UNIVERSITY OF WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 6 Math Circles

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Intro to Functions: Problem Set Solutions

For Questions 1–4, it can be helpful to organise your answers into a table like this:

x	-3	-2	-1	0	1	2	3
$f(x) = x + 3$	0	1	2	3	4	5	6

If an answer is undefined, you can leave your answer as "undefined" or "DNE" (which stands for "Does Not Exist").

For an extra challenge in Questions 1–3, try finding the range of each function!

1. Evaluate each function for $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$. Find the domain of each function.

   (a) $f(x) = x - 2$

   Solution:

x	-3	-2	-1	0	1	2	3
$f(x)$	-5	-4	-3	-2	-1	0	1

   Domain: $\mathbb{R}$
   Range: $\mathbb{R}$

   (b) $g(x) = 3x - 5$

   Solution:

x	-3	-2	-1	0	1	2	3
$g(x)$	-14	-11	-8	-5	-2	1	4

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

(c) $h(x) = -x + 25$

Solution:

x	-3	-2	-1	0	1	2	3
$h(x)$	28	27	26	25	24	23	22

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

(d) $j(x) = \frac{x+5}{2}$

Solution:

x	-3	-2	-1	0	1	2	3
$j(x)$	1	$\frac{3}{2}$	2	$\frac{5}{2}$	3	$\frac{7}{2}$	4

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

(e) $k(x) = \frac{1}{4}x$

Solution:

x	-3	-2	-1	0	1	2	3
$k(x)$	$\frac{-3}{4}$	$\frac{-1}{2}$	$\frac{-1}{4}$	0	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$

Domain: $\mathbb{R}$
Range: $\mathbb{R}$

(f) $l(x) = 100 - 10x$

Solution:

x	-3	-2	-1	0	1	2	3
$l(x)$	130	120	110	100	90	80	70

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

2. Evaluate each function for $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$. Find the domain of each function.

   (a) $f(x) = x^2 + 2$

   Solution:

x	-3	-2	-1	0	1	2	3
$f(x)$	11	6	3	2	3	6	11

Domain: $\mathbb{R}$

Range: $\{f(x) \in \mathbb{R},\ f(x) \geq 2\}$

   (b) $g(x) = \frac{x^2}{2}$

   Solution:

x	-3	-2	-1	0	1	2	3
$g(x)$	$\frac{9}{2}$	2	$\frac{1}{2}$	0	$\frac{1}{2}$	2	$\frac{9}{2}$

Domain: $\mathbb{R}$

Range: $\{g(x) \in \mathbb{R},\ g(x) \geq 0\}$

   (c) $h(x) = -x^2$

   Solution:

x	-3	-2	-1	0	1	2	3
$h(x)$	-9	-4	-1	0	-1	-4	-9

Domain: $\mathbb{R}$

Range: $\{h(x) \in \mathbb{R},\ h(x) \leq 0\}$

(d) $j(x) = 2x^2$

Solution:

x	-3	-2	-1	0	1	2	3
$j(x)$	18	8	2	0	2	8	18

Domain: $\mathbb{R}$

Range: $\{j(x) \in \mathbb{R},\ j(x) \geq 0\}$

(e) $k(x) = 0.5x^2 + x$

Solution:

x	-3	-2	-1	0	1	2	3
$k(x)$	1.5	0	-0.5	0	1.5	4	7.5

Domain: $\mathbb{R}$

Range: $\{k(x) \in \mathbb{R},\ k(x) \geq -0.5\}$

(f) $l(x) = x^2 + 4x + 3$

Solution:

x	-3	-2	-1	0	1	2	3
$l(x)$	0	-1	0	3	8	15	24

Domain: $\mathbb{R}$

Range: $\{l(x) \in \mathbb{R},\ l(x) \geq -1\}$

3. Evaluate each function for $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$. Find the domain of each function.

(a) $f(x) = x^3$

Solution:

x	-3	-2	-1	0	1	2	3
$f(x)$	-27	-8	-1	0	1	8	27

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

(b) $g(x) = x^4$

Solution:

