Intermediate Math Circles Fall 2020

A Math Game

Centre for Education in Mathematics and Computing
Faculty of Mathematics, University of Waterloo

www.cemc.uwaterloo.ca

In this session, we are going to be looking at variations of a game that we call *Addition Magician*.

We will look at the original game and a strategy that will guarantee a win for this game.

Addition Magician

You Will Need:

- Two players
- A piece of paper and a pencil

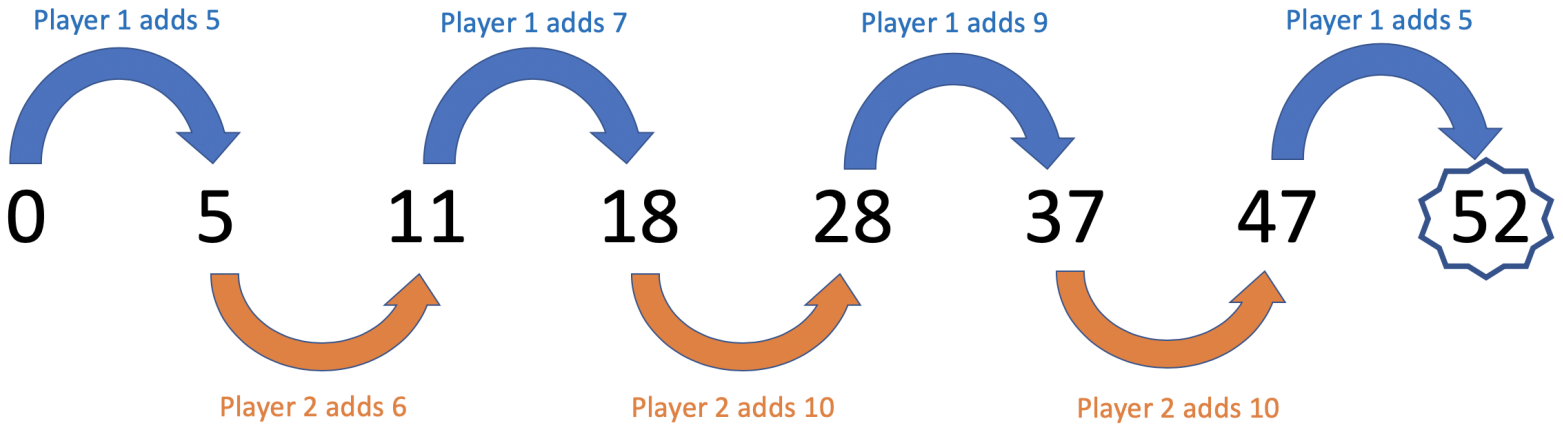
How to Play:

1. Start with a total of 0 (on the paper).
2. The two players will alternate turns changing the total. Decide which player will go first.
3. On your turn, you can add 1, 2, 3, 4, 5, 6, 7, 8, 9, or 10 to the total.
   Numbers may be used more than once throughout the game.
4. The player who brings the total to 52 wins the game!

An example game is shown on the next page.

Addition Magician

Example:



Player 1 wins this game!!

Addition Magician

Play this game a number of times.

Can you come up with a strategy that will win every time?

* A *strategy* is a pre-determined set of rules that a player will use to play the game. The strategy dictates what the player will do for every possible situation in the game. It's a *winning strategy* if the strategy allows the player to always win, regardless of what the other player does.

Addition Magician

Here are a few hints to help develop a strategy for this game:

Do you notice anything special about the number 41?
You may now want to play the game again but to 41.

Addition Magician

We will look at a winning strategy on the next three pages.

Addition Magician

You likely noticed that the player that brings the total to 42, 43, 44, 45, 46, 47, 48, 49, 50, or 51 generally loses the game on the next turn. The next player can reach 52 by adding 10, 9, 8, 7, 6, 5, 4, 3, 2, or 1, respectively, and so will win the game as long as they choose the correct number. Therefore, the player that brings the total to 41 is guaranteed to be able to bring the total to 52 on their next turn.

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Using similar reasoning, the player that brings the total to 30 is guaranteed to be able to bring the total to 41 on their next turn. Also the player that brings the total to 19 is guaranteed to be able to bring the total to 30 on their next turn, and the player that brings the total to 8 is guaranteed to be able to bring the total to 19 on their next turn.

Addition Magician

Notice that the target numbers 8, 19, 30, 41, and 52 all differ by 11. We can describe the strategy more concisely as follows: Go first and start by adding 8. For all turns that follow, if the other player adds n, then add $11 - n$.

Addition Magician

Now try the exercises provided.