



## Grade 6 Math Circles

February 11/12 2020

### *Applications*

## Energy

What is energy?

Energy is the ability to do work. It is a property that allows things to move.  
It takes energy for us to...



- Lift our arms.
- Jump up and down.
- Walk to school.

...and so much more.

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Some types of energy:

- **Kinetic energy:** The energy an object possesses from being in motion, because something had to transfer energy to it in order for it to move.



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- **Gravitational potential energy:** Energy that is stored due to an object being above the ground. When objects aren't moving, they can have the potential to move due to the energy stored in them. For example, a book on a shelf is not moving, but it has gravitational potential energy because if someone were to nudge it, it would fall to the ground due to gravity (therefore it would move).



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- Light energy.
- Heat energy.

Exercises:

1. Write down 2 examples of things that have kinetic energy.
  
  
  
  
  
  
  
  
  
  
2. Write down 2 examples of things that have gravitational potential energy.

## A Closer Look at Kinetic Energy

Harry Potter, 60 kg, is running away from a group snatchers in the woods at a speed of 10 m/s. When he is caught, the snatchers tell him that if he can calculate how much kinetic energy he had while running, they won't capture and bring him to Voldemort. How can Harry do this?

Calculating kinetic energy:

$$KE = \frac{1}{2} \times m \times v^2$$

Where...

$KE$  stands for kinetic energy, measured in joules (J).

$m$  is the mass in kg.

$v$  is the speed in m/s.

So, Harry must use...

$$m = 60 \text{ kg}$$

$$v = 10 \text{ m/s}$$

To calculate...

$$KE = \frac{1}{2} \times 60 \times 10^2$$

$$KE = 3000 \text{ J}$$

Exercise:

You pitch a baseball to a batter. Using a radar gun, you find that the speed at which you pitched the baseball was 27 m/s. If the baseball has a mass of 149 g, what is the kinetic energy of the baseball after you let go of the ball?

We have,

$$v = 27 \text{ m/s}$$

$$m = 0.149 \text{ kg}$$

We use,

$$KE = \frac{1}{2} \times 0.149 \times 27^2$$

To get,

$$KE = 54 \text{ J}$$

## A Closer Look at Gravitational Potential Energy

Harry Potter is in the middle of trying to retrieve an egg from a dragon for his first task in the triwizard tournament. Unfortunately, while flying on his broom he falls and finds himself hanging off the ledge of a building, 45 m above the ground. He eventually recovers and is able to retrieve the egg, but loses points for poor technique because he fell off of his broom during the task. The judges tell him that he may earn back the points he lost if he can calculate what his gravitational potential energy was while he was hanging from the building. How can Harry do this?

Calculating gravitational potential energy:

$$PE = m \times g \times h$$

Where...

$PE$  is the gravitational potential energy, measured in joules (J).

$m$  is the mass in kg.

$g$  is the gravitational constant (9.8 m/s<sup>2</sup>).

$h$  is the height of the object above the ground in meters (m).

So, Harry must use...

To calculate...

$m = 60$  kg (from the previous question)

$g = 9.8$  m/s<sup>2</sup>

$h = 45$  m

$PE = 60 \times 9.8 \times 45$

$PE = 26,460$  J

### Exercise:

You race to get on the Leviathan first thing in the morning at Canada's Wonderland. The total mass of everyone on the ride when you go on is 3360kg. When you get to the top and stop, the car is 93m above the ground. How much potential energy does the car have?

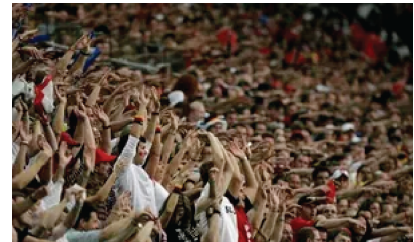


We use,  $m = 3360 \text{ kg}$   
 $h = 93 \text{ m}$   
 $g = 9.8 \text{ m/s}^2$

To get,  $PE = 3360 \times 9.8 \times 93$   
 $PE = 3,062,304 \text{ J}$

Retrieved from [www.themeparktourist.com](http://www.themeparktourist.com)

## The Wave



Retrieved from [www.sciencefocus.com](http://www.sciencefocus.com)

