



Grade 6 Math Circles
February 5th/6th
Arithmetic Tricks

Warm-Up

Attempt the following calculations without a calculator.

1. $5 \times 6 = 30$

11. $11 \times 517 = 5687$

2. $6 \times 8 = 48$

12. $11 \times 5688 = 62568$

3. $12 \times 11 = 132$

13. $99 \times 12 = 1188$

4. $8 \times 12 = 96$

14. $99 \times 458 = 45342$

5. $5 \times 7 = 35$

15. $999 \times 16 = 15984$

6. $9 \times 5 = 45$

16. $999 \times 800 = 799200$

7. $17 \times 5 = 85$

17. $30 \times 15 = 450$

8. $42 \times 5 = 210$

18. $52 \times 25 = 1300$

9. $11 \times 32 = 352$

19. $32 \times 47 = 1504$

10. $11 \times 47 = 517$

20. $67 \times 93 = 6231$

Introduction

We are introduced early on how to add, subtract, multiply, and divide. As we learn more math, we have to deal with bigger numbers requiring more time to compute the answer.

Although many of you may be comfortable with multiplying numbers up to 12×12 , what happens when you try to multiply larger numbers? It can seem overwhelming at first, but today you will learn a few tricks to help speed up your calculations and be confident with your answers.

Being able to calculate quickly alleviates the load on your limited working memory, allowing your brain to concentrate on more difficult mathematics and problem solving.

As always, the trick to learning math is practice, *practice*, **practice!** The goal is to be able to do all these tricks eventually in your head without writing anything down. Today, try to do all of the exercises without using a calculator!

Multiplying by 5

Example. Evaluate 137×5 .

A clever trick is to realize that $5 = 10 \div 2$.

$$\begin{aligned} 137 \times 5 &= 137 \times (10 \div 2) \\ &= (137 \times 10) \div 2 \\ &= 1370 \div 2 \\ &= 685 \end{aligned}$$

Trick: When multiplying by 5, multiply by 10 and divide by 2.

Exercise. Evaluate the following

$$\begin{aligned} 1. \quad 15 \times 5 &= 15 \times (10 \div 2) \\ &= (15 \times 10) \div 2 \\ &= 150 \div 2 \\ &= 75 \end{aligned}$$

$$\begin{aligned} 2. \quad 28 \times 5 &= 28 \times (10 \div 2) \\ &= (28 \times 10) \div 2 \\ &= 280 \div 2 \\ &= 140 \end{aligned}$$

$$\begin{aligned}
3. \quad 90 \times 5 &= 90 \times (10 \div 2) \\
&= (90 \times 10) \div 2 \\
&= 900 \div 2 \\
&= 450
\end{aligned}$$

$$\begin{aligned}
5. \quad 18 \times 5 &= 18 \times (10 \div 2) \\
&= (18 \times 10) \div 2 \\
&= 180 \div 2 \\
&= 90
\end{aligned}$$

$$\begin{aligned}
4. \quad 65 \times 5 &= 65 \times (10 \div 2) \\
&= (65 \times 10) \div 2 \\
&= 650 \div 2 \\
&= 325
\end{aligned}$$

$$\begin{aligned}
6. \quad 22 \times 5 &= 22 \times (10 \div 2) \\
&= (22 \times 10) \div 2 \\
&= 220 \div 2 \\
&= 110
\end{aligned}$$

Dividing by 5

Trick: Dividing by 5 is the same as multiplying by 2, then dividing by 10.

Example. $325 \div 5$

$$\begin{aligned}
325 \div 5 &= 325 \times 2 \div 10 \\
&= 650 \div 10 \\
&= 65
\end{aligned}$$

Why does this work? Recall that $325 \div 5 = \frac{325}{5}$

$$\begin{aligned}
\frac{325}{5} &= 325 \times \frac{1}{5} \\
&= 325 \times \frac{2}{10}
\end{aligned}$$

We can see in the final step that multiplying by $\frac{2}{10}$ is the same as multiplying by 2 and then dividing by 10.

