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Grade 6 Math Circles

February 18/19 2020

Black Holes

Scientific Notation

Let's look at some exponents:

$10^2 = 10 \times 10$

$10^3 = 10 \times 10 \times 10$

$10^5 = 10 \times 10 \times 10 \times 10 \times 10$

What about 10^{-2}?

What do you notice?

What about $10^2 \times 10^3$?

What about $10^5 \div 10^3$?

What about $10^{-2} \times 10^3$?

What about $10^5 \div 10^7$?

What do you notice?

What do you notice?

What about $(10^3)^2$?

Now let's try something else. What if we have 1.1×10? What is this equal to?

So, we see that multiplying by 10^1 moves the decimal place over ONCE to the RIGHT.

What if we have 0.5×10^2?

So, we see that multiplying by 10^2 moves the decimal place over TWICE to the RIGHT.

What if we have 5×10^{-1}?

So, we see that multiplying by 10^{-1} moves the decimal place over ONCE to the LEFT.

What if we have 5×10^{-2}?

So, we see that multiplying by 10^{-2} moves the decimal place over TWICE to the LEFT.

What pattern do you notice?

Let's apply these ideas:

Multiply $(5.5 \times 10^3) \times (6.1 \times 10^4)$.

What if we have $(3.0 \times 10^8)^2$?

Exercises:

1. $(4.4 \times 10^{-4}) \times (8.6 \times 10^{9})$

2. $(2.5 \times 10^{1}) \div (1.4 \times 10^{3})$

3. $(6.67 \times 10^{-11}) \div (3.0 \times 10^{8})^{2}$

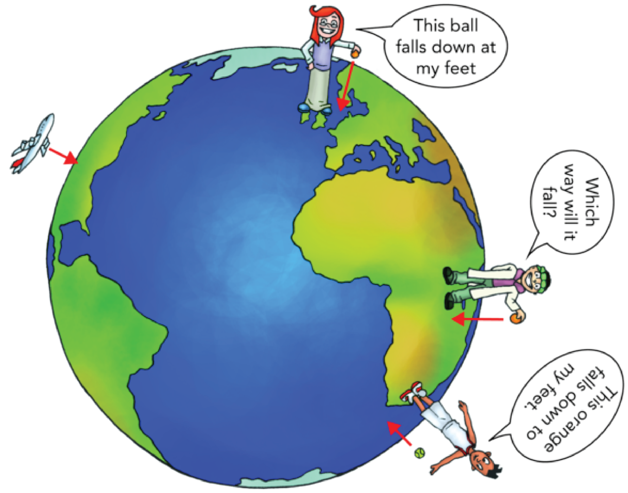
Recap:

$10^m \times 10^n =$	
$\frac{10^m}{10^n} =$	
$(10^m)^n =$	
5.0×10^m, where m is positive.	
5.0×10^{-m}, where m is positive.	

Gravity

What is gravity?

When we talk about the Earth's gravity, we describe it as a ______________________ where the Earth pulls everything towards its ___________. This is how we are able to stay on the ground rather than float, and it is also why when we throw a ball into the air it does not just keep going up forever, but rather it falls back down toward the ground.



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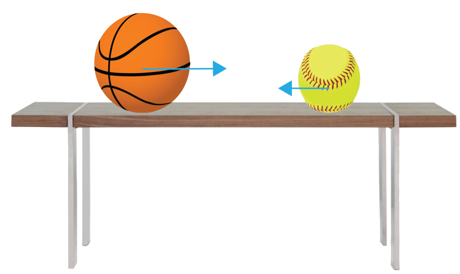
Is the Earth the only object that has gravity?

____

That is, ___________ object has a force of attraction that pulls other objects towards its centre.



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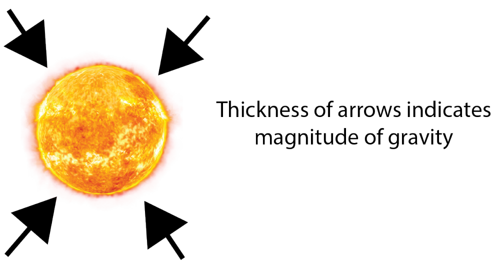
Can you list some examples of objects that have gravity?

