Intermediate Math Circles
Fall 2019
Fun With Inequalities

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What Happened Last Weeks?

- We looked at sets, interval notation, bracket notation, and representing interval on real number line
- We looked at the definition of “less than or equal to”
- We looked at some properties of “less than or equal to”
- We proved one of them
- We used those properties to help us solve linear inequalities with one variable (Algebraically and representation on number line)
- We solved word problems using linear inequalities with one variable

What Happened Last Weeks?

- We went over graphing equalities and inequalities on a number line with a single variable
- We looked at solving Absolute Value Inequalities (Single Variable) and Rational Inequalities (Single Variable).

Plan for Week 3

- Graph two variable linear inequalities.
- Prove some properties of the Absolute Value Function.
- Discuss some complex word problems using linear inequalities covered so far.

Last Absolute Value Question

Solve $|x - 3| + |x + 4| > 9$ algebraically and graphically.

Lines

- Lines contain infinitely many line segments
- Slope
  - Slope measures steepness and direction of a line (upward or downward)
  - Given $A(x_1, y_1)$ and $B(x_2, y_2)$ where $x_1 \neq x_2$

$$\text{slope} = m = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{y_1 - y_2}{x_1 - x_2}$$

  - Slope of a horizontal line is 0
  - Slope of a vertical line is undefined
  - When lines are parallel $m_1 = m_2$
  - When lines are perpendicular $m_2 = -\frac{1}{m_1}$

Equations of Lines

- Horizontal Line: $y = k$ where $k \in \mathbb{R}$
- Vertical Line: $x = h$ where $h \in \mathbb{R}$
- Slope-intercept Equation: $y = mx + b$
- General Equation: $Ax + By + C = 0$
- Intersection Points

  Case 1: no points of intersection
  Case 2: one point of intersection
  Case 3: infinitely many points of intersection
  (i.e. they are collinear)

Graphing linear equations:

The graph of a linear equation in two variables is a line (that's why they call it linear). We outline the procedure of graphing linear equations as follows:

Procedure:

1. Find two solutions, corresponding to the x -intercepts (by setting $y = 0$) and y -intercepts (by setting $x = 0$) of the graph
2. Plot these two points and draw the line connecting them

Practice

Graph the line $2x + 3y = 6$.

Graphing linear inequalities with two variables

Procedure

1. Rearrange the equation so "y" is on the left and everything else on the right
2. Plot the "$y =$" line (make it a solid line for $y \leq$ or $y \geq$, and a dashed line for $y <$ or $y >$)
3. Shade above the line for a "greater than" ($y >$ or $y \geq$) or below the line for a "less than" ($y <$ or $y \leq$)

Practice

Graph the line $y \leq 2x - 1$.

Graphing Systems of Linear Inequalities

Procedure

1. Change your inequality to equality
2. Graph that equation
3. Finally, pick one point that is not on either line ((0,0) is usually the easiest) and decide whether these coordinates satisfy the inequality or not. If they do, shade the half-plane containing that point. If they don't, shade the other half-plane.
4. Graph each of the inequalities in the system in a similar way. The solution of the system of inequalities is the intersection region of all the solutions in the system.

Practice

Graph the region that satisfies all three of these inequalities

$$3x - y \leq 12$$

$$x + y < 5$$

$$x - 2y > 4$$

i.e. graph the region that satisfies

$$3x - y \leq 12 \cap x + y < 5 \cap x - 2y > 4$$

$\leq, \geq \implies$ Graphical representation [thick line, points in line are included in the solution set]

$<, > \implies$ Graphical representation [dashed line, points in line are excluded in the solution set]

Review [Absolute Value Function]

What is *absolute value*?

Definition

The *absolute value* $|b|$ of a real number b is defined to be b if b is positive or zero, and to be $-b$ if b is negative.

What does this look like in "math speak"?

$$|b| = \begin{cases} b & \text{if } b \geq 0 \\ -b & \text{if } b < 0 \end{cases}$$

Another cool way of expressing *absolute value* is as follows

$$|b| = \sqrt{b^2}$$

Proving some properties of the Absolute Value Function

Challenge

Prove the following:

1. $|-x| = x$
2. $|x| - |y| \leq |x - y|$
3. $||x| - |y|| \leq |x - y|$

Triangle Inequality

1. A triangle can be formed having side lengths 4, 5 and 8. It is impossible however, to construct a triangle with side lengths 4, 5 and 10. Using the side lengths 2, 3, 5, 7 and 11, how many different triangles *with exactly two equal sides can be formed*?

References

- https://www.varsitytutors.com/hotmath/hotmath_help/topics/graphing-linear-equations
- https://www.varsitytutors.com/hotmath/hotmath_help/topics/graphing-systems-of-linear-inequalities
- https://www.mathsisfun.com/algebra/graphing-linear-inequalities.html

Thank you!