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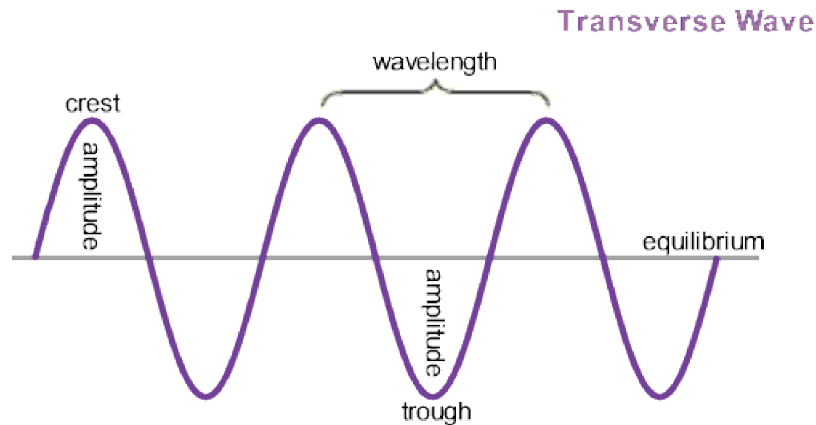
Retrieved from [www.theguardian.com](http://www.theguardian.com)

### What is a wave?

Waves are a method of energy transfer. A wave is something that travels through space and transfers energy from one place to another.

For example, light is a wave. Light travels through the air as a wave and transfers light energy to objects. Think of how plants get energy from the sun during photosynthesis. This is due to the waves of light transferring energy from the sun to the plants, so that they are able to create sugar.

What does a wave look like?



**Crest:** All the highest points (or maxima) of the wave.

**Trough:** All the lowest points (or minima) of the wave.

**Wavelength ( $\lambda$ ):** The horizontal distance between two consecutive crests or two consecutive troughs.

**Equilibrium:** The midpoint between the crests and the troughs.

**Amplitude:** The vertical height from the equilibrium line to a crest or trough.

## Frequency and Wavespeed

**Frequency ( $f$ ):** The number of times one full wavelength passes a fixed point in one second (measured in cycles/s, otherwise known as Hertz (Hz)). Frequency

$$f = \frac{\# \text{ of cycles}}{\text{Time in seconds}}$$

**Wavespeed ( $v$ ):** How fast the wave is travelling in m/s (the same way you would measure the speed of a car).

The Wave Equation

$$v = f \times \lambda$$

Where we measure  $v$  in m/s,  $f$  in Hz and  $\lambda$  in m.

This equation tells us that if we know the frequency and wavelength of the wave, we can find its wavespeed.

Example:

Your teammate is up to bat and hits a homerun! The sound wave from the bat hitting the ball has a frequency of 2000 Hz and a wavelength of 17 cm. How fast is this wave travelling?

We have  $v = f \times \lambda$ , we want to find  $v$ .

The question tells us that  $f = 2000$  Hz and  $\lambda = 0.17$  m.

$$v = 2000 \times 0.17$$

$$v = 340 \text{ m/s}$$

This is close to the speed of sound in air, 343 m/s.