Exercise. Evaluate the following

$$\begin{aligned}
1. \quad 85 \div 5 &= 85 \times 2 \div 10 \\
&= 170 \div 10 \\
&= 17
\end{aligned}$$

$$\begin{aligned}
2. \quad 135 \div 5 &= 135 \times 2 \div 10 \\
&= 270 \div 10 \\
&= 27
\end{aligned}$$

$$\begin{aligned}
 3. \quad 275 \div 5 &= 275 \times 2 \div 10 \\
 &= 550 \div 10 \\
 &= 55
 \end{aligned}$$

$$\begin{aligned}
 5. \quad 360 \div 5 &= 360 \times 2 \div 10 \\
 &= 720 \div 10 \\
 &= 72
 \end{aligned}$$

$$\begin{aligned}
 4. \quad 440 \div 5 &= 440 \times 2 \div 10 \\
 &= 880 \div 10 \\
 &= 88
 \end{aligned}$$

$$\begin{aligned}
 6. \quad 95 \div 5 &= 95 \times 2 \div 10 \\
 &= 190 \div 10 \\
 &= 19
 \end{aligned}$$

Multiplication of 11 With Any 2 Digit Number

To multiply a 2 digit number with 11,

1. Write down the **first digit** of the non-11 number. This is going to be our **first digit**.
2. Write down the **second digit** of the non-11 number. This will be our **last digit**.
3. Add the two digits. This is the **middle digit**.
4. If the sum is greater than or equal to 10, carry over the number in the tens column from the middle digit and add it to the first digit.

Example. 25×11

The non-11 number is 25.

1. The **first digit** of 25 is 2.

The **first digit** of the product of 25×11 will be 2.

2

2. The **second digit** of 25 is 5.

The **last digit** of the product of 25×11 will be 5.

2 5

3. $2 + 5 = 7$ and place it between 2 and 5.

The **middle digit** of the product of 25×11 will be 7.

$$\begin{array}{ccc} & 2 + 5 & \\ \underline{2} & \underline{7} & \underline{5} \end{array}$$

$$\therefore 25 \times 11 = 275$$

Example. 38×11

In this example, we have to carry over from the sum. Since 38 is the non-11 term, we write down 3 as the first digit and 8 as the last digit.

$$\begin{array}{ccc} \underline{3} & & \underline{8} \end{array}$$

All that remains is to find the middle digit. Add the two digits of 38, $3 + 8 = 11$. Since 11 is greater than 10, we have to carry over the 1 from the tens column and add it to 3.

$$\begin{array}{ccc} & 3 + 8 & \\ \underline{3} & \underline{11} & \underline{8} \end{array}$$

We can only have a single digit in the middle, so we have to carry over the 1 over

$$\begin{array}{ccc} \underline{3 + 1} & \underline{1} & \underline{8} \end{array}$$

We carry over by adding the 1 to the digit to the right of it which is 3

$$\begin{array}{ccc} \underline{4} & \underline{1} & \underline{8} \end{array}$$

$$\therefore 38 \times 11 = 418$$

Exercise. Evaluate the following

$$\begin{aligned} 1. \quad & 45 \times 11 \\ & = \underline{4} (4 + 5) \underline{5} \\ & = 495 \end{aligned}$$

$$\begin{aligned} 4. \quad & 85 \times 11 \\ & = \underline{8} (8 + 5) \underline{5} \\ & = \underline{(8 + 1)} \underline{3} \underline{5} \\ & = 935 \end{aligned}$$

$$\begin{aligned} 2. \quad & 12 \times 11 \\ & = \underline{1} (1 + 2) \underline{2} \\ & = 132 \end{aligned}$$

$$\begin{aligned} 5. \quad & 94 \times 11 \\ & = \underline{9} (9 + 4) \underline{4} \\ & = \underline{(9 + 1)} \underline{3} \underline{4} \\ & = 1034 \end{aligned}$$

$$\begin{aligned} 3. \quad & 67 \times 11 \\ & = \underline{6} (6 + 7) \underline{7} \\ & = \underline{(6 + 1)} \underline{3} \underline{7} \\ & = 737 \end{aligned}$$

$$\begin{aligned} 6. \quad & 11 \times 11 \\ & = \underline{1} (1 + 1) \underline{1} \\ & = 121 \end{aligned}$$

Multiplying Any Number by 11

To multiply any number by 11,

1. Write down the number's rightmost digit.
2. Add that digit to the digit on the left, write down the units digit, and carry over the tens digit if it is greater than 1.
3. Proceed to the next digit and repeat the process all over.
4. Once you reach the leftmost digit, write down that digit.