...But wait, if all objects have gravity, why can't I see my water bottle being pulled towards my laptop when I put them both on a table close to each other? Shouldn't the water bottle be pulling my laptop towards it? Or vice versa?

The answer to this is that the water bottle actually ____________ attracting my laptop to it due to its ____________, and vice versa my laptop is also attracting my water bottle towards it due to its gravity! Why can't we see this?



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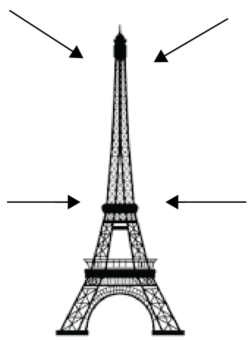
The more ____________ an object is, the ____________ its gravity will be. That is, objects with mass as great as the Earth, sun etc. have very strong forces of gravity, so we can see the effect of the Earth's gravity on a baseball when it falls to the ground easily, as the Earth's gravity is so much stronger than the baseball's.



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Objects that have much ____________ mass than the Earth, such as my laptop, water bottle or even the hugest buildings, do ____________ have enough mass for objects to actually be ____________ towards them. Although, ____________ still ____________ all other objects to it at least a little, just usually not enough for objects to actually move towards it.

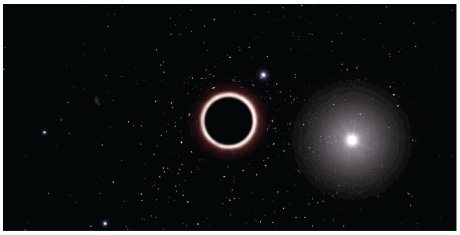
Imagine this:

- The mass of the Earth is __________________________. (That's 5972000000000000000000000 kg.)
- The mass of the sun is __________________________. That's 333,000 times heavier than the Earth.
- Some stars however, have masses that are much greater than the mass of our sun. If a star has a mass of at least _____________ times the mass of our sun, when the star dies (burns out), all of its mass gets compressed into such a small space that it creates a _____________.

What's a black hole?

A black hole is an object in outer space whose gravity is so strong that once another object gets close enough to it, it is trapped inside the black hole forever and can never escape. This is because black holes have so much mass that they pull everything towards it with a huge amount of force.

What does a black hole look like?



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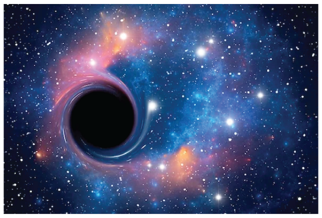

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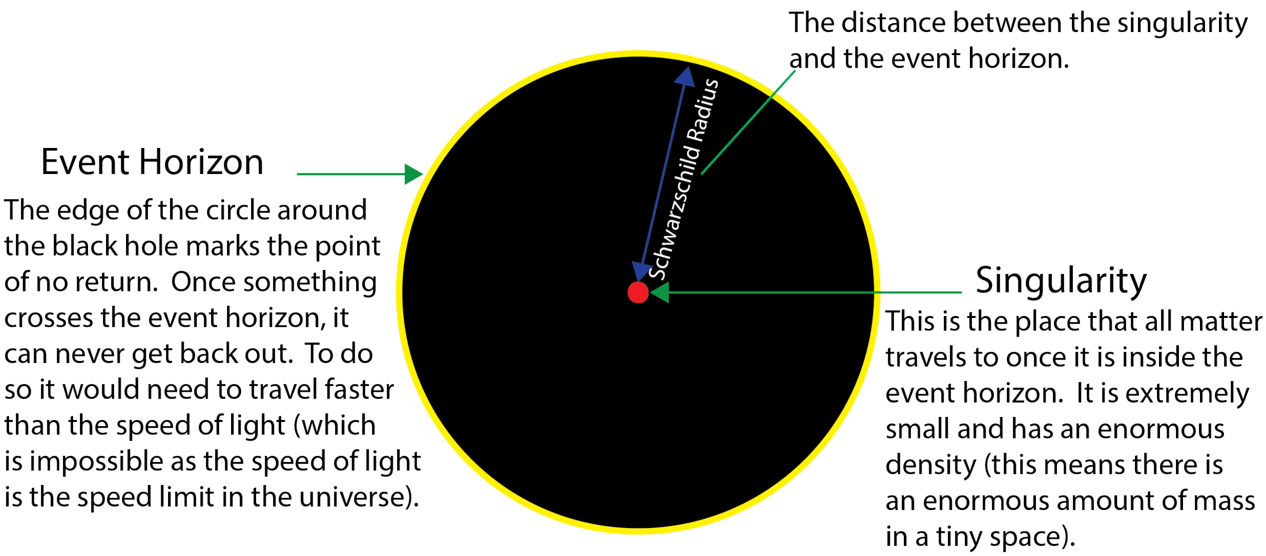
Diagram of a black hole:



