UNIVERSITY OF WATERLOO
Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

Winter 2019 – Mar. 19/20

Exponentiation

Solutions

Blanks (in order of occurrence): *exponentiation*, *square root*, *one*, $\sqrt[3]{8^2} = \sqrt[3]{64} = 4$

"Try it yourself" #1:

1. $4^3 = 4 \times 4 \times 4 = 64$
2. $2^5 = 2 \times 2 \times 2 \times 2 \times 2 = 32$
3. $8^2 = 8 \times 8 = 64$

"Try it yourself" #2:

1. $1^2 = 1 \times 1 = 1$
2. $1^3 = 1 \times 1 \times 1 = 1$
3. $1^{2019} = \underbrace{1 \times \cdots \times 1}_{\text{2019 times}} = 1$
4. $0^2 = 0 \times 0 = 0$
5. $0^3 = 0 \times 0 \times 0 = 0$
6. $0^{2019} = \underbrace{0 \times \cdots \times 0}_{\text{2019 times}} = 0$

"Try it yourself" #3:

1. $2^1 = 2$
2. $3^1 = 3$
3. $2019^1 = 2019$

"Try it yourself" #4:

1. $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$ or 0.125
2. $5^{-2} = \frac{1}{5^2} = \frac{1}{25}$ or 0.04
3. $10^{-4} = \frac{1}{10^4} = \frac{1}{10000}$ or 0.0001

"Try it yourself" #5:

1. $\sqrt{49} = 7$ since $7^2 = 49$
2. $\sqrt{81} = 9$ since $9^2 = 81$
3. $\sqrt[3]{-27} = -3$ since $(-3)^3 = 27$
4. $\sqrt[5]{32} = 2$ since $2^5 = 32$

"Try it yourself" #6:

1. $49^{\frac{1}{2}} = \sqrt{49} = 7$
2. $81^{\frac{1}{2}} = \sqrt{81} = 9$
3. $(-27)^{\frac{1}{3}} = \sqrt[3]{-27} = -3$
4. $32^{\frac{1}{5}} = \sqrt[5]{32} = 2$

"Try it yourself" #7:

1. $2^2 \times 2^2 = 2^4 = 16$
2. $3^2 \times 3 = 3^3 = 27$
3. $5^2 \times 5^2 = 5^4 = 625$
4. $6^1 \times 6^{-1} = 6^0 = 1$

"Try it yourself" #8:

1. $(2^2)^2 = 2^4 = 16$
2. $(3^4)^{\frac{1}{2}} = 3^2 = 9$
3. $(5^2)^{-1} = 5^{-2} = \frac{1}{25}$
4. $(6^1)^{-2} = 6^{-2} = \frac{1}{36}$

End-of-lesson problems:

1. (a) $5^2 = 5 \times 5 = 25$

   (b) $3^3 = 3 \times 3 \times 3 = 27$

   (c) $4^{-1} = \frac{1}{4}$ or 0.25

   (d) $9^{\frac{1}{2}} = \sqrt{9} = 3$

   (e) $64^{-\frac{1}{3}} = \frac{1}{64^{\frac{1}{3}}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ or 0.25

   (f) $8^{-\frac{2}{3}} = \frac{1}{8^{\frac{2}{3}}} = \frac{1}{\sqrt[3]{8^2}} = \frac{1}{\sqrt[3]{64}} = \frac{1}{4}$ or 0.25

2. (a) $2^2 \times 2^3 = 2^{2+3} = 2^5$

   (b) $3^8 \times 3^{-6} = 3^{8-6} = 3^2$

   (c) $\sqrt{5} \times 5^{\frac{3}{2}} = 5^{\frac{1}{2}} \times 5^{\frac{3}{2}} = 5^{\frac{1}{2}+\frac{3}{2}} = 5^2$

   (d) $8^{19} \times \frac{1}{8^{20}} = 8^{19} \times 8^{-20} = 8^{-1}$ or $\frac{1}{8}$

   (e) $\left(11^2 \times \frac{1}{\sqrt[3]{11}} \div 11^{-\frac{2}{3}}\right) = \left(11^2 \times \frac{1}{11^{\frac{1}{3}}} \div \frac{1}{11^{\frac{2}{3}}}\right) = \left(11^2 \times 11^{-\frac{1}{3}} \times 11^{\frac{2}{3}}\right) = 11^{\frac{6}{3}-\frac{1}{3}+\frac{2}{3}} = 11^{\frac{7}{3}} \approx 269.102$