Example. Evaluate 54321×11

$$54321 \times 11 = \underline{5} (5 + 4) (4 + 3) (3 + 2) (2 + 1) \underline{1} = 597531$$

Example. Evaluate 62473×11

$$\begin{aligned}62473 \times 11 &= \underline{6} \underline{(6+2)} \underline{(2+4)} \underline{(4+7)} \underline{(7+3)} \underline{3} \\ &= \underline{6} \underline{8} \underline{6} \underline{(4+7)} \underline{(7+3)} \underline{3} \\ &= \underline{6} \underline{8} \underline{7} \underline{1} \underline{(7+3)} \underline{3} \\ &= 687203\end{aligned}$$

Exercise. Evaluate the following

1. 111×11

$$\begin{aligned}&= \underline{1} \underline{(1+1)} \underline{(1+1)} \underline{1} \\ &= 1221\end{aligned}$$

4. 4389×11

$$\begin{aligned}&= \underline{4} \underline{(4+3)} \underline{(3+8)} \underline{(8+9)} \underline{9} \\ &= \underline{4} \underline{(7+1)} \underline{(1+1)} \underline{7} \underline{9} \\ &= 48279\end{aligned}$$

2. 345×11

$$\begin{aligned}&= \underline{3} \underline{(3+4)} \underline{(4+5)} \underline{5} \\ &= 3795\end{aligned}$$

5. 72831×11

$$\begin{aligned}&= \underline{7} \underline{(7+2)} \underline{(2+8)} \underline{(8+3)} \underline{(3+1)} \underline{1} \\ &= \underline{7} \underline{(9+1)} \underline{(0+1)} \underline{1} \underline{4} \underline{1} \\ &= \underline{(7+1)} \underline{0} \underline{1} \underline{1} \underline{4} \underline{1} \\ &= 801141\end{aligned}$$

3. 2359×11

$$\begin{aligned}&= \underline{2} \underline{(2+3)} \underline{(3+5)} \underline{(5+9)} \underline{9} \\ &= \underline{2} \underline{5} \underline{(8+1)} \underline{4} \underline{9} \\ &= 25949\end{aligned}$$

6. 9527136×11

$$\begin{aligned}&= \underline{9} \underline{(9+5)} \underline{(5+2)} \underline{(2+7)} \underline{(7+1)} \underline{(1+3)} \underline{(3+6)} \\ &\quad \underline{6} \\ &= \underline{(9+1)} \underline{4} \underline{7} \underline{9} \underline{8} \underline{4} \underline{9} \underline{6} \\ &= 104798496\end{aligned}$$

Multiplication with 9, 99, 999, ...

Multiplying numbers with 9, 99, or 999 can seem like a hassle but we can multiply things more easily by taking advantage of the **distributive property** of multiplication.

The distributive property says that when you multiply an addition or multiplication expression by a number, you must multiply all numbers in the bracket by the multiplying values.

$$a \times (b + c) = a \times b + a \times c$$

Example. Evaluate 44×9

We know that $9 = 10 - 1$

$$\begin{aligned} 44 \times 9 &= 44 \times (10 - 1) \\ &= 44 \times 10 - 44 \times 1 \\ &= 440 - 44 \\ &= 440 - 40 - 4 \\ &= 396 \end{aligned}$$

This trick can also be described by, multiply by 10 then subtract the original number.

Example. Evaluate 68×99

$$\begin{aligned} 68 \times 99 &= 68 \times (100 - 1) \\ &= 68 \times 100 - 68 \times 1 \\ &= 6800 - 68 \\ &= 6800 - 60 - 8 \\ &= 6732 \end{aligned}$$

This trick can be described by, multiply by 100 then subtract the original number.