Quick Review:

The radius of a circle is the distance between the centre of the circle and the edge of the circle.

Likewise, the radius of an sphere is the distance from the centre of the sphere to the surface (edge) of the sphere.

What is so special about the Schwarzschild radius?

Anything can become a black hole. That's right, anything!

Write down a few things that we could theoretically compress into a black hole:

The catch here is that, in order to make something into a black hole, we need to compress (squeeze) it into a really small space. The Schwarzchild radius tells us how small we would need to compress an object to turn it into a black hole, depending on its mass.

The Schwarzschild radius equation:

$$R = \frac{2GM}{c^2}$$

Retrieved from https://www.perthobservatory.com.au/news/the-schwarzschild-radius

Where,
R = The Schwarzschild radius in m (the radius that we must squish the object to).
$G = 6.67 \times 10^{-11}\text{Nm}^2/\text{kg}^2$(gravitational constant).
M = The mass of the object in kg.
$c = 3 \times 10^8$ m/s (speed of light).

Example:

The mass of our moon is 7.35×10^{22} kg. What is its Schwarzschild radius?

Exercise:

The mass of the Earth is 5.98×10^{24} kg. What radius must you squish the Earth into in order to turn it into a black hole?

An Alternate Schwarzschild Equation:

Because the value of G and c in the Schwarzschild equation are always the same, we can simplify the equation to:

$$R = 1.48 \times 10^{-27} M$$

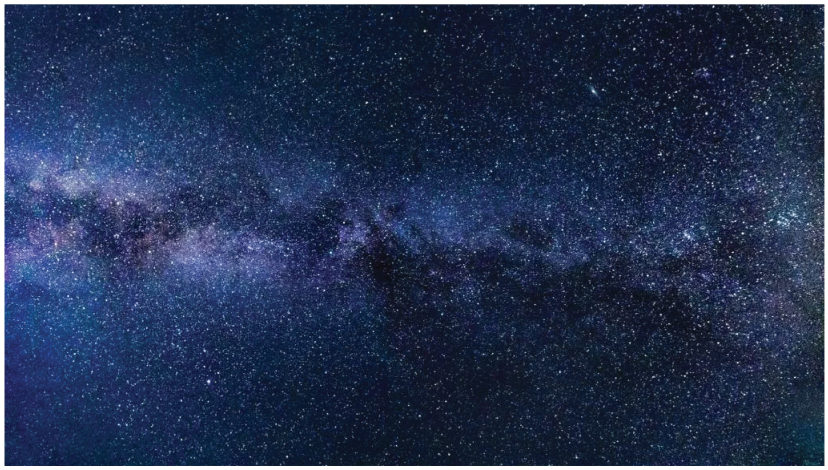
Where,
R = The Schwarzschild radius in m (the radius that we must squish the object to).
M = The mass of the object in kg.
1.48×10^{-27} is just $\frac{2G}{c^2}$.

Try using this simplified formula to calculate the Schwarzschild radius of the moon again (7.35×10^{22} kg). Do you get the same answer?

A Closer Look at The Formation of Black Holes

We mentioned briefly that black holes form when a star dies, if that star had a great enough mass.

What does it mean for a star to die?



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When we look up at the night sky, we see stars as specs of light. This is essentially what a star is, a ball of light in the sky. But a star needs to possess certain chemicals that react in order to emit this light. When a star ____________ of these chemicals, it ____________ and transforms into either a different type of star or a ____________, so we say that it "dies."

