Math Circles - Grade 11/12
October 21-27, 2020
Crocodiles! - Solution

Solution 1:

Let d be initial distance from the train to the bridge, let x be the length of the bridge, and let s be your speed. The two different scenarios given in the problem give us two equations in terms of these variables. We will make use of the fact that

$$\text{time} = \frac{\text{distance}}{\text{speed}}$$

In the first scenario, we are told that the time it takes the train to reach the start of the bridge is equal to the time it takes you to run back to the start of the bridge. Therefore,

$$\frac{d}{40} = \frac{\frac{3}{8}x}{s},$$

which simplifies to

$$ds = 15x. \qquad (1)$$

In the second scenario, we are told that the time it takes the train to reach the start of the bridge and then cross it is equal to the time it takes you to run to the end of the bridge. Therefore,

$$\frac{d+x}{40} = \frac{\frac{5}{8}x}{s},$$

which simplifies to

$$ds + xs = 25x. \qquad (2)$$

Substituting equation (1) into equation (2), we get $15x + xs = 25x$ or $xs = 10x$. We know that $x \neq 0$ and so we divide both sides of the equation by x to obtain $s = 10$. Therefore, your speed is 10 km/h.

Solution 2:

Consider the scenario where you run away from the train. When the train reaches the start of the bridge, where are you? We know that if you run towards the train, then you will be at the start of the bridge when the train reaches the bridge. In other words, you run $\frac{3}{8}$ of the bridge in the time it takes for the train to reach the bridge. So if you run away from the train, when the train reaches the bridge you will be $\frac{3}{8} + \frac{3}{8} = \frac{6}{8} = \frac{3}{4}$ of the way across the bridge. You have $\frac{1}{4}$ of the bridge left to run. In the time it takes you to run what's left, the train will go the length of the bridge. Therefore, your speed is $\frac{1}{4}$ the speed of the train and so your speed is 10 km/h.

Discussion: Isn't the second solution here amazing? Many students are really excited when they see it! We often solve problems by introducing variables, creating equations and solving for unknowns like we did in the first solution. In many situations this process is necessary to solve the problem. However, in this problem, if we use the information in a different way, we can find a simple solution. As you continue to study mathematics you will add more and more tools to your problem solving toolbox. These tools are very powerful and very useful, but always be on the lookout for alternative ways to tackle problems.