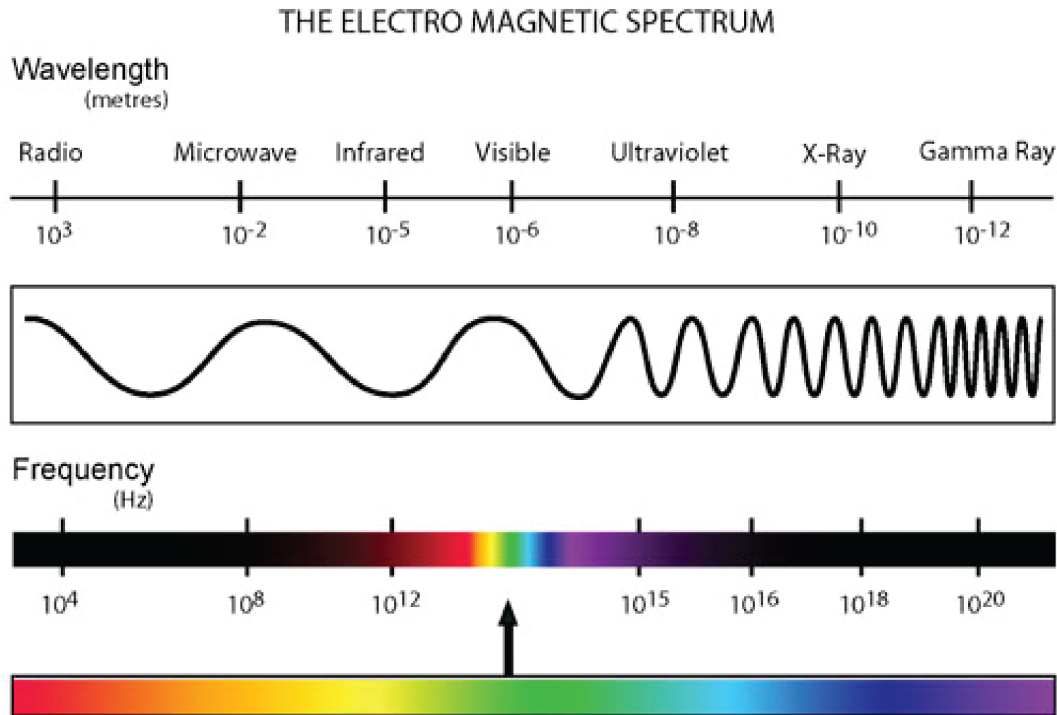
Exercise:

1. The wavelength of a beam of red light is 0.00000068 m. The frequency of this beam of light is 441,000,000,000,000 Hz. What is the speed of this light wave?

$$v = 0.00000068 \times 441,000,000,000,000$$

$$v = 299,880,000 \text{ m/s}$$

# The Electromagnetic Spectrum



Retrieved from: <https://mydarksky.org/2008/10/29/what-is-a-gamma-ray-burst-its-history/>

Here we see waves that have a large wavelength on the left, and a much smaller wavelength on the right. These are all light waves, but the type of light wave depends on the wavelength.

The visible section on the spectrum shows us the wavelengths of light that we are able to see, which is why we refer to it as visible light.

All of the other types of light here, we cannot see with our eyes. However, there are some devices that can be used to detect other types of light.

Summary Video: Waves

Exercises:

Order the visible wave colours from largest to smallest wavelength. Hint: It is the same as the order of colours in the rainbow.



Red, orange, yellow, green, blue, indigo, violet.  
A helpful acronym to remember this: ROY G BIV.

Which has the smallest wavelength: microwaves, visible waves or X-rays?

X-rays.

## Problem Set

\* Indicates challenge questions.

1. List the types of energy (kinetic or gravitational potential energy) that the following items possess:

- (a) A car driving along the road.

Kinetic energy.

- (b) A ball sitting still on the top of a hill.

Gravitational potential energy.

- (c) A ball soaring through the air.

Kinetic energy and gravitational potential energy. The ball is moving *and* it is above the ground.

- (d) A phone I hold still in my hand at eye level.

Gravitational potential energy.

- (e) A skier skiing down the middle of a hill.

Kinetic energy and gravitational potential energy. The skier is moving *and* is above ground level.

- (f) A bowling ball rolling along the alley.

Kinetic energy.

2. (a) A sprinter is running along the track at 4 m/s. If the mass of this sprinter is 70 kg, what is their kinetic energy?

Answer: 560 J

$v = 4 \text{ m/s}$ ,  $m = 70 \text{ kg}$ .

Following the kinetic energy equation,  $KE = \frac{1}{2} \times 70 \times 4$ .

$KE = 560 \text{ J}$

- (b) \* If another sprinter has a kinetic energy of 398 J and a mass of 65,000 g, how fast are they running?

Answer: 3.5 m/s

We have to rearrange the  $KE$  equation for  $v$  to find the speed.

$KE = \frac{1}{2} \times m \times v^2$ . Move everything on the right side of the equation to the left side, except  $v$ .

$2 \times KE = 2 \times \frac{1}{2} \times m \times v^2$ . This cancels out the  $\frac{1}{2}$  on the right side ( $2 \times \frac{1}{2} = 1$ ).

Now we have  $2 \times KE = m \times v^2$ .

