Grade 6 Math Circles
October 21, 2020
Counting Part I - Problem Set

1. Spiderman decided that he wants to fight today. He needs to pick a villain, a location, and a teammate. Here's are Spiderman's options:

   - **Villain**: Scorpion, Carnage, or Mysterio
   - **Location**: The Bronx, Queens, Brooklyn, or Manhattan
   - **Teammate**: Iron Man, Thor, Captain America, Black Widow, Hulk, or Black Panther

   How many different fights could Spiderman choose from?

2. Plankton is only a few steps away in getting his hands on the Krabby Patty secret formula. He needs to open a vault that has a 5-digit numerical password. How many different password combinations could he try?

3. A simplified version of a Canadian postal code is made up of 6 characters of the format "A1A 1A1" where A is a letter and 1 is a digit (from 0 to 9). How many different postal codes could there be?

4. Solve these factorial problems:

   (a) $8! =$

   (b) $\dfrac{6!}{3!} =$

   (c) $\dfrac{17!}{14!3!2!} =$

   (d) $_7P_3 =$

5. Write these expressions using factorials:

   (a) $1 =$

   (b) $6 \times 5 \times 4 \times 3 \times 2 \times 1 =$

(c) $100 \times 99 \times 98 \times 97 =$

(d) $15 \times 14 \times 3 \times 2 \times 1 =$

6. The NBA has to schedule basketball matches between 4 different teams. The Toronto Raptors, the Boston Celtics, the Los Angeles Lakers, and Miami Heat. Each team plays each of the other teams once.

   (a) How many matches in total are there?

   (b) How many different ways are there to schedule the matches?

7. Spongebob, Patrick, Mr. Krabs, Squidward, Plankton, Sandy, and Gary, are having a race in their Boatmobiles. There is a prize of have a small, a medium, and a large Magic Conch. The first, second, and third place winners will get the large, medium, and small magic conch respectively. How many ways can the Magic Conches be given out?

8. (a) How many ways can the letters of the word GUITAR be arranged?

   (b) How many 3-letter words can you make using letters of the word GUITAR?

   (c) How many ways can the letters of the word GUITAR be arranged if the first letter has to be a G?

9. (a) How many 7-digit passwords are there with no repeating digits?

   (b) How many 6-digit numbers are there with no repeating digits? (**Hint:** Can a 6-digit number start with a 0?)

   (c) How many 6-digit **even** numbers are there with repeating digits?(**Hint:** Use the Fundamental Counting Principle)

10. **Challenge problem:** Luke, Princess Leia, Han Solo, Chewbacca, C-3PO, Yoda, and R2D2 are taking a ride on the Millennium Falcon. How many ways ways can they sit if R2D2 and C-3PO must sit together and Han Solo cannot sit beside Luke? Assume the seats are arranged in a line.

11. **Challenge problem:** How many different 10-letter "words" can you make using the letters in the word:

STATISTICS

Note: These do not have to be real words but do need to be made up of the above 10 letters arranged with no spaces or other symbols in between.

Hint: Think about the repeated letters in the word "STATISTICS".