UNIVERSITY OF
WATERLOO

Faculty of Mathematics
Waterloo, Ontario N2L 3G1

Centre for Education in
Mathematics and Computing

Grade 7/8 Math Circles

February 24, 2021

Confidence Intervals - Problem Set Solutions

1. Identify the point estimator and point estimate.

    (a) The mean height for the sample of classmates you took from your grade is 140cm.

    Solution: The point estimator is the sample mean, and the point estimate is 140cm.

    (b) Quality control inspectors note that 6 out of the 100 batteries they tested did not meet production standards.

    Solution: The point estimator is the sample proportion, and the point estimate is $\frac{6}{100} = 6\%$.

    (c) Polling shows that 62% of surveyed voters from around the city voted for the new candidate.

    Solution: The point estimator is the sample proportion, and the point estimate is 62%.

    (d) Apples are packed into crates of 50 apples each. To estimate the mean weight of individual apples, one crate is weighed and found to be 35kg.

    Solution: The point estimator is the sample mean, and the point estimate is $\frac{35}{50} = 0.7$kg.

2. What is the point estimate and margin of error for each interval?

    (a) $[60, 70]$ *(this notation means the interval from 60–70.)*

    Solution: The point estimate is 65, and the margin of error is 5, giving us a confidence interval of 65 ± 5.

    (b) $[3, 6]$

    Solution: The point estimate is 4.5, and the margin of error is 1.5, giving us a confidence interval of 4.5 ± 1.5.

(c) $[0.2, 1.6]$

Solution: The point estimate is 0.9, and the margin of error is 0.7, giving us a confidence interval of 0.9 ± 0.7.

(d) $[\frac{1}{6}, \frac{5}{6}]$

Solution: The point estimate is $\frac{3}{6} = \frac{1}{2}$, and the margin of error is $\frac{2}{6} = \frac{1}{3}$, giving us a confidence interval of $\frac{3}{6} \pm \frac{2}{6}$.

(e) $[31, 243]$

Solution: The point estimate is 137, and the margin of error is 106, giving us a confidence interval of 137 ± 106.

(f) $[\frac{1}{8}, \frac{9}{4}]$

Solution: The point estimate is $(\frac{1}{8} + \frac{9}{4}) \div 2 = \frac{1+18}{8} \times \frac{1}{2} = \frac{19}{16}$, and the margin of error is $\frac{19}{16} - \frac{1}{8} = \frac{17}{16}$, giving us a confidence interval of $\frac{19}{16} \pm \frac{17}{16}$.

3. For a given sample, would a confidence interval have to get bigger or smaller:

(a) when C increases? Why?

Solution: The interval would get **bigger**. When the confidence level (C%) increases, that means that we are more confident that our interval may capture the parameter value—this would happen if there were more values included in our confidence interval! In other words, we are trading for higher accuracy in exchange for lower precision.

(b) when C decreases? Why?

Solution: The interval would get **smaller**. Similarly to part (a), when the confidence interval decreases, we are less confident that our interval may capture the parameter value. When C decreases, this means that our guess is less reliably accurate—this happens when the interval contains less values, which also increases the precision.

4. If researchers are aiming to have approximately 48 out of their 50 repetitions of an experiment to produce confidence intervals that capture the true parameter value, what confidence level should they be using to calculate their intervals?

Solution: They should be constructing $\frac{48}{50} = 96\%$ confidence intervals.

5. If researchers were able to take 200 samples and calculated 85% confidence intervals based off the sample statistic for each of them, how many of the intervals would *not* capture the true parameter value?

Solution: Approximately $200 - (0.85 \times 200) = 30$ of the intervals would not capture the true parameter value.

6. Octagon News reports that based on their analysis of the game, the Prisms will score 3 ± 2 more goals in the last quarter, 4 times out of 5. The team needs to score 6 goals to win the game. Based on this estimation, is it plausible for the Prisms to win?

   Solution: No, it is not still plausible for the Prisms to win the game. The confidence interval [1, 5] is given with a confidence level of $\frac{4}{5} = 80\%$. Since 6 is not included in the confidence interval, it is implausible for them to score enough goals to win based on this analysis by Octagon News.

7. According to a survey done with a sample of students at your school, a 95% confidence interval for the percentage of students who would support later lunchtimes is $69\% \pm 8$. Is it plausible that at least three-quarters of students at the school want later lunchtimes?

   Solution: Yes, based on the data collected, it is plausble for at least three-quarters of students to want later lunchtimes. The 95% confidence interval constructed for the population proportion is [61%, 77%], which contains 75%, or three-quarters.

8. Interpret the confidence interval and the confidence level.

   (a) A study suggests that many adolescents lie about their age online to access certain websites. In a survey conducted, 83% of the participants answered "Yes" to the question, "Have you ever provided false information about your age to access an online website?" The researchers gave a 95% confidence interval of $[76\%, 90\%]$ for the true proportion of adolescents who lie about their age.