A quick rundown:

- We have three types of black holes, classified by their size: ________________________ black holes, ________________________ black holes, and ________________________ black holes.
- Stars that are ____________ than ________________________ times as massive as our sun, will ____________ form a black hole when they die.
- Stars that are between ________________________ and ________________________ times as massive as our sun will form a ________________________ black hole when they die.
- Stars ____________ form black holes bigger than this, however there are other ways that even bigger black holes can be created.

- We call a black hole that is between ________________________ and ________________________ times as massive as our sun an ________________________ black hole.
- We call a black hole that is greater than ________________________ times the mass of our sun a ________________________ black hole.

<u>Example:</u>

The mass of our sun is 1.99×10^{30} kg. Will a star with a mass of 2.189×10^{31} kg turn into a black hole when it dies?

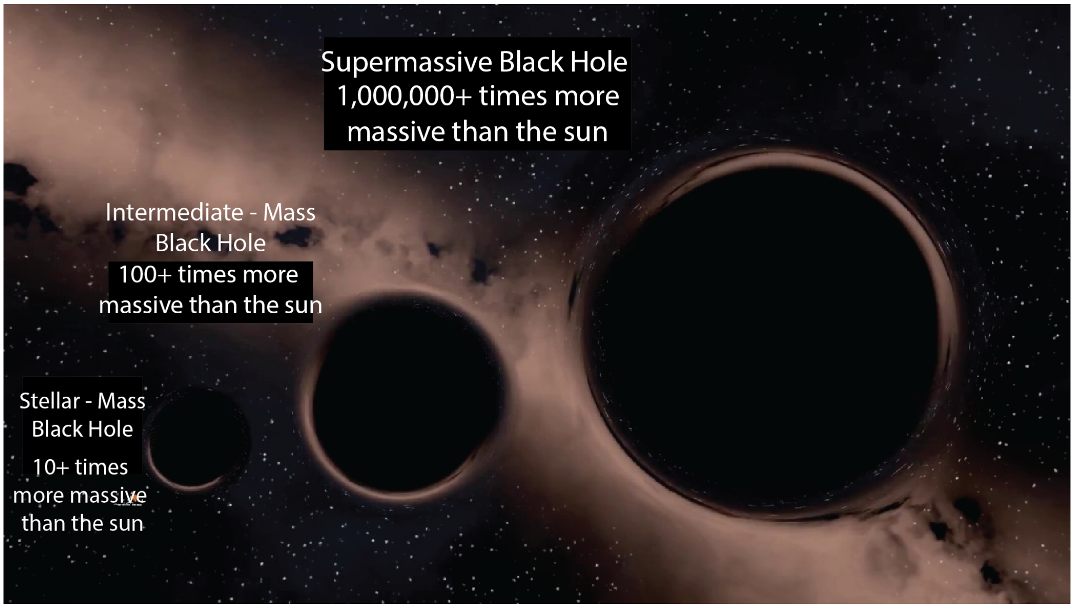
<u>Example:</u>

The mass of a black hole is found to be 1.592×10^{32} kg. Is this black hole massive enough to be considered an intermediate-mass black hole?

Example:

While searching through our Milky Way galaxy, you detect a black hole with a mass of 7.96×10^{36} kg. That's a really massive black hole, is it a supermassive black hole?

Recap!



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Exercises:

Classify each black hole as either a stellar-mass, intermediate or supermassive black hole.

1. A black hole with a mass of 9.95×10^{31} kg.

2. A black hole with a mass of 3.98×10^{36} kg.

3. A black hole with a mass of 1.17×10^{33} kg.

Problem Set

* Indicates challenge problems.

1. Use our scientific notation exponent rules to simplify the following:
   Ex. We would write $10^3 \times 10^6$ as 10^9.