$\frac{2 \times KE}{m} = \frac{m \times v^2}{m}$ . This cancels out the  $m$  on the right side ( $\frac{m}{m} = 1$ ).

Now we have  $\frac{2 \times KE}{m} = v^2$ .

Now we have to make  $v^2$  on the right side just  $v$ . To do this, we do the opposite operation on the left side of the equation.

We get  $\sqrt{\frac{2 \times KE}{m}} = v$ .

Plugging in our  $KE$  and  $m$  ( $m = 65 \text{ kg}$ , watch your units!!) from the question into our calculator, we get  $v = 3.5 \text{ m/s}$ .

3. (a) Mickey is standing at the top of a ladder that is 7.5 m from the ground. If Mickey has a mass of 50 kg, what is Mickey's gravitational potential energy at the top of the ladder?

Answer: 3675 J

$h = 7.5 \text{ m}$ ,  $m = 50 \text{ kg}$ ,  $g = 9.8 \text{ m/s}^2$ .

Following the gravitational potential energy equation,  $PE = 50 \times 9.8 \times 7.5$ .

$PE = 3675 \text{ J}$

- (b) Mickey throws Minnie a birthday party in which he orders 50 tons of cheesecake. As a result of this, Mickey eats a lot of leftover cheesecake everyday for multiple weeks. He gains 10 kg.

- i. How much does Mickey weigh now?

Answer: 60 kg

Mickey weighed 50 kg in part a), so if he gained 10 kg he now weighs 60 kg.

- ii. Mickey is at the top of the same ladder again. What is his new gravitational potential energy?

Answer: 4410 J

We plug our numbers into the same equation as part a), only instead of  $m = 50 \text{ kg}$ , we use  $m = 60 \text{ kg}$ .

- (c) \*Mickey signs up for a gym membership once all the cheesecake is gone. As a result, he has lost weight and is in great shape! When Mickey is at the top of his ladder again, his gravitational potential energy is 3.307 KJ. How much weight did Mickey lose since going to the gym?

Answer: 15 kg

We need to find Mickey's new weight first. To do this, we rearrange our  $PE$  equation for  $m$ .

$\frac{PE}{g} = \frac{1}{g} \times m \times g \times h$ . This cancels out  $g$  on the right side.

Now we have  $\frac{PE}{g} = m \times h$ .

$\frac{PE}{g \times h} = \frac{1}{h} \times m \times h$ . This cancels out the  $h$  on the right side.

Now we have  $\frac{PE}{g \times h} = m$ .

Now we plug in our values for  $PE$  ( $PE = 3307$  J, watch your units!),  $h$  and  $g$  from the question into our calculator and we get  $m = 45$  kg.

Since the question does not ask for Mickey's new weight, but rather how much weight he lost, we subtract 45 from his weight before going to the gym (60 kg) to get 15 kg.

4. Which of the following has more gravitational potential energy: A 0.5 kg ball sitting on top of a shelf that is 1 m above the ground, or a 0.5 kg ball sitting on top of a shelf that is 2 m above the ground?

Answer: The ball that is 2 m above the ground.

The greater the height, the more gravitational potential energy an object has. From looking at our equation for  $PE$ , we can see that increasing  $h$  will increase the number on the right side of the equation, hence increasing the number on the left side of the equation as well.

5. What happens to the gravitational potential energy when the height is doubled? (Hint: Use the equation  $PE = m \times g \times h$ ).
- (A) It is doubled.    (B) It is tripled.    (C) It is halved.    (D) Changing the height has no effect on the gravitational potential energy.

Answer: A

Whatever we do to one side of the equation, we must do to the other. If we multiply the height in our  $PE$  equation by 2 (double the height, we are multiplying the entire right side of the equation by 2. This means that we must also multiply the left side of the equation by 2. If we multiply the left side of the equation by 2, we are doubling the  $PE$ .

