

## Grade 6 Math Circles

November 11, 2020

### *Logic Puzzles - Solutions*

### Introduction

This week we will be looking at logic puzzles! Logic puzzles are puzzles or games that can be solved with deductive reasoning and logical thinking. Logic puzzles are a great way to practice thinking like a mathematician, as logical thinking is used in mathematics every day.

### Warm-up

Let's start with a couple of shorter logic puzzles to help get a feel for what logic puzzles may look like. Try to solve both problems on your own and when you think you have the answer, click on the link to watch a video that briefly goes through the solutions.

#### Warm-up 1. Flower Gardens

Three neighbours, Lily, Petunia and Rose are all growing flowers in their gardens. One is growing lilies, another is growing petunias and the third is growing roses. Lily notes that none of the three are growing the flower that corresponds with their name. The person who is growing petunias looks around at the gardens and agrees with Lily. Who is growing which flower?



*Solution:*

Lily is growing roses, Petunia is growing lilies and Rose is growing petunias.

#### Warm-up 2. Filling Water

Felix needs 4L of water to water his garden. However, he only has a 3L and a 5L container. He can fill each as many times as he needs, empty them or pour from one container to the other. How can Felix get exactly 4L of water? This online activity: <https://www.geogebra.org/m/vc6vje74> may help to visualize the problem. Warm-up Problem Solutions Video: <https://youtu.be/5zYNRfOIWWE>

*Solution:*

One solution is: Fill 5L, Pour 5L into 3L, Empty 3L, Pour 5L into 3L, Fill 5L, Pour 5L into 3L. The 5L container now holds 4L of water.

## Strategies

Although logic puzzles can vary greatly, there are some strategies that are useful for many puzzles. Below, a few such strategies are listed on how to approach logic puzzles. Please read through the following strategies. We will put some of them into use this week.

- Read through the question and clues multiple times. Information that you weren't able to use earlier may become usable or you may notice something that you hadn't before.
- Draw a chart or diagram to help organize the given information and what you can learn from it.
- Break the puzzle into pieces. Focus on one clue or step at a time.
- Check your solution to ensure that it satisfies all given conditions.
- Have fun! Some of these problems may be pretty tricky and take persistence. Don't forget that logic puzzles are meant to be fun!

## River-Crossing Puzzles

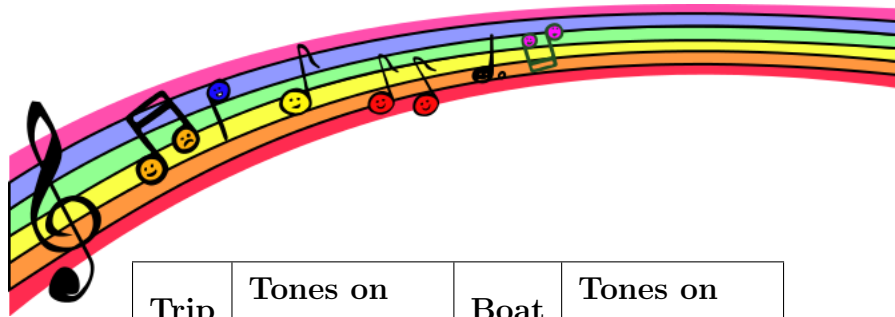
River-crossing puzzles are puzzles which ask the solver to carry people or objects between two places. Often, there are certain constraints to the crossing, such as the number of objects able to be transported at once or objects that can't stay together.

### Example 1. Carrying a Tune

While exploring in the woods, you have found and captured five *Pure Tones*: magical objects that each produce a single, pure musical note. You have put these Tones in glass jars labelled 1, 2, 3, 4, and 5, organized from lowest note to highest note. In order to take these Tones home, you have to transport them across a river, from the south side to the north side. However, your boat only has storage space for two Tones at a time, plus a seat for you, the driver.

The problem is that these Tones only stay quiet while you are watching them. If they are left alone on one side of the river, they will start making noise. If Tones that are one note apart are left together (like 1 and 2, or 4 and 5), their combined noise will shatter their glass jars, and they will escape.

Design a set of trips back and forth across the river so that you and the five Tones end up on the north side together, without any of them escaping. The table below may help organize your thinking.