This trick continues for all other numbers composed of all nines. Just add one to the number with all nines to get an easy number to multiply, multiply, then subtract the original non-nine number.

This is known as the distributive property as you are writing the number you are multiplying by as two parts,

$$99 = 100 - 1$$

then **distributing** the original number to both parts by multiplying each one by it,

$$\begin{aligned}42 \times 99 &= 42 \times (100 - 1) \\ &= (42 \times 100) - (42 \times 1) \\ &= 4200 - 42 \\ &= 4158\end{aligned}$$

Exercise. Evaluate the following

$$\begin{aligned}1. \quad 18 \times 9 &= 18 \times (10 - 1) \\ &= 18 \times 10 - 18 \times 1 \\ &= 180 - 18 \\ &= 180 - 10 - 8 \\ &= 162\end{aligned}$$

$$\begin{aligned}5. \quad 13 \times 99 &= 13 \times (100 - 1) \\ &= 13 \times 100 - 13 \times 1 \\ &= 1300 - 13 \\ &= 1300 - 10 - 3 \\ &= 1287\end{aligned}$$

$$\begin{aligned}2. \quad 45 \times 9 &= 45 \times (10 - 1) \\ &= 45 \times 10 - 45 \times 1 \\ &= 450 - 45 \\ &= 450 - 40 - 5 \\ &= 405\end{aligned}$$

$$\begin{aligned}6. \quad 112 \times 99 &= 112 \times (100 - 1) \\ &= 112 \times 100 - 112 \times 1 \\ &= 11200 - 112 \\ &= 11200 - 100 - 10 - 2 \\ &= 11088\end{aligned}$$

$$\begin{aligned}3. \quad 93 \times 99 &= 93 \times (100 - 1) \\ &= 93 \times 100 - 93 \times 1 \\ &= 9300 - 93 \\ &= 9300 - 90 - 3 \\ &= 9207\end{aligned}$$

$$\begin{aligned}7. \quad 178 \times 999 &= 178 \times (1000 - 1) \\ &= 178 \times 1000 - 178 \times 1 \\ &= 178000 - 178 \\ &= 178000 - 100 - 70 - 8 \\ &= 177822\end{aligned}$$

$$\begin{aligned}4. \quad 78 \times 99 &= 78 \times (100 - 1) \\ &= 78 \times 100 - 78 \times 1 \\ &= 7800 - 78 \\ &= 7800 - 70 - 8 \\ &= 7722\end{aligned}$$

$$\begin{aligned}8. \quad 24 \times 999 &= 24 \times (1000 - 1) \\ &= 24 \times 1000 - 24 \times 1 \\ &= 24000 - 24 \\ &= 24000 - 20 - 4 \\ &= 23976\end{aligned}$$

Multiplying 2 Digit Numbers Up to 100

Example. Evaluate 21×31

1. Multiply the **first digit** of the **first number** by the **first digit** of the **second number**.
This number becomes the **first digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \underline{2 \times 3 = 6} \\ \text{6} _ _ \end{array}$$

6 is the first digit

2. Multiply the **last digit** of the **first number** by the **last digit** of the **second number**.
This number becomes the **last digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \underline{1 \times 1 = 1} \\ \text{6} _ \text{1} \end{array}$$

1 is the last digit

3. Multiply the **inner digits** and **outer digits** of the two numbers and add them. This sum becomes the **middle digit**.

$$\begin{array}{r} \text{21} \times \text{31} \\ \underline{1 \times 3 + 2 \times 1 = 5} \\ \text{6} \text{5} \text{1} \end{array}$$

Example. 42×63

1. We multiply the **first digit** of the **first number** by the **first digit** of the **second number**.

$$\begin{array}{r} \text{42} \times \text{63} \\ \text{4} \times \text{6} = \text{24} \\ \text{24 is the first two digit} \\ \text{24} \quad _ _ \end{array}$$