x	-3	-2	-1	0	1	2	3
$g(x)$	81	16	1	0	1	16	81

Domain: $\mathbb{R}$

Range: $\{g(x) \in \mathbb{R},\ g(x) \geq 0\}$

(c) $h(x) = x^5$

Solution:

x	-3	-2	-1	0	1	2	3
$h(x)$	-243	-32	-1	0	1	32	243

Domain: $\mathbb{R}$

Range: $\mathbb{R}$

4. Evaluate each function for $x = -3$, $x = -2$, $x = -1$, $x = 0$, $x = 1$, $x = 2$, and $x = 3$. Find the domain of each function.

(a) $f(x) = \frac{1}{x+1}$

Solution:

x	-3	-2	-1	0	1	2	3
$f(x)$	$-\frac{1}{2}$	-1	DNE	1	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$

Domain: $\{x \in \mathbb{R},\ x \neq -1\}$

(b) $g(x) = \frac{3}{2x}$

Solution:

x	-3	-2	-1	0	1	2	3
$g(x)$	$-\frac{1}{2}$	$-\frac{3}{4}$	$-\frac{3}{2}$	DNE	$\frac{3}{2}$	$\frac{3}{4}$	$\frac{1}{2}$

Domain: $\{x \in \mathbb{R},\, x \neq 0\}$

(c) $h(x) = \frac{x}{x-4}$

Solution:

x	-3	-2	-1	0	1	2	3
$h(x)$	$\frac{3}{7}$	$\frac{1}{3}$	$\frac{1}{5}$	0	$-\frac{1}{3}$	-1	-3

Domain: $\{x \in \mathbb{R},\, x \neq 4\}$

(d) $j(x) = \frac{x+1}{x+2}$

Solution:

x	-3	-2	-1	0	1	2	3
$j(x)$	2	DNE	0	$\frac{1}{2}$	$\frac{2}{3}$	$\frac{3}{4}$	$\frac{4}{5}$

Domain: $\{x \in \mathbb{R},\, x \neq -2\}$

(e) $k(x) = \frac{5x}{5x+2}$

Solution:

x	-3	-2	-1	0	1	2	3
$k(x)$	$\frac{15}{13}$	$\frac{5}{4}$	$\frac{5}{3}$	0	$\frac{5}{7}$	$\frac{5}{6}$	$\frac{15}{17}$

Domain: $\{x \in \mathbb{R},\, x \neq -\frac{2}{5}\}$

(f) $l(x) = \frac{x^2}{x}$

Solution:

x	-3	-2	-1	0	1	2	3
$l(x)$	-3	-2	-1	DNE	1	2	3

Domain: $\{x \in \mathbb{R},\, x \neq 0\}$

5. The cost of a taxi ride is a base rate of \$3.50 plus \$1.50 per kilometre travelled.

   (a) Express the cost of a taxi as a function. Use C as the function name (to represent "Cost"), and use d as the variable name (to represent "distance").

   Solution: $C(d) = 3.5 + 1.5d$, where d is the number of kilometres travelled.

   (b) Calculate the cost of a 10km trip.

   Solution: The trip is 10km long, so we have $d = 10$.

   $C(10)$
   $= 3.5 + 1.5(10)$
   $= 3.5 + 15$
   $= 18.5$ dollars.

   (c) Calculate the cost of a 175km trip.

   Solution: The trip is 175km long, so we have $d = 175$.

   $C(175)$
   $= 3.5 + 1.5(175)$
   $= 3.5 + 262.5$
   $= 266$ dollars.

   (d) What is the domain of the function $C(d)$?

   Solution: It doesn't make sense to travel a negative distance (but any non-negative distance is fine), so the domain is $\{d \in \mathbb{R},\, d \geq 0\}$.

   (e) What is the range of the function $C(d)$?