Trip	Tones on South Side	Boat	Tones on North Side
	1, 2, 3, 4, 5		None
1		→	
2		←	
3		→	
4		←	
5		→	
6		←	
⋮			

Carrying a Tune Solution Video: [https://youtu.be/3YgJh9OC\\_pA](https://youtu.be/3YgJh9OC_pA)

*Solution:*

The table below shows one possible way to get the Tones across successfully in exactly seven trips.

Trip	Tones on South Side	Boat	Tones on North Side
	1, 2, 3, 4, 5		None
1	1, 3, 5	2, 4 →	2, 4
2	1, 3, 5	←	2, 4
3	3	1, 5 →	1, 2, 4, 5
4	2, 3, 4	← 2, 4	1, 5
5	2, 4	3 →	1, 3, 5
6	2, 4	←	1, 3, 5
7	None	2, 4 →	1, 2, 3, 4, 5

## Grid Puzzles

Many logic puzzles are in the form of grid puzzles. In a grid puzzle, the solver is given clues and asked to find certain information, given those clues. They are called grid puzzles as one of the most common ways to solve them is to organize the information into a grid. Try the below grid puzzle. Once you have tried it, click on the link to see a video solution, along with some strategies for solving these types of problems.

### **Example 2.** Out for Dinner

Four friends, Alex, Blake, Jess and Quinn decide to go out for dinner at a new restaurant in town. They each order a different food (pizza, a burger, pasta or a sandwich) and drink (water, apple juice, hot cocoa or milk). They all also have different last names, Baker, Miller, Smith or Green. Determine each person's full name, along with what they chose to eat and to drink at the restaurant.

Below, a grid, to organize your work, has been given. When using the grid, each square represents the possibility of a certain matching occurring. For example, in the grid below, the box in the top left would symbolize the possibility of Alex's last name being Smith. If it is possible for Alex's last name to be Smith but we don't know for sure that this is the case, we can leave the square blank. If we know Alex can't have the last name Smith, we typically denote that with an  $X$ . If Alex's last name is Smith, we typically use a checkmark.

**Clues:**

1. Alex Baker does not like pizza.
2. The person who had the apple juice, the one whose last name is Green, Quinn and the one who ate a burger are all different people.
3. Somebody enjoyed a meal with hot cocoa and pizza.
4. Jess, whose last name is not Miller, drank water.
5. The person who had milk, who was not Blake, had the sandwich.

	Smith	Green	Baker	Miller	Water	Apple Juice	Hot Cocoa	Milk	Pizza	Burger	Sandwich	Pasta
Alex												
Blake												
Jess												
Quinn												
Pizza												
Burger												
Sandwich												
Pasta												
Water												
Apple Juice												
Hot Cocoa												
Milk												

Fill in the grid above or use this online activity: <https://www.geogebra.org/m/h6zpygnj>.

Out for Dinner Solution Video: <https://youtu.be/UglhpwabcD8>

Solution:

	Smith	Green	Baker	Miller	Water	Apple Juice	Hot Cocoa	Milk	Pizza	Burger	Sandwich	Pasta
Alex	X	X	√	X	X	√	X	X	X	X	X	√
Blake	X	√	X	X	X	X	√	X	√	X	X	X
Jess	√	X	X	X	√	X	X	X	X	√	X	X
Quinn	X	X	X	√	X	X	X	√	X	X	√	X
Pizza	X	√	X	X	X	X	√	X				
Burger	√	X	X	X	√	X	X	X				
Sandwich	X	X	X	√	X	X	X	√				
Pasta	X	X	√	X	X	√	X	X				
Water	√	X	X	X								
Apple Juice	X	X	√	X								
Hot Cocoa	X	√	X	X								
Milk	X	X	X	√								

Reset

⌂

Alex Baker had apple juice and pasta.  
 Blake Green ate hot chocolate and pizza.  
 Jess Smith enjoyed water with a burger.  
 Quinn Miller had milk with a sandwich.

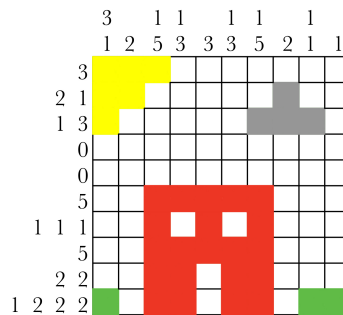
# Nonograms

Nonograms are a type of logic puzzle that ask the solver to fill in squares in a box. Often, nonograms will create a picture.