2. We multiply the **last digit** of the **first number** by the **last digit** of the **second number**.

$$\begin{array}{r} \text{42} \times \text{63} \\ \text{2} \times \text{3} = \text{6} \\ \text{6 is the last digit} \\ \text{24} \quad _ \quad \text{6} \end{array}$$

3. Multiply the **inner digits** and **outer digits** of the two numbers and add them. This sum becomes the **middle digit**. However, since 24 is greater than 10, the 2 from the tens column must be added to the first number.

$$\begin{array}{r} \text{42} \times \text{63} \\ \text{2} \times \text{6} + \text{4} \times \text{3} \\ = \text{12} + \text{12} \\ = \text{24} \\ \text{24} \quad \text{24} \quad \text{6} \end{array}$$

We can only have a single digit in the middle so we have to carry the 2 over

$$\begin{array}{r} \text{24} + \text{2} \quad \text{4} \quad \text{6} \\ \text{26} \quad \text{4} \quad \text{6} \end{array}$$

We carry the 2 over by adding it to 24

Exercise. Evaluate the following

1. 21×22

$$\begin{aligned} &= \underline{(2 \times 2)} \underline{(1 \times 2)} + \underline{(2 \times 2)} \underline{(1 \times 2)} \\ &= \underline{4} \underline{(2 + 4)} \underline{2} \\ &= 462 \end{aligned}$$

2. 25×31

$$\begin{aligned} &= \underline{6} \underline{(15 + 2)} \underline{5} \\ &= \underline{(6 + 1)} \underline{7} \underline{5} \\ &= 775 \end{aligned}$$

3. 34×13

$$\begin{aligned} &= \underline{(3 \times 1)} \underline{(4 \times 1)} + \underline{(3 \times 3)} \underline{(4 \times 3)} \\ &= \underline{3} \underline{(4 + 9)} \underline{12} \\ &= \underline{(3 + 1)} \underline{(3 + 1)} \underline{2} \\ &= 442 \end{aligned}$$

4. 42×24

$$\begin{aligned} &= \underline{(4 \times 2)} \underline{(2 \times 2)} + \underline{(4 \times 4)} \underline{(2 \times 4)} \\ &= \underline{8} \underline{(4 + 16)} \underline{8} \\ &= \underline{(8 + 2)} \underline{0} \underline{8} \\ &= 1008 \end{aligned}$$

5. 65×14

$$\begin{aligned} &= \underline{(6 \times 1)} \underline{(5 \times 1)} + \underline{(6 \times 4)} \underline{(5 \times 4)} \\ &= \underline{6} \underline{(5 + 24)} \underline{20} \\ &= \underline{(6 + 2)} \underline{(9 + 2)} \underline{0} \\ &= \underline{8 + 1} \underline{1} \underline{0} \\ &= 910 \end{aligned}$$

6. 87×53

$$\begin{aligned} &= \underline{(8 \times 5)} \underline{(7 \times 5)} + \underline{(8 \times 3)} \underline{(7 \times 3)} \\ &= \underline{40} \underline{(35 + 24)} \underline{21} \\ &= \underline{(40 + 5)} \underline{(9 + 2)} \underline{1} \\ &= \underline{(45 + 1)} \underline{1} \underline{1} \\ &= 4611 \end{aligned}$$

Problem Set

1. Try the warm-up questions again. See if you can do them faster than you did before!

2. Multiplying by 5

$$\begin{aligned} \text{(a)} \quad 6 \times 5 &= 6 \times (10 \div 2) \\ &= (6 \times 10) \div 2 \\ &= 60 \div 2 \\ &= 30 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 42 \times 5 &= 42 \times (10 \div 2) \\ &= (42 \times 10) \div 2 \\ &= 420 \div 2 \\ &= 210 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 17 \times 5 &= 17 \times (10 \div 2) \\ &= (17 \times 10) \div 2 \\ &= 170 \div 2 \\ &= 85 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 99 \times 5 &= 99 \times (10 \div 2) \\ &= (99 \times 10) \div 2 \\ &= 990 \div 2 \\ &= 495 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 26 \times 5 &= 26 \times (10 \div 2) \\ &= (26 \times 10) \div 2 \\ &= 260 \div 2 \\ &= 130 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad 75 \times 5 &= 75 \times (10 \div 2) \\ &= (75 \times 10) \div 2 \\ &= 750 \div 2 \\ &= 375 \end{aligned}$$

