CEMC Math Circles - Grade 11/12

March 3-9, 2021
Trugs and Falths

You are an ambassador of The Republic of Logica and you have been sent on a mission to Duoterra, one of the islands of Logica. This island is entirely inhabited by two societies of people: the Trugs, who always tell the truth and the Falths, who always lie. You must figure out to which society the people you encounter belong. To do so you will have to use the skills you learned at the University of Logica.

One of these skills is the process of elimination which is more formally known as "Proof by Contradiction" which we talked about last week.

On each day, you meet at least two new people on the island. Let's see what happens on Day 1.

Day 1: You meet Algorn and Birk.

Algorn: Birk and I are Trugs.
Birk: Algorn is a Falth.

From this information you must determine to which society Algorn and Birk belong.

Solution for Day 1: Algorn must be either a Trug or a Falth.

- *Suppose that Algorn is a Trug.* This means that Algorn is telling the truth and so both Algorn and Birk must be Trugs. Therefore, Birk is also telling the truth and so Algorn is a Falth. This contradicts our initial assumption that Algorn is a Trug. So Algorn must be a Falth.

- *Algorn is a Falth.* This means Algorn is lying, which is already clear because she said that both Birk and her are Trugs. Birk's statement is then true and so Birk must be a Trug.

In summary, Algorn is a Falth and Birk is a Trug.

Logical Connectives

During your travels you will encounter sentences that involve the words "and", "or", and "not". Here is a summary of how to interpret the truth of statements like this:

AND	A statement of the form "P and Q" is true if *both* P and Q are true. A statement of the form "P and Q" is false if *at least one* of P and Q is false.

OR	A statement of the form "P or Q" is true if *at least one* of P and Q is true. A statement of the form "P or Q" is false if *both* P and Q are false.

Note that the word "or" is sometimes used differently than described above in everyday English. Sometimes people interpret "P or Q" as being true when *exactly one* of P and Q is true. We will not use this interpretation in what follows. You will also encounter statements involving "not". This has the usual meaning of the "opposite". In particular, "not P" is true exactly when P is false.

Day 2: You meet Crozul and Dek.

Crozul: I am a Trug or Dek is a Falth.
Dek: Exactly one of Crozul and I is a Trug.

From this information you must determine to which society Crozul and Dek belong.

Solution for Day 2: Crozul must be either a Trug or a Falth.

- *Suppose that Crozul is a Trug.* Therefore, Crozul is telling the truth. Since Crozul is a Trug, then Dek could be a Trug or a Falth and Crozel's statement would still be true.
  - *Suppose Dek is a Trug.* Then they are both Trugs and Dek's statement is false, which is not possible since Dek is a Trug. So Dek is a Falth.
  - *Dek is a Falth.* Then exactly one of Crozul and Dek is a Trug and so Dek's statement is true, which is not possible since Dek is a Falth.

  So, if Crozul is a Trug, then there are no possibilities for Dek to belong to either society, which is not possible. So it must be the case that Crozul is a Falth.

- *Crozul is a Falth.* Then Crozul is lying and so his statement tells us that he is not a Trug (this is consistent with our assumption) and that Dek is not a Falth (he is a Trug). We need to verify that this assignment of societies is consistent with Dek's statement. Exactly one of Crozul and Dek is a Trug and so Dek is telling the truth, which is consistent with the fact that he is a Trug.

In summary, Cozul is a Falth and Dek is a Trug.

Your journey continues

On your third, fourth and fifth days you have three more interactions. Use these interactions to determine to which society the people you meet belong.

Day 3: You meet Gup and Hoken.

Gup: Hoken is a Falth.
Hoken: I am a Trug or Gup is a Trug.

Day 4: You meet Ized and Jeke.

Ized: Jeke is not a Falth and I am a Trug.
Jeke: Ized and I are from the same society.

Day 5: You meet Kip, Lolo and Moy.

Kip: I am not a Falth and Lolo is not a Falth.
Lolo: Kip is a Falth.
Moy: Lolo is a Trug.

More Info:

These problems are based on a famous type of logic puzzles called "Knights and Knaves". They were made popular in the late 1970s by Raymond Smullyan, an American mathematician. An internet search for "knights and knaves" will lead to many other problems of this type.