Rules:

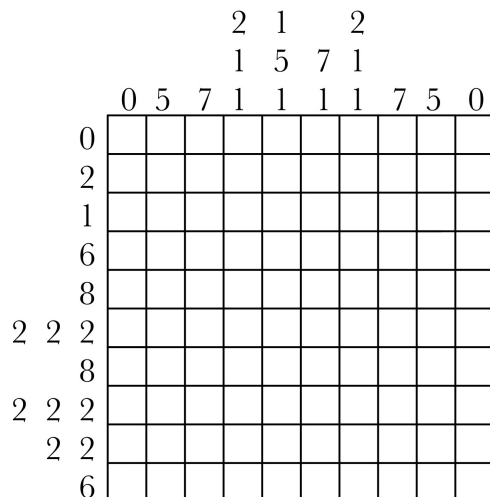
- Each square must be either coloured or uncoloured.
- Above each row and column, there is a list of numbers. The numbers represent the groups of coloured squares adjacent to each other in that row or column. For example, in our completed nonogram below, the 2nd row from the top has some number of uncoloured squares, followed by 2 squares that are coloured, at least 1 uncoloured square, 1 square coloured and then the rest of the row is uncoloured.
- There must be at least one uncoloured space between groups of coloured spaces.

Below is an example of a completed nonogram:



## Example 3. Nonogram

Fill in the following nonogram.

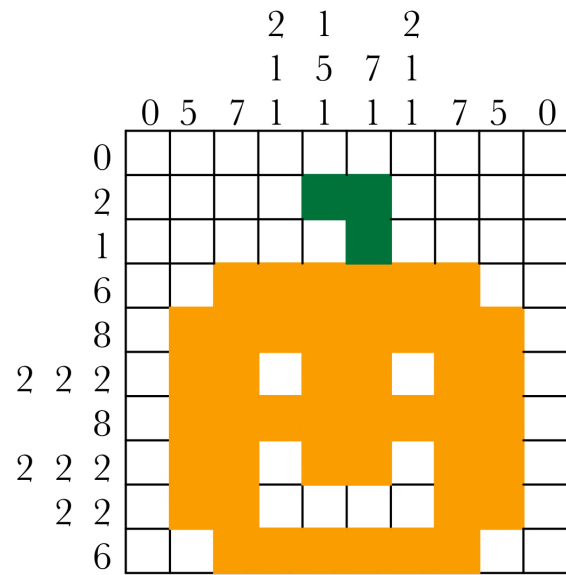


Fill in the grid to the left or use this online activity: <https://www.geogebra.org/m/kehztqb8>.

Nonogram Solution Video:

<https://youtu.be/54wBjyHRGyc>

*Solution:*





# Problem Set

## 1. Pet Race

The results of the Orangetown Annual Pet race have gone missing! Can you help by determining each pet's name, owner and place they came in?

### Clues:

- (a) Fluffy wasn't awarded second place, but the rabbit placed 4th.
- (b) The turtle placed lower than Zippy the sloth, but higher than Riley's pet.
- (c) Parker's pet, which wasn't a sloth, was awarded first place.
- (d) Sidney's pet wasn't named Spot.
- (e) In no particular order, the four pets were the pet named Whiskers, Charlie's pet, the pet who won 3rd place and the lizard.

	Fluffy	Spot	Whiskers	Zippy	Lizard	Rabbit	Sloth	Turtle	1st	2nd	3rd	4th
Charlie												
Parker												
Sidney												
Riley												
1st												
2nd												
3rd												
4th												
Lizard												
Rabbit												
Sloth												
Turtle												

