Problem Set 1

Intermediate Math Circles Fall 2019
Fun With Inequalities

Linear Inequalities - Single Variable

Solve each of the following.

1. $x + 5 < \frac{7}{2}$

   Solution: $x + 5 < \frac{7}{2}$

   $x < \frac{7}{2} - 5$

   $x < \frac{7}{2} - \frac{10}{2}$

   $x < \frac{-3}{2}$

   Therefore $x < \frac{-3}{2}$ satisfies the inequality.

2. $3 - \frac{x}{2} \geq -8$

3. $-1 - 3x \leq 4x + 10$

4. $2x + 5 > 4x - 7$

5. $-\frac{2}{3}x + \frac{3}{7} \leq 5 - \frac{x}{2}$

Properties

1. Which of the eight properties of $\leq$ also hold for $<$?

2. Use whichever of the properties (1) to (8) that you need to prove the following

   (a) If $a \leq b$ and $c \leq d$, then $a + c \leq b + d$.

   (b) If $0 \leq a \leq b$ and $0 \leq c \leq d$, then $0 \leq ac \leq bd$.

3. (a) If $a \leq b$ and $c \leq d$, is it true that $ac \leq bd$?

   (b) If $a \leq b$, is it true that $\frac{1}{b} \leq \frac{1}{a}$?

4. Show that if $a < b$, then $a < \frac{1}{2}(a+b) < b$.

5. Show that the sum of a positive number and its reciprocal is at least 2. In other words show that
$$a + \frac{1}{a} \geq 2$$

6. If $a \geq b$ and $c \geq d$, is it true that $a - c \geq b - d$?

Word Problems

1. Write this mathematical sentence using algebra:
6 is subtracted from a number x. Then the result is multiplied by 4. The final answer is less than or equal to 15.
A. $6(x-4) \leq 15$
B. $4(x-6) < 15$
C. $4(x-6) \leq 15$
D. $4x - 6 \leq 15$

2. Sandy and Mandy play in the same soccer team. Last Saturday Sandy scored 4 more goals than Mandy, but together they scored less than 12 goals. What are the possible number of goals Sandy scored?

3. Of 12 puppies, there are more girls than boys. How many girl puppies could there be? We assume that there is at least one boy puppy.

4. Alex went to the carnival with 27.50 dollars. He bought a hot dog and a drink for 6.50 dollars, and he wanted to spend the rest of his money on ride tickets which cost 1.50 dollar each. What is the maximum number of ride tickets that he can buy?