



Grade 6 Math Circles
Wednesday, February 24, 2021
Set Theory - Problem Set

- Describe the following sets using set notation.
 - Avengers movies
 - Canadian NHL teams
 - 3-letter names
 - Great Lakes
 - digits of ϕ
 - Fibonacci numbers
- Let $A = \{10, 11, 12, 13, \dots, 97, 98, 99\}$. Find one way to describe the set A .
- Give an example of each of the following:
 - a numerical finite set
 - a non-numerical finite set
 - a numerical infinite set
 - a non-numerical infinite set
- A prime number is a number that is only divisible by 1 and itself. For example, 2 is the smallest prime number because it is only divisible by 1 and 2. However, 4 is not a prime number since it is divisible by 1, 2, and 4. Let P represent the set of prime numbers. Are the following statements true or false?
 - $1 \in P$
 - $27 \notin P$
 - $73 \in P$
 - $89 \notin P$
 - P is an infinite set
- Let P be the set of planets in the solar system.
 - Describe the set using set notation.
 - What is the cardinality of P ?
- What is the smallest possible set? What is the cardinality of the set?
- A set has only one item in it. Are there any possible subsets? If yes, what are they?

8. Suppose a set had the following children's books by Dr. Seuss: The Lorax, Horton Hears a Who, and Fox in Socks. What is a possible larger set that this might be a subset of?

9. Given the set $A = \{1, 2, 3, 4, 5, 6, 7\}$:

(a) Is it possible to find a unique subset? In other words, does A have only one subset? Why or why not?

(b) What is the largest possible subset of A ? What is a largest possible proper subset of A ?

10. Consider the following sets:

- $T = \{\text{is, and, but, it, the, my, they, weekend, hurry, noodles, jacket, a, all}\}$
- $U = \{\text{I, math, bigger, pie, nature, younger, no, have, play, legs, feel, as, that}\}$
- $V = \{\text{always, fun, better, secret, love, hill, inside, time, like, library, to, back}\}$

The set X is a proper subset of the union of T , U , and V such that, the elements in X spell out a message. What is one possible set X ? Share your answer on Piazza!

11. Let X be the set of numbers divisible by 2. Let Y be the set of numbers divisible by 3. A number is said to be divisible by 6 if it is divisible by both 2 and 3. If Z is the set of numbers divisible by 6, how can you write it using the sets X and Y ? (Hint: use set operators!)

12. A librarian put together the following sets of books:

- $A = \{\text{Harry Potter, Percy Jackson, Hunger Games, Divergent, Mortal Instruments}\}$
- $B = \{\text{Hunger Games, Divergent, The Selection, Uglies}\}$
- $C = \{\text{Harry Potter, Hunger Games, Divergent, Lord of the Rings, The Chronicles of Narnia}\}$

(a) Define the following sets:

- i. $A \cup C$ ii. $B \cap C$ iii. $A \cup B$ iv. $A \cap B \cap C$

(b) Calculate the cardinality of each set. Will the cardinality of the union of the three sets, i.e. $A \cup B \cup C$, equal the sum of the cardinality of each set i.e. $|A| + |B| + |C|$? Explain.

- (c) Let the universal set, U , be equal to the union of all three sets. What is $(A \cup C)^C$?
13. If A and B are two sets such that $A \cap B = \emptyset$, what does that tell you about the sets? Recall that the symbol \emptyset represents the empty set, a set with no elements.
14. The union of two sets, C and D , results in the empty set i.e. $C \cup D = \emptyset$. What are C and D ?
15. Sally is planning a Zoom party for her birthday. She polls her friends to find which day works best for everyone. Here are the results:

Friend	March 5	March 6	March 7	March 12	March 13	March 15
Rory	✓	✓			✓	✓
Mercury		✓	✓			✓
Lamar		✓	✓	✓	✓	
Sonia		✓	✓			✓
Amber		✓		✓		✓

- (a) Write the results as a set. Let the initial of each friend's name represent the set and list the dates that they are available e.g. the set $R = \{\text{March 5, March 6, March 13, March 15}\}$ represents the dates Rory is available.
- (b) What set operator can be used to find which date works for everyone? Use the set operator to find the date when everyone can attend.
16. Consider the universal set of integers from 0 to 100. The set E is a subset of U containing only even integers. The set O is the complement of E or $O = E^C$. Describe the set O using set notation.
17. Using your knowledge of number systems, categorize the following numbers as a whole number, natural number, integer, rational number, or irrational number.
Note: use the number system that the number best falls into e.g. while -2 is technically a rational number, it is better described as an integer.

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|-------------------|------------|-------------|--------------------|
| (a) $\frac{4}{5}$ | (c) 3 | (e) 3.14159 | (g) $-\frac{1}{3}$ |
| (b) $\sqrt{7}$ | (d) ϕ | (f) -6 | (h) 0 |