*Solution:*

	Fluffy	Spot	Whiskers	Zippy	Lizard	Rabbit	Sloth	Turtle	1st	2nd	3rd	4th
Charlie	X	X	X	✓	X	X	✓	X	X	✓	X	X
Parker	X	✓	X	X	✓	X	X	X	✓	X	X	X
Sidney	✓	X	X	X	X	X	X	✓	X	X	✓	X
Riley	X	X	✓	X	X	✓	X	X	X	X	X	✓
1st	X	✓	X	X	✓	X	X	X				
2nd	X	X	X	✓	X	X	✓	X				
3rd	✓	X	X	X	X	X	X	✓				
4th	X	X	✓	X	X	✓	X	X				
Lizard	X	✓	X	X								
Rabbit	X	X	✓	X								
Sloth	X	X	X	✓								
Turtle	✓	X	X	X								

Parker's lizard, Spot, placed 1st.  
 Charlie's sloth, Zippy, placed 2nd.  
 Sidney's turtle, Fluffy, came in 3rd.  
 Riley's rabbit, Whiskers, came in 4th.

## 2. Across the River

Three athletes, Alex (whose coach is Liam), Ben (whose coach is Mike), and Carly (whose coach is Nora), are trying to get across a river. None of the coaches trust any of the other coaches to stay with their athlete while they are not there. For example, Mike cannot be with Alex while the boat is in the water and Liam is not there. But, Mike could drop off Ben on the side that Alex is on, so long as Mike immediately turns back for the other shore, not carrying Alex back with him. Either a coach or an athlete can drive the boat, but someone must drive the boat for it to cross. At most two people can be in the boat at any one time. How can they all get across?

*Solution:*

A sample solution of how all coaches and athletes could get across in 9 trips:

Trip	People on Initial Side	Boat	People on Final Side
	Alex, Liam, Ben Mike, Carly and Nora		None
1	Alex, Liam, Ben, Mike	Carly, Nora →	Carly, Nora
2	Alex, Liam, Ben, Mike, Nora	← Nora	Carly
3	Ben, Mike, Nora	Alex, Liam →	Carly, Alex, Liam
4	Ben, Mike, Nora, Liam	← Liam	Carly, Alex
5	Mike, Ben	Nora, Liam →	Carly, Alex, Liam, Nora
6	Ben, Mike, Carly	← Carly	Alex, Liam, Nora
7	Carly	Mike, Ben →	Alex, Liam, Nora, Ben, Mike
8	Carly, Nora	← Nora	Alex, Liam, Ben, Mike
9	None	Carly, Nora →	Alex, Liam, Ben, Mike, Carly, Nora

### 3. Places Please!

Nine friends, Alex, Blacke, Casey, Drew, Eddie, Finn, Gale, Harper and Jamie rent an apartment together. There are nine rooms to assign - laid out in three storeys as shown below. Each friend will be assigned exactly one room. The friends have some requirements, listed below. Can you help them pick rooms so that they are all happy?

#### Requirements:

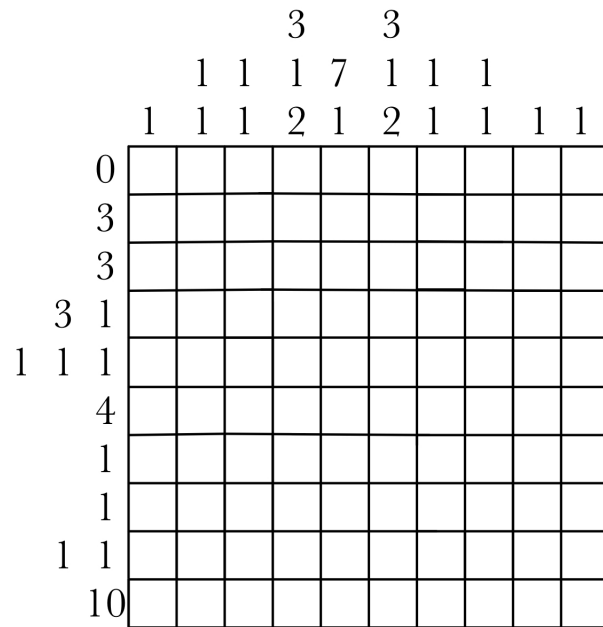
- (a) Alex wants to live directly to the left of Finn and right under Harper.
- (b) Drew wants to live right under Finn and directly to the right of Jamie.
- (c) Eddie wants to live below Casey, who wants to live below Gale.
- (d) Blake wants to live on the right side of the building.

