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# Grade 6 Math Circles

November 18, 2020

## *Bijections and Binary Strings - Problem Set*

1. Give an example of each of the following type of relations:
   (a) Function
   (b) Injective, but not surjective function
   (c) Surjective, but not injective function
   (d) Bijective Function
   (e) Relation but not a function

2. Give an example of a set with a cardinality of 10.

3. Let A represent the set of binary strings of length 3 and B represent the set of odd numbers greater than 4 but less than 18. Explain why it's not possible to find a bijection between these two sets.

4. Find a bijective function that follows a pattern between the following pairs of sets. Explain the pattern and draw a chart showing the connections between the domain and codomain.
   (a) $\{000, 00110, 110, 1, 001\}$ and $\{111, 11001, 001, 0, 110\}$
   (b) $\{000, 001, 010, 011, 100, 101, 110, 111\}$ and $\{aaa, aab, aba, abb, baa, bab, bba, bbb\}$
   (c) $\{43, 26, 32, 10, 67, 88, 91\}$ and $\{78, 95, 89, 01, 54, 33, 20\}$
   (d) $\{\{1\}, \{2, 3\}, \{1, 3\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 2, 3, 4\}\}$ and $\{1000, 0110, 1010, 1110, 0111, 1111\}$

   *Hint: Match the elements according to their listed order in the set. Try to find a pattern to the functions found.*

5. Determine if the following binary strings are in the set of binary strings created by this decomposition: $\{0, 00\}\{1, 10, 100\}^*\{001\}$
   (a) 10001
   (b) 0010111001
   (c) 0001
   (d) 0110101
   (e) 0111010010001
   (f) 0110001001

6. The following decompositions are ambiguous. Find a binary string that can be created in two different ways with the decompositions to show this fact.
   (a) $\{11, 111\}^*$
   (b) $\{1\}\{0, 00\}^*\{1, 11\}$
   (c) $\{1\}^*\{0, 10\}^*$
   (d) $\{0\}^*\{1\}^*\{0\}^*\{1\}^*$