



**Grade 6 Math Circles**  
February 4/5 2020  
*Mental Math and Conversions*

Calculators away! We will not be using them for this lesson.

## Adding in Parts

General Rule: Add from the largest place values to the smallest place values (ie. Move from left to right). To do this, we expand each number, add all of our numbers together that have the same number of digits, and then add the sums of these answers up.

### Adding Examples:

1.  $32 + 44$

(a) Expanding these numbers, we get...

(b) Add the 2-digit numbers first:

(c) Add the 1-digit numbers:

(d) Add the sums from b and c:

2.  $1355 + 206$

(a) Expanding these numbers, we get...

(b) Add the 4-digit numbers first:

(c) Add the 3-digit numbers:

(d) Add the 2-digit numbers:

(e) Add the 1-digit numbers:

(f) Add the sums from b,c,d and e:

Exercises:

1.  $3561 + 1223$

2.  $6577 + 5643$

## Multiplying In Parts

For multiplying big numbers in your head, it is important to know your times tables (up to  $12 \times 12$ ) very well.

General Rule for multiplying two digit numbers and beyond: Like adding and subtracting, we will multiply from left to right. Multiply the largest place value of the first number by EACH place value in the second number and move right until you have multiplied the smallest place value in the first number by EACH place value in the second number. This makes more sense when we think of multiplication as grouping.

Example:

1.  $32 \times 46$

Exercises:

1. Multiply your age by 62

2.  $321 \times 505$

3.  $45 \times 2.2$

# Divisibility Tricks

How can we tell if a number is divisible by another number?

The following tricks are useful in determining if a number is divisible by different digits:

1. Is your number divisible by 2?

- If your number is an even number, then yes!
  - (a) Is my number an even number? If the last digit is even, the whole number is even!
- Example: Is 572 divisible by 2?

2. Is your number divisible by 3?

- Add up all the digits in your number. If this sum is divisible by 3, your number is also divisible by 3.
- Example: Is 755 divisible by 3?

3. Is your number divisible by 4?

- If the number formed by the last two digits of your number is divisible by 4, so is your number.
- Example: Is 612 divisible by 4?

4. Is your number divisible by 5?

- If your number ends in a 5 or 0, it is divisible by 5.
- Example: Is 98 divisible by 5?

5. Is your number divisible by 6?

- If your number is divisible by 2 AND 3, it is divisible by 6.
- Example: Is 6 a factor of 4323?

6. Is your number divisible by 7?

- Double the last digit in your number.
- Subtract this value from the rest of your digits.
- If the resulting number is divisible by 7, your original number is divisible by 7.
- Can repeat if resulting number still too large.
- Example: Is 2331 divisible by 7?

7. Is your number divisible by 8?

- If the number formed by the last three digits are divisible by 8, yes.
- If the number is odd, it will not be divisible by 8.
- Example: Is 52,016 divisible by 8?

8. Is your number divisible by 9?

- Add up all the digits in the number, if the sum is divisible by 9, yes.
- Example: Is 7222 divisible by 9?

9. Is your number divisible by 10?

- If it ends in a zero, yes.
- Example Is 56,785 divisible by 10?

Exercise:

1. List all the numbers from 1 to 10 that are factors of 2346.

# Dividing in Parts

$$\begin{array}{ccc} 96 & \div & 2 = 48 \\ \uparrow & & \uparrow \\ \text{dividend} & & \text{divisor} \quad \text{quotient} \end{array}$$
  
$$\begin{array}{r} 48 \leftarrow \text{quotient} \\ \text{divisor} \rightarrow 2 \overline{)96} \leftarrow \text{dividend} \end{array}$$

Retrieved from [digitalpreprod.totvs.com](http://digitalpreprod.totvs.com)

## Splitting the dividend:

If we split the dividend into two or more numbers that add up to the dividend and can more easily be divided by the divisor, we can just divide each of these two (or more) numbers by the divisor and **add** up our results. This will give us the quotient.

