



## Grade 6 Math Circles

February 5th/6th  
*Arithmetic Tricks*

### Introduction

We are introduced early on how to add, subtract, multiply, and divide. As we learn more math, we have to deal with bigger numbers requiring more time to compute the answer.

Although many of you may be comfortable with multiplying numbers up to  $12 \times 12$ , what happens when you try to multiply larger numbers? It can seem overwhelming at first, but today you will learn a few tricks to help speed up your calculations and be confident with your answers.

Being able to calculate quickly alleviates the load on your limited working memory, allowing your brain to concentrate on more difficult mathematics and problem solving.

As always, the trick to learning math is practice, *practice*, **practice!** The goal is to be able to do all these tricks eventually in your head without writing anything down. Today, try to do all of the exercises without using a calculator!

### Multiplying by 5

**Example.** Evaluate  $137 \times 5$ .

A clever trick is to realize that  $5 = 10 \div 2$ .

**Trick:** When multiplying by 5, multiply by 10 and divide by 2.

**Exercise.** Evaluate the following

1.  $15 \times 5$

4.  $65 \times 5$

2.  $28 \times 5$

5.  $18 \times 5$

3.  $90 \times 5$

6.  $22 \times 5$

## Dividing by 5

**Trick:** Dividing by 5 is the same as multiplying by 2, then dividing by 10.

**Example.**  $325 \div 5$

**Why does this work?** Recall that  $325 \div 5 = \frac{325}{5}$

**Exercise.** Evaluate the following

1.  $85 \div 5$

4.  $440 \div 5$

2.  $135 \div 5$

5.  $360 \div 5$

3.  $275 \div 5$

6.  $95 \div 5$

## Multiplication of 11 With Any 2 Digit Number

To multiply a 2 digit number with 11,

1. Write down the **first digit** of the non-11 number. This is going to be our **first digit**.
2. Write down the **second digit** of the non-11 number. This will be our **last digit**.
3. Add the two digits. This is the **middle digit**.
4. If the sum is greater than or equal to 10, carry over the number in the tens column from the middle digit and add it to the first digit.

**Example.**  $25 \times 11$

The non-11 number is 25.

1. The **first digit** of 25 is 2.

The **first digit** of the product of  $25 \times 11$  will be 2.

2

2. The **second digit** of 25 is 5.

The **last digit** of the product of  $25 \times 11$  will be 5.

$$\begin{array}{ccc} 2 & & 5 \\ \hline \end{array}$$

3.  $2 + 5 = 7$  and place it between 2 and 5.

The **middle digit** of the product of  $25 \times 11$  will be 7.

$$\begin{array}{ccc} & 2 + 5 & \\ 2 & 7 & 5 \\ \hline \end{array}$$

$$\therefore 25 \times 11 = 275$$

**Example.**  $38 \times 11$

In this example, we have to carry over from the sum. Since 38 is the non-11 term, we write down 3 as the first digit and 8 as the last digit.

$$\begin{array}{ccc} 3 & & 8 \\ \hline \end{array}$$

All that remains is to find the middle digit. Add the two digits of 38,  $3 + 8 = 11$ . Since 11 is greater than 10, we have to carry over the 1 from the tens column and add it to 3.

$$\begin{array}{ccc} & 3 + 8 & \\ 3 & 11 & 8 \\ \hline \end{array}$$

We can only have a single digit in the middle, so we have to carry over the 1 over

$$\begin{array}{ccc} 3 + 1 & 1 & 8 \\ \hline \end{array}$$

We carry over by adding the 1 to the digit to the right of it which is 3

$$\begin{array}{ccc} 4 & 1 & 8 \\ \hline \end{array}$$

$$\therefore 38 \times 11 = 418$$

**Exercise.** Evaluate the following

1.  $45 \times 11$

4.  $85 \times 11$

2.  $12 \times 11$

5.  $94 \times 11$

3.  $67 \times 11$

6.  $11 \times 11$

## Multiplying Any Number by 11

To multiply any number by 11,

1. Write down the number's right most digit.
2. Add that digit to the digit on the left, write down the units digit, and carry over the tens digit if it is greater than 1.
3. Proceed to the next digit and repeat the process all over.
4. Once you reach the leftmost digit, write down that digit.

**Example.** Evaluate  $54321 \times 11$

**Example.** Evaluate  $62473 \times 11$

**Exercise.** Evaluate the following

1.  $111 \times 11$

4.  $4389 \times 11$

2.  $345 \times 11$

5.  $72831 \times 11$

3.  $2359 \times 11$

6.  $9527136 \times 11$

## Multiplication with 9, 99, 999, ...

Multiplying numbers with 9, 99, or 999 can seem like a hassle but we can multiply things more easily by taking advantage of the **distributive property** of multiplication.

The distributive property says that when you multiply an addition or multiplication expression by a number, you must multiply all numbers in the bracket by the multiplying values.

$$a \times (b + c) = a \times b + a \times c$$

**Example.** Evaluate  $44 \times 9$

We know that  $9 = 10 - 1$

This trick can also be described by, multiply by 10 then subtract the original number.

**Example.** Evaluate  $68 \times 99$

This trick can be described by, multiply by 100 then subtract the original number.

