Intermediate Math Circles
October 16, 2019
Problem Set Counting Part II

1. If 50 different students try out for a team of 30 players, in how many different ways can the coach choose the team?

2. How many groups can be formed from 8 adults and 5 children if:

   a.) the group must have exactly 2 adults and 2 children?

   b.) the group can be any size, but must have at least one member and an equal number of adults and children?

3. How many groups containing seven different numbers can be formed by selecting the numbers from the set $\{1, 2, \ldots, 20\}$ such that

   a.) 19 is the largest number in the group?

   b.) 9 is the middle number in the group?

   c.) the difference between the largest and smallest number in the group is equal to 14?

4. With a standard deck of 52 cards, a subset of 5 cards is called a hand.

   a.) How many hands are there?

   b.) How many hands contain exactly one pair? (2 of a kind and 3 different cards)

   c.) How many hands have 4 of a kind?

5. Evaluate the following **without** the use of a calculator.

   a) $\dfrac{\binom{7}{3}}{\binom{7}{4}}$ b) $\dfrac{\binom{12}{8}}{\binom{9}{4}}$ c) $\dfrac{\binom{n}{3}}{\binom{n}{2}}$, $n \geq 3$.

6.   a) How many non-negative integer solutions are there for the equation $x + y + z = 10$?

   b) Given the equation $x + y + z + w = 20$ where x, y, z, w are integers, $x \geq -2$, $y \geq -1$, $z \geq 0$, $w \geq 1$, determine the number of different solutions.

Answers are provided on the reverse side of this page. If you disagree with an answer, try again. Next week, difficulties can be taken up in class.

Full solutions will be posted on our website by the end of this week.

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Answers Only

1. You can make the selection in $\binom{50}{30}$ ways.

2. a.) There are 280 ways to make a group that has exactly 2 adults and 2 children.

   b.) There are 1 286 ways to make a group that has an equal number of adults and children.

3. a.) There are 18 564 ways to make a group of seven from this number set so that 19 is the largest number in the group.

   b.) There are 9 240 ways to make a group of seven where 9 is the middle number in the group.

   c.) There are 7 722 ways to make a group of seven in which the difference between the largest value and smallest value is 14.

4. a.) There are 2 598 960 hands.

   b.) There are 1 098 240 hands that contain exactly one pair and three other cards, each with a different value.

   c.) There are 624 hands that contain 4 of a kind.

5. a) 1

   b) $\dfrac{55}{14}$

   c) $\dfrac{n-2}{3}$

6. a) 66

   b) 2300