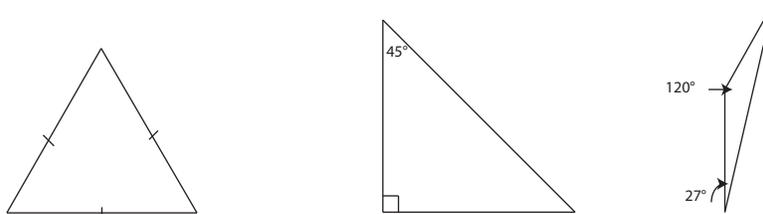
Problem Set

1. Classify the angle.

a)

b)

c)

d)

e)

f)
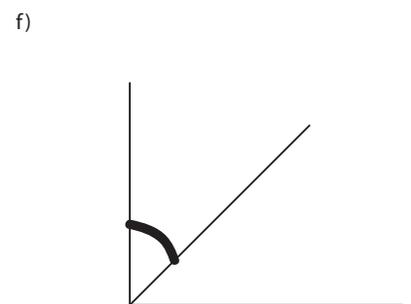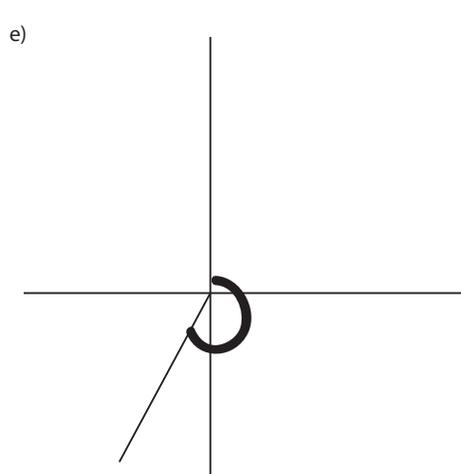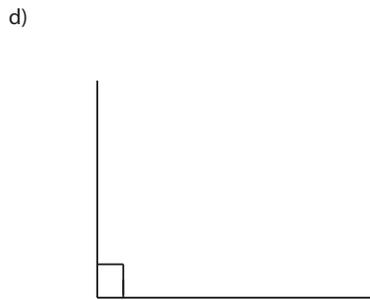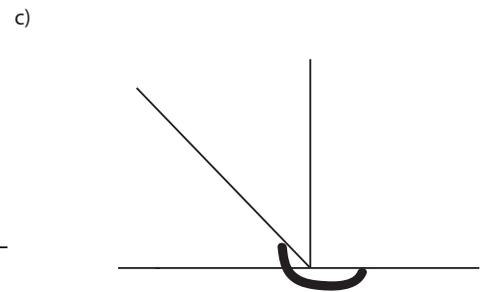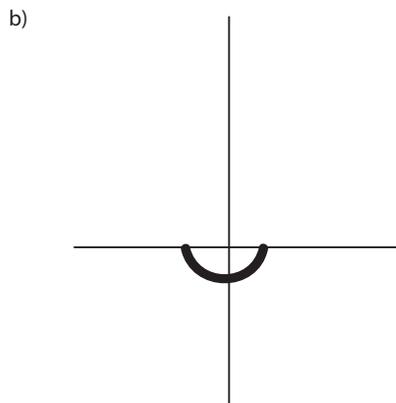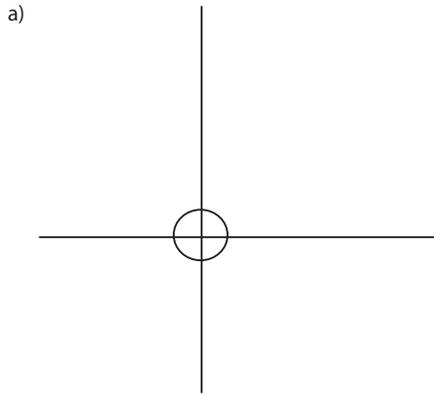
2. Find the missing angle, x, in each diagram.
   (Note: Diagrams may not be to scale)

a)

x

36°

b)