This trick continues for all other numbers composed of all nines. Just add one to the number with all nines to get an easy number to multiply, multiply, then subtract the original non-nine number.

This is known as the distributive property as you are writing the number you are multiplying by as two parts,

$$99 = 100 - 1$$

then **distributing** the original number to both parts by multiplying each one by it,

$$\begin{aligned} 42 \times 99 &= 42 \times (100 - 1) \\ &= (42 \times 100) - (42 \times 1) \\ &= 4200 - 42 \\ &= 4158 \end{aligned}$$

**Exercise.** Evaluate the following

1.  $18 \times 9$

5.  $13 \times 99$

2.  $45 \times 9$

6.  $112 \times 99$

3.  $93 \times 99$

7.  $178 \times 999$

4.  $78 \times 99$

8.  $24 \times 999$

## Multiplying 2 Digit Numbers Up to 100

**Example.** Evaluate  $21 \times 31$

1. Multiply the **first digit** of the **first number** by the **first digit** of the **second number**.  
This number becomes the **first digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \hline 2 \times 3 = 6 \\ \text{6 is the first digit} \\ \text{6} \quad \_ \quad \_ \end{array}$$

2. Multiply the **last digit** of the **first number** by the **last digit** of the **second number**.  
This number becomes the **last digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \hline 1 \times 1 = 1 \\ \text{1 is the last digit} \\ \text{6} \quad \_ \quad \text{1} \end{array}$$

3. Multiply the **inner digits** and **outer digits** of the two numbers and add them. This sum becomes the **middle digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \hline 1 \times 3 + 2 \times 1 = 5 \\ \text{6} \quad \text{5} \quad \text{1} \end{array}$$

**Example.**  $42 \times 63$

1. We multiply the **first digit** of the **first number** by the **first digit** of the **second number**.

$$\begin{array}{r} \text{42} \times \text{63} \\ \hline 4 \times 6 = 24 \\ \text{24 is the first two digit} \\ \text{24} \quad \_ \quad \_ \end{array}$$



2. We multiply the **last digit** of the **first number** by the **last digit** of the **second number**.

$$\begin{array}{r}
 \text{42} \times \text{63} \\
 \text{2} \times \text{3} = \text{6} \\
 \text{6 is the last digit} \\
 \text{24} \quad \underline{\quad} \quad \text{6}
 \end{array}$$

3. Multiply the **inner digits** and **outer digits** of the two numbers and add them. This sum becomes the **middle digit**. However, since 24 is greater than 10, the 2 from the tens column must be added to the first number.

$$\begin{array}{r}
 \text{42} \times \text{63} \\
 \text{2} \times \text{6} + \text{4} \times \text{3} \\
 = \text{12} + \text{12} \\
 = \text{24} \\
 \text{24} \quad \text{24} \quad \text{6}
 \end{array}$$

We can only have a single digit in the middle so we have to carry the 2 over

$$\begin{array}{r}
 \text{24} + \text{2} \quad \text{4} \quad \text{6} \\
 \text{26} \quad \text{4} \quad \text{6}
 \end{array}$$

We carry the 2 over by adding it to 24

**Exercise.** Evaluate the following

- |                   |                   |
|-------------------|-------------------|
| 1. $21 \times 22$ | 4. $42 \times 24$ |
| 2. $25 \times 31$ | 5. $65 \times 14$ |
| 3. $34 \times 13$ | 6. $87 \times 53$ |

## Problem Set

1. Try the warm-up questions again. See if you can do them faster than you did before!

2. Multiplying by 5

(a)  $6 \times 5$

(d)  $42 \times 5$

(b)  $17 \times 5$

(e)  $99 \times 5$

(c)  $26 \times 5$

(f)  $75 \times 5$

3. Division by 5

(a)  $660 \div 5$

(d)  $70 \div 5$

(b)  $110 \div 5$

(e)  $240 \div 5$

(c)  $145 \div 5$

(f)  $625 \div 5$

4. Multiplication of 2 digit number with 11

(a)  $86 \times 11$

(d)  $23 \times 11$

(b)  $15 \times 11$

(e)  $78 \times 11$

(c)  $57 \times 11$

(f)  $55 \times 11$

5. Multiplication of any number with 11

(a)  $1234 \times 11$

(d)  $823 \times 11$

(b)  $5890 \times 11$

(e)  $881 \times 11$

(c)  $4583 \times 11$

(f)  $2401 \times 11$

6. Multiplication of 9, 99, 999...

(a)  $26 \times 9$

(d)  $72 \times 99$

(b)  $89 \times 9$

(e)  $34 \times 99$

(c)  $890 \times 9$

(f)  $93 \times 999$

7. Product of Two Numbers

(a)  $83 \times 45$

(d)  $82 \times 14$

(b)  $28 \times 31$

(e)  $35 \times 34$

(c)  $45 \times 12$

(f)  $32 \times 85$

### CHALLENGE

8. Prove the “Multiply Any Number by 11” rule for a 3 digit integer.
  
9. Using the distributive law, what is another trick for multiplying by 11 that would also be useful?
  
10. Are there any other numbers that you think are hard to multiply by? Try to find a trick that works for a number that you struggle with.