3. (a) $(2^2)^{\frac{1}{2}} = 2^{\left(2\times\frac{1}{2}\right)} = 2^1 = 2$

   (b) $(3^{-1})^{-3} = 3^{((-1)\times(-3))} = 3^3 = 27$

   (c) $\left(\left(\frac{1}{5}\right)^2\right)^{-\frac{3}{2}} = \left((5^{-1})^2\right)^{-\frac{3}{2}} = \left(5^{((-1)\times 2)}\right)^{-\frac{3}{2}} = (5^{-2})^{-\frac{3}{2}} = 5^{\left((-2)\times(-\frac{3}{2})\right)} = 5^3 = 125$

   (d) $\left(8^{\frac{1}{3}}\right)^{\sqrt[3]{27}} = \left(8^{\frac{1}{3}}\right)^3 = 8^{\left(\frac{1}{3}\times 3\right)} = 8^1 = 8$

   (e) $\left(\left(\frac{1}{\sqrt{11}}\right)^{\frac{1}{4}}\right)^{-\sqrt{16}} = \left(\left(\frac{1}{11^{\frac{1}{2}}}\right)^{\frac{1}{4}}\right)^{-\sqrt{16}} = \left(\left(11^{-\frac{1}{2}}\right)^{\frac{1}{4}}\right)^{-\sqrt{16}} = \left(11^{\left((-\frac{1}{2})\times\frac{1}{4}\right)}\right)^{-\sqrt{16}} = \left(11^{-\frac{1}{8}}\right)^{-\sqrt{16}} = \left(11^{-\frac{1}{8}}\right)^{-4} = 11^{\left((-\frac{1}{8})\times(-4)\right)} = 11^{\frac{1}{2}}$ or $\sqrt{11} \approx 3.317$

4. (a) $10^x - 10 = 9990 \implies 10^x = 10000$, so $x = 4$

   (b) $4^x = 64^2 \implies 4^x = (4^3)^2 \implies 4^x = 4^6$, so $x = 6$

   (c) $3^6 = 27^x \implies 3^6 = (3^3)^x \implies 3^6 = 3^{3x} \implies 3x = 6$, so $x = 2$

   (d) $5^{33} = 125^x \implies 5^{33} = (5^3)^x \implies 5^{33} = 5^{3x} \implies 3x = 33$, so $x = 11$

   (e) $6^{x+2} = 216 \implies 6^{x+2} = 6^3 \implies x + 2 = 3$, so $x = 1$

   (f) $8^{x-1} = 2^6 \implies (2^3)^{x-1} = 2^6 \implies 2^6 = 2^{3x-3} \implies 3x - 3 = 6$, so $x = 3$

5. (a) If $x = -3$,
$$\begin{aligned} x^2 - 9 &= (-3)^2 - 9 \\ &= 9 - 9 \\ &= 0 \end{aligned}$$

(b) If $x = 2$,
$$\begin{aligned} x^2 - 2x + 1 &= (2)^2 - 2(2) + 1 \\ &= 4 - 4 + 1 \\ &= 1 \end{aligned}$$

(c) If $x = 4$,
$$\begin{aligned} x^2 + 4x - 5 &= (4)^2 + 4(4) - 5 \\ &= 16 + 16 - 5 \\ &= 27 \end{aligned}$$

(d) If $x = -1$,
$$\begin{aligned} x^3 + x^2 + x + 1 &= (-1)^3 + (-1)^2 + (-1) + 1 \\ &= -1 + 1 - 1 + 1 \\ &= 0 \end{aligned}$$

(e) If $x = 0$,
$$\begin{aligned} 3x^4 - 5x^3 + x^2 + 12x - 8 &= 3(0)^4 - 5(0)^3 + (0)^2 + 12(0) - 8 \\ &= 0 + 0 + 0 + 0 - 8 \\ &= -8 \end{aligned}$$

(f) If $x = 5$,
$$\begin{aligned} 2x^5 - 3x^4 + 31x^3 - 8x^2 + x - 10 &= 2(5)^5 - 3(5)^4 + 31(5)^3 - 8(5)^2 + (5) - 10 \\ &= 2(25 \times 125) - 3(25 \times 25) + 31(125) - 8(25) + 5 - 10 \\ &= 2(3125) - 3(625) + 31(125) - 8(25) + 5 - 10 \\ &= 6250 - 1875 + 3875 - 200 + 5 - 10 \\ &= 8045 \end{aligned}$$

6. The only two distinct positive integers that satisfy this equation are 2 and 4.