36°

x

c)

180°

36°

36°

x
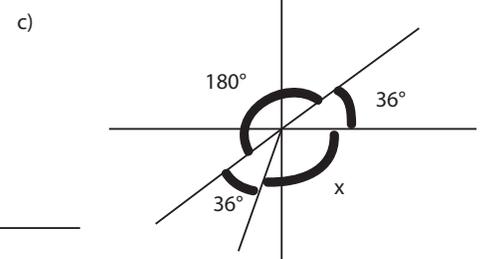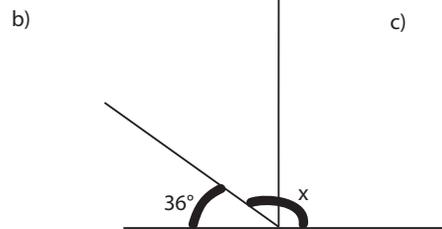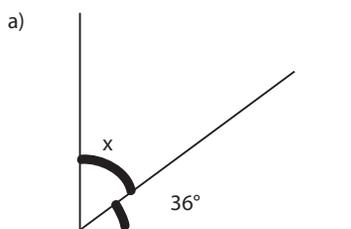
3. Find angles a, b, c, d, f, g and h.
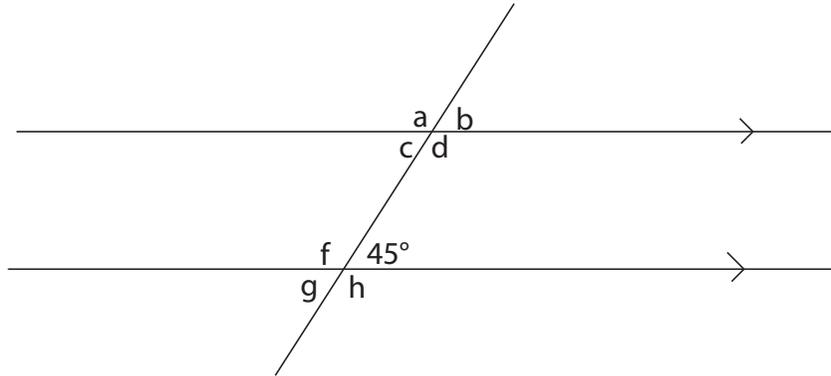
4. Find angles a, b, c, d, e, f, g, h, i, j, k and l.
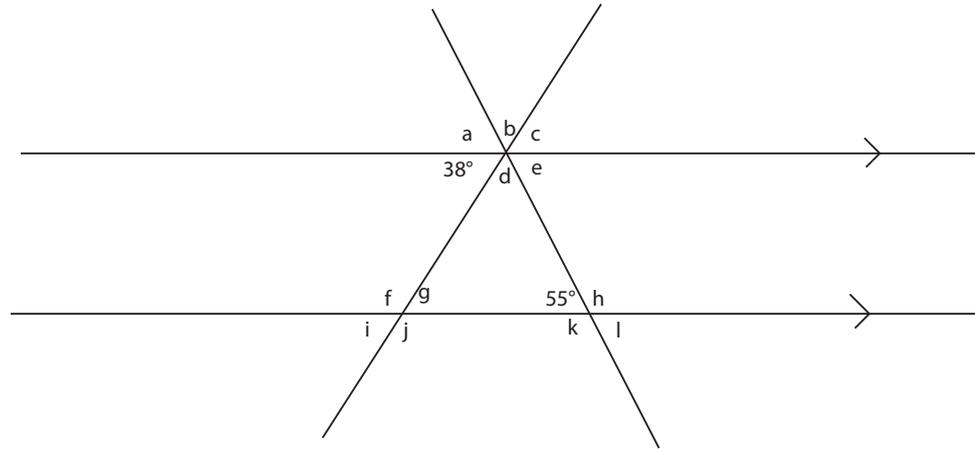

5. Classify the triangle and find the missing angle(s):
   (Note: Diagrams may not be to scale)

a)

160°

b)