*Solution:*

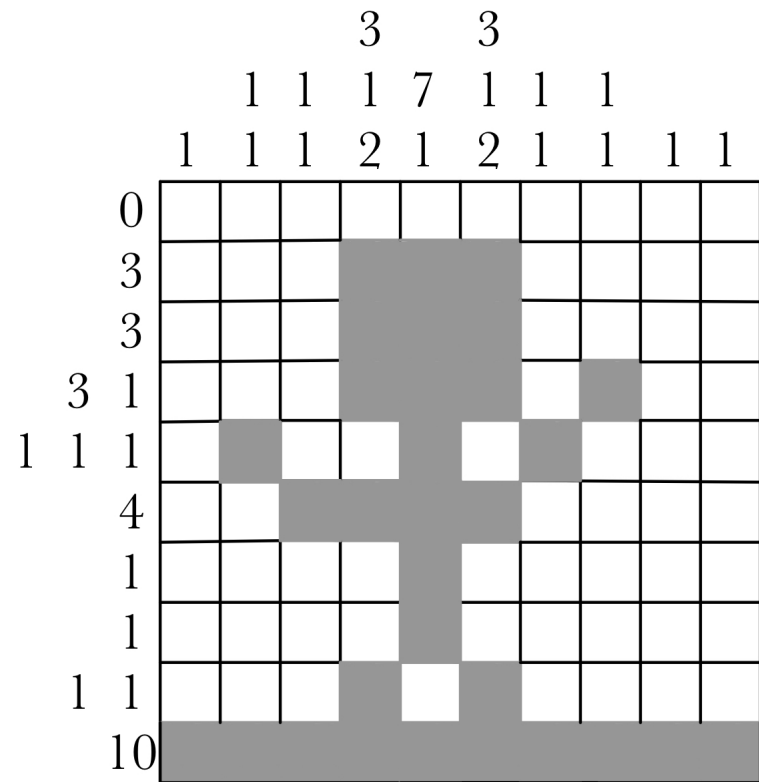
Gale	Harper	Blake
Casey	Alex	Finn
Eddie	Jamie	Drew

#### 4. Nonograms

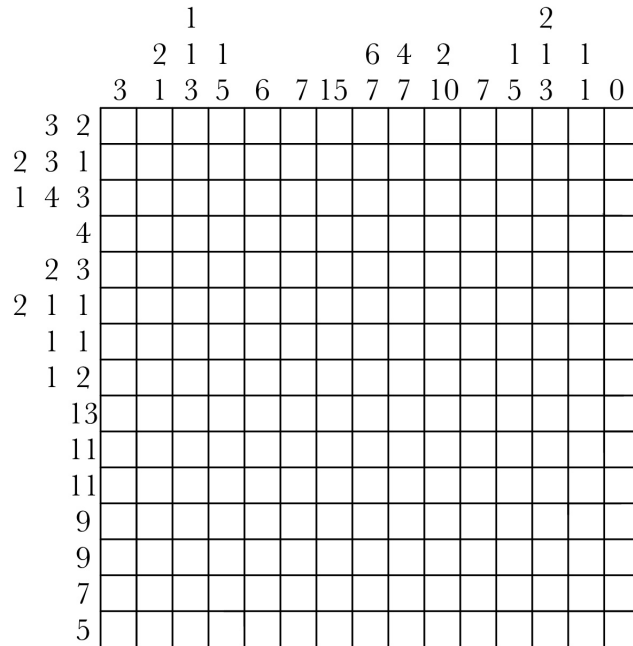
a)