   (A) $10^5 \times 10^9$ (B) $10^7 \times 10^{55}$ (C) $10^{-8} \times 10^{-10}$ (D) $10^5 \times 10^{-3}$

   (E) $\frac{10^{77}}{10^{60}}$ (F) $\frac{10^{43}}{10^{80}}$ (G) $\frac{10^{-22}}{10^5}$ (H) $\frac{10^{-25}}{10^{-3}}$

   (I) $(10^4)^2$ (J) $(10^{12})^5$ (K) $(10^3)^{-4}$

2. Write the following as a single number:
   Ex. We would write 1.0×10^4 as 10,000.

   (A) 3.8×10^2 (B) 5.6×10^6 (C) 8.9×10^1 (D) 7.5×10^{-3}

   (E) 6.2×10^{-1}

3. Put the following in scientific notation:

   (A) 45,000 (B) 6,899,000 (C) 0.57 (D) 0.00032

   (E) 50 (F) 468.9

4. Calculate the following and express your answers in scientific notation.

   (A) $5.8 \times 10^{-27} \times 5.5 \times 10^{30}$ (B) $6.4 \times 10^{44} \times 1.5 \times 10^{-8}$ (C) $\frac{5.7\times10^{19}}{4.2\times10^{6}}$

   (D) $\frac{4.4\times10^{9}}{3.9\times10^{-5}}$ (E) $\frac{9.2\times10^{10}\times7.6\times10^{-3}}{8.1\times10^{4}}$ (F) $\frac{1.6\times10^{-10}\times5.6\times10^{-3}}{3.1\times10^{-6}}$

   (G) $(5.4 \times 10^6)^2$ (H) $\frac{4.5\times10^{4}}{(6.3\times10^{3})^2}$

5. Give an example of a scenario where we could see the effects of Earth's gravity.

6. Why do we not see the effects of a pencil's gravity?

7. Which has a greater force of gravity: the Earth or the sun? Why?

8. You tell your friend that you can turn a piece of paper into a black hole. Your friend doesn't believe you, so you start crushing the paper up into a smaller and smaller space, trying to create a black hole. If your sheet of paper has a mass of 4.5×10^{-3} kg, what radius would you have to condense the paper to in order to create a black hole?

9. Pluto is upset about being deemed "not a planet" anymore. To get its revenge, Pluto decides to try to become a black hole so that it can swallow up the rest of the planets. Pluto has a mass of 1.31×10^{22} kg. To what radius must Pluto condense to in order to become a black hole?

10. *The Schwardzchild radius of my dog is 4.74×10^{-26} m. What is the mass of my dog in kg?

11. *What is the effect of decreasing the mass of the object on the Schwardzchild radius?

12. The following is a list of masses of different stars. Which of these stars will turn into a black hole when they die? (Remember that the mass of the sun is 1.99×10^{30} kg).

    (A) 1.22×10^{31} kg (B) 1.99×10^{31} kg (C) 8.32×10^{31} kg (D) 1.12×10^{31}kg

    (E) 1.32×10^{32} g

13. The following is a list of masses of black holes. Classify which type of black hole each one is.

    (A) 2.5×10^{37} kg (B) 1.7×10^{36} kg (C) 6.5×10^{33} kg (D) 7.8×10^{34} kg

    (E) 9.0×10^{34} g

14. *You find a black hole that is 55 times as massive as the sun. What is the mass of this black hole?

15. *Because the Earth has a strong force of gravity, in order to leave the Earth, rocketships have to be moving very fast. This speed that a rocketship must move to leave the Earth is called the *escape velocity.* Because the rocketship's speed has to counteract the Earth's force of gravity, we can imagine that the stronger the force of gravity of an object, the faster you would need to move to counteract that object's gravity and escape from the object. So, for example, the sun has a greater escape velocity than the Earth (Meaning that rocketships would need to move faster to leave the sun than they do to leave the Earth).

    The formula to find the escape velocity to leave any particular object is $v = \sqrt{\frac{2GM}{R}}$.

    Where,
    v is the escape velocity.
    G is the gravitational constant ($6.67 \times 10^{-11} \text{Nm}^2/\text{kg}^2$).
    M is the mass of the object that you (or another object) are trying to escape from.
    R is the radius of the object that you (or another object) are trying to escape from.

    Use this equation and the Schwardzchild equation to show that light cannot escape from a black hole.