3. Division by 5

$$\begin{aligned} \text{(a)} \quad 660 \div 5 &= 660 \times 2 \div 10 \\ &= 1320 \div 10 \\ &= 132 \end{aligned}$$

$$\begin{aligned} \text{(d)} \quad 70 \div 5 &= 70 \times 2 \div 10 \\ &= 140 \div 10 \\ &= 14 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad 110 \div 5 &= 110 \times 2 \div 10 \\ &= 220 \div 10 \\ &= 22 \end{aligned}$$

$$\begin{aligned} \text{(e)} \quad 240 \div 5 &= 240 \times 2 \div 10 \\ &= 480 \div 10 \\ &= 48 \end{aligned}$$

$$\begin{aligned} \text{(c)} \quad 145 \div 5 &= 145 \times 2 \div 10 \\ &= 290 \div 10 \\ &= 29 \end{aligned}$$

$$\begin{aligned} \text{(f)} \quad 625 \div 5 &= 625 \times 2 \div 10 \\ &= 1250 \div 10 \\ &= 125 \end{aligned}$$

4. Multiplication of 2 digit number with 11

(a) 86×11

$$= \underline{8} \underline{(8+6)} \underline{6}$$

$$= \underline{(8+1)} \underline{4} \underline{6}$$

$$= 946$$

(b) 15×11

$$= \underline{1} \underline{(1+5)} \underline{5}$$

$$= 165$$

(c) 57×11

$$= \underline{5} \underline{(5+7)} \underline{7}$$

$$= \underline{(5+1)} \underline{2} \underline{7}$$

$$= 627$$

(d) 23×11

$$= \underline{2} \underline{(2+3)} \underline{3}$$

$$= 253$$

(e) 78×11

$$= \underline{7} \underline{(7+8)} \underline{8}$$

$$= \underline{(7+1)} \underline{5} \underline{8}$$

$$= 858$$

(f) 55×11

$$= \underline{5} \underline{(5+5)} \underline{5}$$

$$= \underline{(5+1)} \underline{0} \underline{5}$$

$$= 605$$

5. Multiplication of any number with 11

(a) 1234×11

$$= \underline{1} \underline{(1+2)} \underline{(2+3)} \underline{(3+4)} \underline{4}$$

$$= 13574$$

(b) 5890×11

$$= \underline{5} \underline{(5+8)} \underline{(8+9)} \underline{(9+0)} \underline{0}$$

$$= \underline{(5+1)} \underline{(3+1)} \underline{7} \underline{9} \underline{0}$$

$$= 64790$$

(c) 4583×11

$$= \underline{4} \underline{(4+5)} \underline{(5+8)} \underline{(8+3)} \underline{3}$$

$$= \underline{4} \underline{(9+1)} \underline{(3+1)} \underline{1} \underline{3}$$

$$= \underline{(4+1)} \underline{0} \underline{4} \underline{1} \underline{3}$$

$$= 50413$$

(d) 823×11

$$= \underline{8} \underline{(8+2)} \underline{(2+3)} \underline{3}$$

$$= \underline{(8+1)} \underline{0} \underline{5} \underline{3}$$

$$= 9053$$

(e) 881×11

$$= \underline{8} \underline{(8+8)} \underline{(8+1)} \underline{1}$$

$$= \underline{(8+1)} \underline{6} \underline{9} \underline{1}$$

$$= 9691$$

(f) 2401×11

$$= \underline{2} \underline{(2+4)} \underline{(4+0)} \underline{(0+1)} \underline{1}$$