*Solution:*

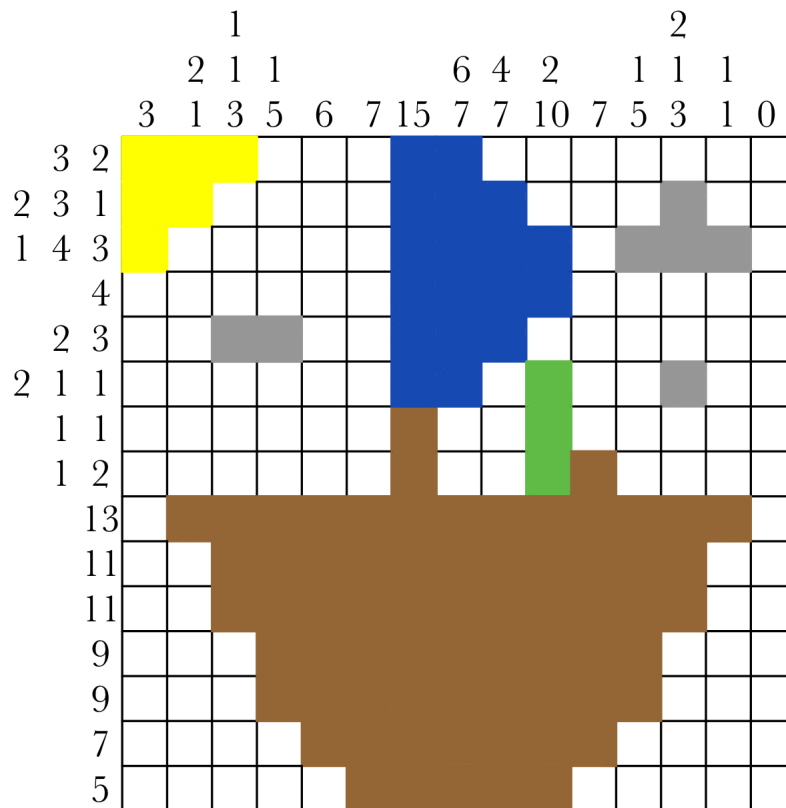


b)



*Hint: As is sometimes the case, for this one, the spaces that you cannot colour are equally as important as those you do colour.*

*Solution:*



## 5. Einstein's Riddle

There are 5 people and 5 houses in a row. Each person is of a different nationality (British, Swedish, Danish, Norwegian, and German), and each of their houses is a different colour (Green, White, Red, Yellow, and Blue). Additionally, each person eats one type of candy (Snickers, Starburst, Skittles, Jelly Beans, or Mike & Ike) and each raises a different type of pet (cat, birds, horse, dog, and fish). Finally, each owner drinks a different beverage (milk, coffee, tea, water or juice). Who raises the fish?

### Clues

- (a) The Brit lives in the red house.
- (b) The Swede has a dog.
- (c) The Dane drinks tea.
- (d) The green house is directly on the left of the white house.
- (e) The green house owner drinks coffee.
- (f) The person who eats Snickers raises birds.
- (g) The owner of the yellow house eats Skittles.
- (h) The person in the centre house drinks milk.
- (i) The Norwegian lives in the first house.
- (j) The person who eats Starburst lives next door to the one who has a cat.
- (k) The person who keeps a horse lives next to the person who eats Skittles.
- (l) The person who eats Jelly Beans drinks juice.
- (m) The German eats Mike & Ike.
- (n) The Norwegian lives next to the blue house.
- (o) The person who eats Starburst has a neighbour who drinks water.

	Cat	Dog	Horse	Fish	Birds	Snickers	Starburst	Skittles	Jelly Beans	Mike & Ike	Juice	Coffee	Tea	Water	Milk	British	Norwegian	Swedish	Danish	German	Blue	Green	Red	White	Yellow
1 (Far Left)																									
2																									
3																									
4																									
5 (Far Right)																									
Blue																									
Green																									
Red																									
White																									
Yellow																									
British																									
Norwegian																									
Swedish																									
Danish																									
German																									
Juice																									
Coffee																									
Tea																									
Water																									
Milk																									
Snickers																									
Starburst																									
Skittles																									
Jelly Beans																									
Mike & Ike																									

*Solution:*

House Location	Nationality	House Colour	Candy	Drink	Pet
1	Norwegian	Yellow	Skittles	Water	Cat
2	Danish	Blue	Starburst	Tea	Horse
3	British	Red	Snickers	Milk	Birds
4	German	Green	Mike & Ike	Coffee	Fish
5	Swedish	White	Jelly Beans	Juice	Dog

So, the German raises the fish.

6. Looking for more problems?

Try out one of the previous Math Circles: [https://cemc.uwaterloo.ca/events/mathcircle\\_presentations\\_gr6.html](https://cemc.uwaterloo.ca/events/mathcircle_presentations_gr6.html) based on Logic Puzzles. Terms that have such sessions include Fall & Winter 2012, Fall 2015 and Fall & Winter 2017. Or, check Piazza for a new puzzle each day!