   Solution: The study results are 95% confident that the population proportion of adolescents who have provided false information about their age to access and online website falls between 76% and 90%.

   If this study were to be repeated many, many times, approximately 95% of the confidence intervals constructed through the same process taken in the original study would capture the true parameter value.

   (b) A study on car usage by Canadians suggests that the average driver adds around 20 000 km to their car mileage each year. According to the researchers, the resulting 99% confidence interval from the study is $20\,000 \pm 8200$km.

   Solution: The study results are 99% confident that the population mean of number of kilometres driven each year in Canadian cars falls between 11 800–28 200km.

If this study were to be repeated many, many times, approximately 99% of the confidence intervals constructed through the same process taken in the original study would capture the true parameter value.

(c) Octagon News reports that up to 7 out of every 8 people don't wash their hands after using the washroom. A 90% confidence interval of 80%±7.5 is cited in an annotation for the article.

Solution: The study results are 90% confident that the population proportion of people who don't wash their hands after using the washroom falls between 72.5–87.5%. This is where the 7 out of 8 comes from; $\frac{7}{8} = 0.875 = 87.5\%$, the upper boundary of the confidence interval.

If this study were to be repeated many, many times, approximately 90% of the confidence intervals constructed through the same process taken in the original study would capture the true parameter value.

9. True or False? Why?

(a) If you take samples from the same population over and over again and construct C% confidence intervals from each sample for the unknown population parameter, about C% of those intervals will contain the sample statistic.

Solution: **False.**

In our usage of the sample statisic as the point estimate, 100% of confidence intervals constructed will contain the sample statistic. This is also true for other construction methods, though the form of the confidence interval itself may differ.

(b) If you take samples from the same population over and over again and construct C% confidence intervals from each sample for the unknown population parameter, about C% of those intervals will contain the parameter value.

Solution: **True.**

This is the definition of what a C% confidence interval is!

(c) Decreasing the margin of error also makes the confidence interval more precise.

Solution: **True.**

Decreasing the margin of error makes the confidence interval smaller, decreasing the distance between the highest and lowest guesses, which makes it more precise. This is like shrinking the size of the circle you draw around your dart in the target analogy!

(d) The more precise an interval is, the more accurate it is.

Solution: **False.**

Intervals become more precise by becoming smaller. As we noticed when considering Question 3, decreasing the size of a confidence interval also decreases the confidence level. A lower confidence level means that the guess is less accurate.

(e) Since samples are all taken from the same population, any random sample you take will have the same statistic values.

Solution: **False.**

Samples comprise of only a portion of the population as a whole. This means that different individual units will be included in each possible sample, generally leading to different values contributing to the sample statistic value.

(f) Having a 95% confidence interval of 100 ± 30 for the population mean means that there is a 95% chance that the population mean is between 70 and 130.

Solution: **False.**

Before selecting a sample, we would've had a 95% chance of selecting a sample of our chosen size that would lead to the construction of a confidence interval that captures the true parameter value. However, once we have constructed a confidence interval, that confidence interval either does or doesn't contain the unknown parameter value. Thus, the probability that a 95% confidence interval captures the parameter is *not* 95%—the confidence level refers to the success rate over all possible samples that could be drawn.

(g) Plausible values are all possible values that the unknown parameter could be.

Solution: **False.**

Plausible values are just the values that we believe it is likely for the unknown parameter to be, based off of the data we have. If we selected a different sample, our range of plausible values could change! The set of all possible values would be derived from the context of the data, and would quite literally be any possible value the parameter could take be.

(h) Usually, if a value is inside of the confidence interval, then it is plausible.

Solution: **True.**

This is the basis off of which confidence intervals are interpreted!

(i) A wider 95% confidence interval has a better chance of capturing the parameter value than a narrower 95% confidence interval.

Solution: **False.**

All 95% confidence intervals have the same success rate, regardless of how large the interval is!

(j) For any given sample, if a confidence interval is made wider, then C increases.

Solution: **True.**

You can think back to Question 3 again! Increasing the margin of error for a given confidence interval decreases precision but raises accuracy, increasing the confidence level.

10. How big would a 100% confidence interval have to be to? Why?

    Solution: A 100% confidence interval would have to contain every possible value for the unknown parameter. That's the only way to be 100% certain that the true parameter value is captured inside of the interval! It's worth noting that we already knew that the parameter value had to be possible, so a 100% confidence interval isn't actually helpful when trying to draw conclusions from data—that's why we exchange accuracy for better precision to narrow down the possibilities!

11. Try to find a real-life example of a confidence interval! It might be helpful to look in the news.