## Example:

1.  $210 \div 3$

## Exercises:

1.  $472 \div 4$

2.  $180 \div 45$

## Splitting the Divisor:

If we split our divisor into two or more numbers that multiply to give us our divisor and that we are more easily able to divide into the dividend, we can just divide the dividend by one of these numbers and divide this result by our other number. This will give us the quotient.

## Example:

1.  $654 \div 6$

Exercises:

1.  $520 \div 8$

2.  $288 \div 12$

Dividing by Multiplying:

When dividing with large numbers, it can be helpful to start with one number to multiply the divisor by. Then, depending on how close your answer is to your dividend, multiply your divisor by a new number. Repeat the process until you get an answer that is very close to your dividend. Once this happens, you know that the number you multiplied your divisor by is approximately your quotient. You can then take your remainder.

Example:

1.  $4593 \div 54$

Exercises:

1.  $1452 \div 44$

2.  $2366 \div 23$

# Squaring Numbers Trick

Quick Review: Squaring numbers is when we multiply a number by itself

$$4^2 = 4 \times 4 = 16$$

Say we want to square the number 408. How do we do this?

1. Pick a number close to 408 that is easier to multiply with:
2. To get from 408 to 400, we went down by 8, so now we must go up by 8:
3. Multiply  $400 \times 416$
4. Now, since we went up and down by 8, we square 8
5. Add 64 to 166,400
6. So  $408^2 =$

## Exercise:

1. What is the square of 598?
2. What is the square of 709?

# Useful Conversions

Different units can be used to describe the same quantities. For example...

- metres, centimetres, millimetres, kilometres and miles can all be used to describe **distance**.
- seconds, minutes, hours, days and years can all be used to describe **time**.
- mg, g, kg and lbs can all be used to represent **mass**.

There also exists different systems of measurement in different countries. In Canada we use the metric system, whereas in the United States they use the imperial system of measurement.

Why do we want to be able to convert between units?

- So that we can correctly use equations.
  - For example, if you know distance (d) in metres and time (t) in hours and you want to find velocity (v), where  $v = \frac{d}{t}$  in m/s or km/h, you want to be able to convert either metres to kilometres or hours to seconds in order to calculate a velocity that we can understand (m/h is not common therefore it is hard to understand just how fast this is).
- So that we can understand what Americans are talking about.

Converting between Fahrenheit and Celsius:

When an American tells you the temperature.....but they tell you in Fahrenheit...

1. Subtract 32 from the temperature in Fahrenheit.
2. Multiply your result by 5.
3. Divide this by 9.



Example:

1. An American tells you it is  $78^{\circ}$  F outside, what does that mean??? Let's convert to Celsius.

Converting between pounds and kilograms:

$$1 \text{ kg} \approx 2.2 \text{ lbs}$$

So to convert from kg to lbs, we multiply our number by 2.2.

To convert from lbs to kg, we divide our number by 2.2.

Example:

1. You buy 4.41 kg of apples at the superstore. The price for apples is \$3/lbs. How much money will you have to pay? (Do not include tax).

Converting between miles and kilometres:

1 mile  $\approx$  1.6 km

So to convert to from miles to km, multiply the number of miles by 1.6. To convert from km to miles, divide the number of miles by 1.6.

Example:

1. You see a sign while driving in Chicago that indicates that the speed limit is 35 miles/hr, but your Canadian car shows you your speed in km/hr. Convert the speed limit to km/hr.

Converting between inches and cm:

1 inch = 2.54 cm

So to convert from inches to cm, multiply the number of inches by 2.54.

To convert from cm to inches, divide the number of cm by 2.54.

Example:

1. You buy a box in the United States with dimensions 11 in  $\times$  11 in  $\times$  11 in, you want to know what the dimensions are in cm. What are they?

## Problem Set

The following questions should be done in your head with minimal writing on the page, unless otherwise specified.

\* indicates challenge questions.

