Intermediate Math Circles

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Perfect Squares - Solutions

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Problem Set #1

Question:

What is the sum of the first 99 consecutive odd positive integers?

Answer:

The sum of the first 99 consecutive odd positive integers is equal to $99^2 = 9801$.

Problem Set #2

Question:

If 1225 is the sum of the first m consecutive odd positive integers, what is the value of m?

Answer:

Since the sum of the first m consecutive odd positive integers is equal to m^2, it must be the case that $m^2 = 1225$. Since m is positive, $m = \sqrt{1225} = 35$.

Problem Set #3

Question:

What is the sum of the odd integers from 1 to 50?

Answer:

There are 25 odd integers between 1 and 50 and their sum is equal to $25^2 = 625$.

Problem Set #4

Question:

What is the value of the sum $1 + 3 + 5 + \ldots + 141 + 143 + 145$?

Answer:

The i^{th} term in this sum is equal to $2i - 1$.

To find out how many terms are in this sum, set 145 equal to $2i - 1$ and solve for i.

$145 = 2i - 1$ which makes $2i = 146$ and $i = 73$.

Therefore,

$1 + 3 + 5 + \ldots + 141 + 143 + 145 = 73^2 = 5329$

Problem Set #5

Question:

What is the value of the sum $17 + 19 + 21 + \ldots + 207 + 209 + 211$?

Answer:

$$
\begin{aligned}
&17 + 19 + 21 + \ldots + 207 + 209 + 211 \\
&= (1 + 3 + 5 + \ldots + 207 + 209 + 211) - (1 + 3 + 5 + \ldots + 11 + 13 + 15) \\
&= 106^2 - 8^2 \\
&= 11\,236 - 64 \\
&= 11\,172
\end{aligned}
$$

Problem Set #6

Question:

What is the value of the sum $3 + 9 + 15 + \ldots + 423 + 429 + 435$?

Answer:

$$
\begin{aligned}
&3 + 9 + 15 + \ldots + 423 + 429 + 435 \\
&= 3(1 + 3 + 5 + \ldots + 141 + 143 + 145) \\
&= 3(73^2) \\
&= 3(5329) \\
&= 15\,987
\end{aligned}
$$

Problem Set #7

Question:

What is the value of the sum $2 + 4 + 6 + \ldots + 296 + 298 + 300$?

Answer:

$$2 + 4 + 6 + \ldots + 296 + 298 + 300$$
$$= (1+1)+(3+1)+(5+1)+\ldots+(295+1)+(297+1)+(299+1)$$
$$= (1+3+5+\ldots+295+297+299)+(1+1+1+\ldots+1+1+1)$$
$$= 150^2 + 150$$
$$= 22\,500 + 150$$
$$= 22\,650$$