$$= 26411$$

6. Multiplication of 9, 99, 999...

(a) $26 \times 9 = 26 \times (10 - 1)$

$$= 26 \times 10 - 26 \times 1$$

$$= 260 - 26$$

$$= 260 - 20 - 6$$

$$= 234$$

(b) $89 \times 9 = 89 \times (10 - 1)$

$$= 89 \times 10 - 89 \times 1$$

$$= 890 - 89$$

$$= 890 - 80 - 9$$

$$= 801$$

$$\begin{aligned}
 \text{(c)} \quad 890 \times 9 &= 890 \times (10 - 1) \\
 &= 890 \times 10 - 890 \times 1 \\
 &= 8900 - 890 \\
 &= 8900 - 800 - 90 \\
 &= 8010
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad 72 \times 99 &= 72 \times (100 - 1) \\
 &= 72 \times 100 - 72 \times 1 \\
 &= 7200 - 72 \\
 &= 7200 - 70 - 2 \\
 &= 7128
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad 34 \times 99 &= 34 \times (100 - 1) \\
 &= 34 \times 100 - 34 \times 1 \\
 &= 3400 - 34 \\
 &= 3400 - 30 - 4 \\
 &= 3366
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad 93 \times 999 &= 93 \times (1000 - 1) \\
 &= 93 \times 1000 - 93 \times 1 \\
 &= 93000 - 93 \\
 &= 93000 - 90 - 3 \\
 &= 92907
 \end{aligned}$$

7. Product of Two Numbers

$$\begin{aligned}
 \text{(a)} \quad 83 \times 45 & \\
 &= \underline{(8 \times 4) (3 \times 4) + (8 \times 5) (3 \times 5)} \\
 &= \underline{32 (12 + 40) 15} \\
 &= \underline{(32 + 5) (2 + 1) 5} \\
 &= 3735
 \end{aligned}$$

$$\begin{aligned}
 \text{(b)} \quad 28 \times 31 & \\
 &= \underline{(2 \times 3) (8 \times 3) + (2 \times 1) (8 \times 1)} \\
 &= \underline{6 (24 + 2) 8} \\
 &= \underline{(6 + 2) 6 8} \\
 &= 868
 \end{aligned}$$

$$\begin{aligned}
 \text{(c)} \quad 45 \times 12 & \\
 &= \underline{(4 \times 1) (5 \times 1) + (4 \times 2) (5 \times 2)} \\
 &= \underline{4 (5 + 8) 10} \\
 &= \underline{(4 + 1) (3 + 1) 0} \\
 &= 540
 \end{aligned}$$

$$\begin{aligned}
 \text{(d)} \quad 82 \times 14 & \\
 &= \underline{(8 \times 1) (2 \times 1) + (8 \times 4) (2 \times 4)} \\
 &= \underline{8 (2 + 32) 8} \\
 &= \underline{(8 + 3) 4 8} \\
 &= 1148
 \end{aligned}$$

$$\begin{aligned}
 \text{(e)} \quad 35 \times 34 & \\
 &= \underline{(3 \times 3) (5 \times 3) + (3 \times 4) (5 \times 4)} \\
 &= \underline{9 (15 + 12) 20} \\
 &= \underline{(9 + 2) (7 + 2) 0} \\
 &= 1190
 \end{aligned}$$

$$\begin{aligned}
 \text{(f)} \quad 32 \times 85 & \\
 &= \underline{(3 \times 8) (2 \times 8) + (3 \times 5) (2 \times 5)} \\
 &= \underline{24 (16 + 15) 10} \\
 &= \underline{(24 + 3) (1 + 1) 0} \\
 &= 2720
 \end{aligned}$$

CHALLENGE

8. Prove the “Multiply Any Number by 11” rule for a 3 digit integer.

$$\begin{array}{r}
 \begin{array}{r}
 a \quad b \quad c \\
 + \quad \quad 1 \quad 1 \\
 \hline
 a \quad b \quad c \\
 a \quad b \quad c \quad 0 \\
 \hline
 a \quad (a+b) \quad (b+c) \quad c
 \end{array}
 \end{array}$$

9. Using the distributive law, what is another trick for multiplying by 11 that would also be useful? $11 = 10 + 1$
Multiply by 10 and then add the original number.
10. Are there any other numbers that you think are hard to multiply by? Try to find a trick that works for a number that you struggle with. *Answers will vary.*