1. Which of the following numbers is a multiple of 9? ie. Which number is divisible by 9? (From grade 7 Gauss contest)  
(A) 50                      (B) 40                      (C) 35                      (D) 45  
(E) 55
2. Write down all of the numbers from 1 to 10 that are factors of...  
(A) 651                      (B) 6654                      (C) 8740                      (D) 3672
3. How many positive whole numbers, including 1, divide exactly into both 40 and 72? (From 2007 grade 7 Gauss contest)
4. A regular hexagon is a shape with six sides. Each side has the same length and this length is a whole number. The perimeter (total of all six side lengths) of this hexagon *cannot* equal  
(A) 1992                      (B) 2322                      (C) 3454                      (D) 4542
5. \*The positive integer  $n$  has exactly 8 positive divisors including 1 and  $n$ . Two of these divisors are 14 and 21. What is the sum of all 8 positive divisors of  $n$ ? It will be helpful to write on a piece of paper for this question to keep track of your numbers. Hint: If 14 and 21 are factors of  $n$ , factors of 14 and 21 are also factors of  $n$ . (From 2019 grade 7 Gauss contest).
6. \*Each of the integers 334 and 419 has digits whose product is 36. How many 3-digit positive integers have digits whose product is 36? You will need to write this one down. Hint: Start by finding all 1 digit numbers that are factors of 36. (From 2019 grade 7 Gauss contest)
7. Calculate.... (In your head!!)  
(A)  $56 + 98$                       (B)  $683 + 255$                       (C)  $974 + 665$                       (D)  $1234 + 5678$   
(E)  $6890 + 4388$

8. The following T-charts show the points scored by two different basketball teams over the course of 10 games. The team with the most overall points at the end of the season wins a vacation to Hawaii. Which team has the most overall points so far?

<u>The Lions</u>		<u>The Tigers</u>	
Game	Points	Game	Points
1	77	1	88
2	82	2	83
3	103	3	59
4	91	4	76
5	68	5	67
6	71	6	99
7	78	7	111
8	77	8	73
9	97	9	89
10	84	10	90

9. Calculate: (in your head!!!)

(A)  $23 \times 9$       (B)  $32 \times 76$       (C)  $445 \times 65$       (D)  $777 \times 234$   
(E)  $7638 \times 2948$       (F)  $9283 \times 2783$

10. The fence that encloses my yard has a length of 33 m and a width of 56 m. What is the area of my yard?

11. Calculate: (in your head!)

(A)  $150 \div 3$       (B)  $819 \div 9$       (C)  $132 \div 4$       (D)  $328 \div 8$   
(E)  $5690 \div 50$       (F)  $3456 \div 64$

12. A regular octagon is a shape with eight sides. All side lengths are equal. If the perimeter of this octagon is 216 cm, what is the length of each side of the octagon?

13. Find the missing number:  $12 \times 24 \times 34 = 34 \times 36 \times \underline{\hspace{2cm}}$

14. A goat, a horse and a giraffe go to Walmart. In total, they spent \$456 all together. If the goat spent one third of this total and the horse spent one fourth of this total, how much money did the giraffe spend?
15. Bobby is 13 years old. If he multiplies his age by 14, then adds 28, then divides by 7 and finally subtracts twice his age, what number does he get?
16. Square...
- (A) 14                      (B) 43                      (C) 101                      (D) 643
- (E) 989                      (F) 1203
17. Convert the following temperatures to Celsius
- (A) 75° F                      (B) 53° F                      (C) 108° F                      (D) 82° F
- (E) 32° F                      (F) 80° F
18. \*The temperature outside is 23° C, what is this temperature in Fahrenheit?
19. While at the vet, I have to write down my dog's weight in lbs on a form. I know my dog weighs 45 kg, how much does he weigh in lbs?
20. \*If I buy 32 lbs of beef at the grocery store, how many kg of beef did I buy?
21. Janice sets a goal to run 5 miles once a week. She keeps track of how many miles she has ran on her phone, but her phone tells her how many km she has ran. Janice tries to convert 5 miles to km by calculating  $5 \times 1.6$  with the following steps:
- (a)  $5 \times 1 = 5$
- (b)  $5 \times 6 = 30$
- (c)  $30 + 5 = 35$

So Janice concludes that 5 miles = 35 km and that she must run 35 km once a week (yikes!), what did Janice do wrong in her calculation?