70°
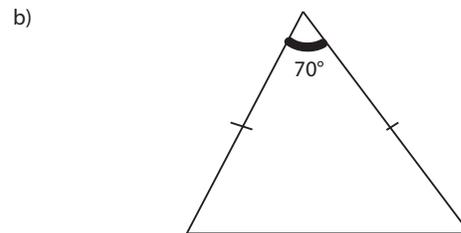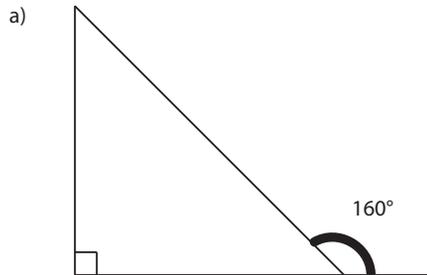
6. Find angles a, b, and c if the top and bottom lines are parallel.

60°
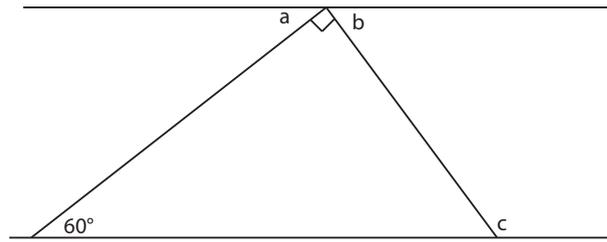
7. In the diagram shown, PQR is a straight line segment. What is the measure of $\angle QSR$?

S

75°   30°

P Q R
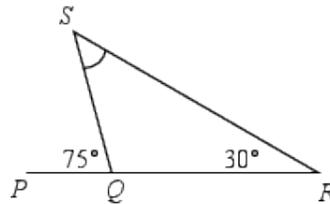
8. Four points B, A, E, and L are on a straight line as shown. G is a point off the line so that $\angle BAG = 120°$ and $\angle GEL = 80°$. If the reflex angle at G is x, then what does x equal?

G

$x°$

120°   80°

B A E L
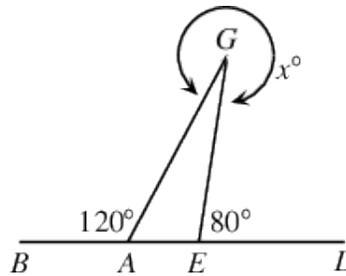
CHALLENGE

9. In the diagram, AB is parallel to DC and ACE is a straight line. What is the value of x?

A B

75°

$x°$

115°

D C 105°

E
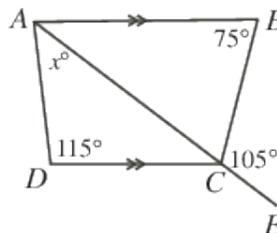
10. In the diagram, points R and S lie on QT. Also, $\angle PTQ = 62°$, $\angle RPS = 34°$, and $\angle QPR = x°$. What is the value of x?
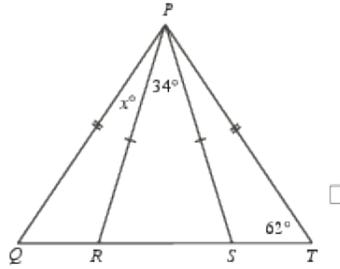
11. In the diagram, PQ is parallel to RS. Also, Z is on PQ and X is on RS. If Y is located between PQ and RS so that $\angle YXS = 20°$ and $\angle ZYX = 50°$, what is the measure of $\angle QZY$?
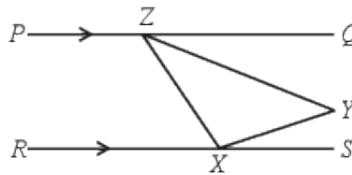
12. In the diagram, $\triangle QUR$ and $\triangle SUR$ are equilateral triangles. Also, $\triangle QUP$, $\triangle PUT$ and $\triangle TUS$ are isosceles triangles with $PU = QU = SU = TU$ and $QP = PT = TS$. What is the measure of $\angle UST$ in degrees?