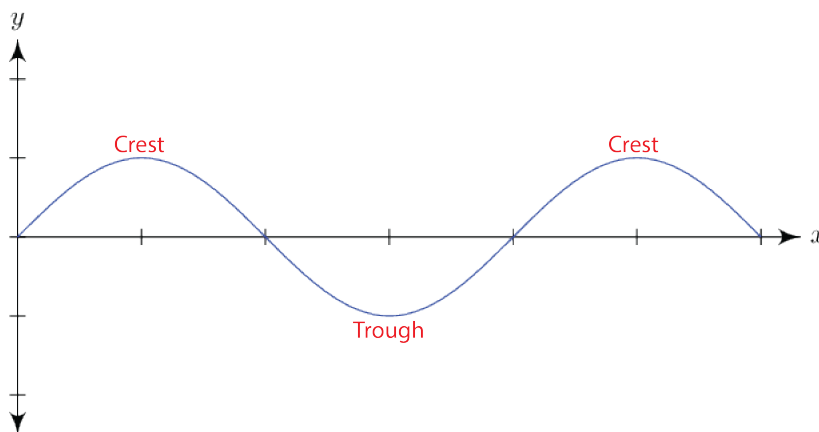
6. \*What happens to the kinetic energy when the speed is halved?
- (A) It is halved.    (B) It is doubled.    (C) It decreases by a factor of  $(\frac{1}{2})^2$ .    (D) It decreases by a factor of  $\sqrt{\frac{1}{2}}$ .

Answer: C

Looking at our  $KE$  equation, whatever we do to one side we must do to the other. When we halve the speed (divide the speed by 2), we are dividing the whole right side of the equation by  $2^2$  because the speed is squared (which is the same as multiplying by  $(\frac{1}{2})^2$ ). This means that we must also multiply the left side of the equation by  $(\frac{1}{2})^2$ . Hence, when the speed is halved, the  $KE$  is multiplied by  $(\frac{1}{2})^2$  ( $=\frac{1}{4}$ ) which causes it to decrease.

7. How many crests can be seen in the following wave? How many troughs?

Answer: 2 crests, 1 trough.



Retrieved from <https://www.shmoop.com/waves/quiz-3-true-false.html>

8. If the amplitude of a wave is 2 cm, what is the vertical height between a trough and a crest of this wave?

Answer: 4 cm

Because the amplitude is the vertical height between the equilibrium point and any crest in the wave *and* it is the vertical height between the equilibrium point and any trough in the wave, the vertical height between a crest and a trough is *two* amplitudes.

9. (a) If one full wavelength of a wave passes a fixed point 5 times in one second, what is the frequency of this wave?

Answer: 5 Hz

Because the frequency is the number of cycles per second, if the wave goes through 5 cycles in one second, then the frequency is 5 Hz.

- (b) \*If it takes one full wavelength of another wave 5 seconds to pass a fixed point 20 times, what is the frequency of this wave?

Answer: 4 Hz

Because frequency is the number of cycles per second, we can calculate the frequency using  $f = \frac{\# \text{ of cycles}}{\text{Seconds}}$ . Plugging in 20 for cycles and 5 for seconds, we get  $f = 4 \text{ Hz}$ .

10. (a) A wave has a frequency of 10 Hz and a wavelength of 80 cm, what is the wavespeed of this wave?

Answer: 8 m/s

Plug  $f = 10$  Hz and  $\lambda = 0.8$  m into our equation  $v = f \times \lambda$ .

- (b) \*Another wave has a frequency of 103 Hz and a wavespeed of 60 m/s, what is the wavelength of this wave?

Answer: 0.58 m

We have to rearrange our wave equation for  $\lambda$ .

$\frac{v}{f} = \frac{1}{f} \times f \times \lambda$ . This cancels out  $f$  on the right side.

Now we have  $\frac{v}{f} = \lambda$ . Plug in the numbers for  $v$  and  $f$  into this equation to get  $\lambda$ .

11. (a) A beam of yellow light on a traffic light tells cars when to slow down. Based on the electromagnetic spectrum, what type of light (or wave) is this?

Answer: Visible light/wave.

All of the colours we see are examples of visible light.

- (b) According to the electromagnetic spectrum, which type of wave has the largest wavelength?

Answer: Radio waves.

- (c) You detect a wave with a wavelength of  $2.0 \times 10^{-5}$  m. What type of wave do you conclude this is?

Answer: Infrared wave.

Waves with a wavelength of magnitude  $10^{-5}$  m are infrared waves, as seen on the electromagnetic spectrum.