   Solution: The domain is $\{d \in \mathbb{R},\, d \geq 0\}$, so the lowest that a taxi trip could cost would be $C(0) = 3.5$ dollars. In addition to this, there's no theoretical maximum to how far the taxi could travel, so the taxi trip could get arbitrarily expensive! Thus, the range is $\{C(d) \in \mathbb{R},\, C(d) \geq 3.5\}$.

6. At a particular buffet restaurant, it costs each person \$26 to eat dinner. The restaurant also charges a fee of \$0.30 for every 100g of leftover food at the end of the meal.

(a) Express the total cost of a meal as a function. Use C as the function name (to represent "Cost"), and use f as the variable name (to represent "food").

Solution: $C(f) = 26 + 0.3f$, where f is the number of 100g of leftover food.

(b) Calculate the total cost of a meal, where the person leaves 100g of leftover food.

Solution: The person had 100g of leftover food, so $f = 1$.

$C(1)$
$= 26 + 0.3(1)$
$= 26 + 0.3$
$= 26.3$

(c) Calculate the total cost of a meal, where the person leaves no leftover food.

Solution: The person had 0g of leftover food, so $f = 0$.

$C(0)$
$= 26 + 0.3(0)$
$= 26 + 0$
$= 26$

(d) Calculate the total cost of a meal, where the person leaves 670g of leftover food.

Solution: The person had 670g of leftover food, so $f = 6.7$.

$C(6.7)$
$= 26 + 0.3(6.7)$
$= 26 + 2.01$
$= 28.01$

(e) What is the domain of the function $C(f)$?

Solution: It doesn't make sense to leave behind a negative amount of food (but you can leave behind any non-negative amount of food), so the domain is $\{f \in \mathbb{R},\ f \geq 0\}$.

(f) What is the range of the function $C(f)$?

Solution: The domain is $\{f \in \mathbb{R}, f \geq 0\}$, so the lowest total cost of a meal at the restaurant would be \$26, when the person has no leftovers. In addition to this, there's no theoretical maximum to how much food a person could leave behind, so the fee charged for leftovers could become arbitrarily expensive! Thus, the range is $\{C(f) \in \mathbb{R}, C(f) \geq 26\}$.

7. A farmer is planning to purchase some straight fencing to build an enclosed pasture for their chickens. Each metre of fencing costs \$10 to purchase and install.

   (a) Construct a function that takes in the farmer's budget, which is the amount of money they plan to spend purchasing fencing, and outputs the maximum area of the pasture that could be built. Use A as the function name (to represent "Area"), and use b as the variable name (to represent "budget"). (Hint: The area of the pasture can be maximised by building it in the shape of a square.)

   Solution: $A(b) = (b \div 10 \div 4)^2 = (\frac{b}{40})^2$
   The area is the side length of the pasture squared. Since a square has four sides, the side length is the total amount of fencing purchased, divided by four. Since each metre of fencing costs \$10, the total amount of fencing purchased is the amount of money spent, divided by ten.

   (b) What is the area of the largest pasture that could be constructed with a budget of \$200?

   Solution: The amount of money spent would be \$200, so $b = 200$.

   $A(200)$
   $= (\frac{(200)}{40})^2$
   $= 5^2$
   $= 25\,m^2$

   (c) What is the area of the largest pasture that could be constructed with a budget of \$1000?

Solution: The amount of money spent would be \$1000, so $b = 1000$.

$A(1000)$
$= (\frac{(1000)}{40})^2$
$= 25^2$
$= 625\, m^2$

(d) What is the domain of the function $A(b)$?

Solution: It doesn't make sense to spend a negative amount of money (but any non-negative amount of money is fine), so the domain is $\{b \in \mathbb{R},\, b \geq 0\}$.

(e) What is the range of the function $A(b)$?

The domain is $\{b \in \mathbb{R},\, b \geq 0\}$, so the least amount of fencing that could be purchased is 0m. (Of course, that is only enough for $0^2 = 0m^2$ of pasture, which isn't useful at all!) There is also no theoretical maximum to how large the farmer's budget could be, so the size of the pasture could become arbitrarily large. Thus, the range is $\{A(b) \in \mathbb{R},\, A(b) \geq 0\}$.